Final Report

Risk analysis of critical loading and blackouts with cascading events

Prepared by

Ian Dobson
Power Systems Engineering Research Center
University of Wisconsin-Madison
Electrical and Computer Engineering Department
1415 Engineering Drive
Madison WI 53706 USA
dobson@engr.wisc.edu

Benjamin A. Carreras
Oak Ridge National Laboratory
Oak Ridge, TN USA
carreras@fed.ornl.gov

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CONTENTS

1 INTRODUCTION

This document is the final report of a two-year CERTS (Consortium for Electric Reliability Technology Solutions) project studying large-scale blackouts and cascading failures of electric power transmission systems. The project is devising new methods, models and analysis tools from complex systems, criticality, probability, and power systems engineering so that the risks of large blackouts and cascading failures can be understood and mitigated from global and top-down perspectives. The work was performed by close collaboration between Oak Ridge National Laboratory and the Power Systems Engineering Research Center at the University of Wisconsin-Madison.

Section 2 explains topics providing background to the project and sections 3 and 4 summarize the project achievements, deliverables and budget. The details of the technical achievements of the project are documented in Section 6 and in preprints available on the web at http://eceserv0.ece.wisc.edu/~dobson/home.html. A comprehensive review of much of the project work is documented in section 6.4.

2 PROJECT BACKGROUND

2.1 GENERAL BACKGROUND

The United States electrical energy supply infrastructure is experiencing rapid changes and will continue to be operated close to a stressed condition in which there is substantial risk of cascading outages and blackouts. The rapid changes in this highly complex system present significant challenges for maintaining its operational stability and reliability.

Avoiding large blackouts and especially those involving most or all of an entire interconnected power transmission system is vital to the United States. Large blackouts typically involve complicated series of cascading rare events that are hard to anticipate. The enormous number and rarity of possible events, interactions and dependencies has previously made the analysis of large blackouts intractable, except by an intricate case-by-case, postmortem analysis. However, we can now exploit the new models and ideas we have previously developed by CERTS to address the risk of large blackouts caused by cascading failures.

In the past we the focus of our work has been the development of models to study blackout dynamics in the power transmission grid. We have developed the OPA model that incorporates self-organization processes based on the engineering response to blackouts and the long-term economic response to customer load demand. It also incorporates the apparent critical nature of the transmission system. The combination of these mechanisms leads to blackouts that range in size from single load shedding to the blackout of the entire system. This model shows a probability distribution of blackout sizes with power tails similar to that observed in NERC blackout data from North America.

We have developed a probabilistic model of cascading failure called CASCADE. CASCADE shows a critical threshold in the overall system loading that leads to large cascading failures. The corresponding threshold in the power system is a threshold in overall system loading or stress that gives a sharply increased risk of large blackouts. This type of threshold has been observed in the OPA power system models and operation near this threshold is consistent with the NERC data. However, this threshold is not well understood in OPA or in real systems, and the parameters controlling it are not easy to identify. Moreover, practical methods to monitor the proximity of the power system to this threshold to assess the risks of large blackouts have not been developed.

We have also developed an approximation to the CASCADE model using the theory of branching processes that yields further insights into cascading failure. The branching process model opens the door to measuring the system overall stress with respect to the extent to which failures propagate after they are started.

One perspective is that in the past, the n-1 criterion and generous operating margins were used to provide some protection against cascading failure and large blackouts. Economic and competitive pressures are now inexorably causing changes in these practices and we seek to assess the risks of these evolving practices with respect to cascading failure. Assessing and mitigating the risk of large blackouts from a global, complex systems perspective is preferable to the direct experimental approach of waiting for large blackouts to occur and then reacting exclusively on a case-by-case basis.

2.2 BLACKOUT RISK ANALYSIS AND POWER TAILS

Figure 1 shows power tails in NERC blackout data. Note that a straight line on a log-log plot such as Figure 1 yields a power law relation between the variables with the exponent given by the line slope. This section, which is based on [Carreras03], reviews some of the consequences of this for blackout risk analysis, because this underpins much of the project work.

To evaluate the risk of a blackout, we need to know both the frequency of the blackout and its costs. It is difficult to determine blackout costs, and there are several approaches to estimate them, including customer surveys, indirect analytic methods, and estimates for particular blackouts [Billington96]. The estimated direct costs to electricity consumers vary by sector and increase with both the amount of interrupted power and the duration of the blackout. [Billington87] defines an interrupted energy assessment rate IEAR in dollars per kilowatt-hour that is used as a factor multiplying the unserved energy to estimate the blackout cost. That is, for a blackout with size measured by unserved energy S,

$$\text{direct costs} = (\text{IEAR})\, S \quad \$ \tag{1}$$

There are substantial nonlinearities and dependencies not accounted for in Eq. (1), but expressing the direct costs as a multiple of unserved energy is a commonly used crude approximation. However, studies of individual large blackouts suggest that the indirect costs of large blackouts, such as those resulting from social disorder, are much higher than the direct costs. Also, the increasing and complicated dependencies of other infrastructures mentioned earlier on electrical energy tend to increase the costs of all blackouts [Rinaldi01], [NERC01].

For our purposes, let the frequency of a blackout with unserved energy S be $F(S)$ and the cost of the blackout be $C(S)$. The risk of a blackout is then the product of blackout frequency and cost:

$$\text{risk} = F(S)\, C(S)$$

The NERC data indicate a power law scaling of blackout frequency with blackout unserved energy as

$$F(S) \sim S^{a}$$

where a ranges from -0.6 to -1.9. If we take a = -1.2, and only account for the direct costs in $C(S)$ according to (1), then

$$\text{risk} \sim S^{-0.2}$$

This gives a weak decrease in risk as blackout size increases, which means that the total cost of blackouts is very heavily dominated by the largest sizes. If we also account for the indirect costs of large blackouts, we expect an even stronger weighting of the cost for larger blackouts relative to smaller blackouts. From this one can clearly see that, although large blackouts are much rarer than small blackouts, the **total** risk associated with the large blackouts is much great than the risk of small blackouts.

In contrast, consider the same risk calculation if the blackout frequency decreases exponentially with size so that

$$F(S) = A^{-S}$$

With the simple accounting for direct costs only, we get

$$\text{risk} \sim S\, A^{-S}$$

for which the risk peaks for blackouts of some intermediate size and decreases exponentially for larger blackouts. Then, unless one deals with an unusual case in which the peak risk occurs for blackouts comparable to the network size, we expect the risk of larger blackouts to be much smaller than the peak risk. This is likely to remain true even if the indirect blackout costs are accounted for unless they are very strongly weighted (exponentially, for example) toward the large sizes.

While there is some uncertainty in assessing blackout costs, and especially the costs of large blackouts, the analysis above suggests that, when all the costs are considered, power tails in the blackout size frequency distribution will cause the risk of large blackouts to exceed the risk of the more frequent small blackouts. This is strong motivation for investigating the causes of power tails.

We now put the issue of power tails in context by discussing other aspects of blackout frequency that impact risk. The power tails are of course limited in extent in a practical power system by a finite cutoff near system size corresponding to the largest possible blackout. More importantly, the frequency of smaller blackouts and hence the shape of the frequency distribution away from the tail impacts the risk. Also significant is the absolute frequency of blackouts. When we consider the effect of mitigation on blackout risk, we need to consider changes in both the absolute frequency and the shape of the blackout frequency distribution. That is, rather than seeking to deterministically avoid all blackouts (which may be unachievable and is certainly too costly), a better question is: How do we assess and manage the risk of all sizes of blackouts?

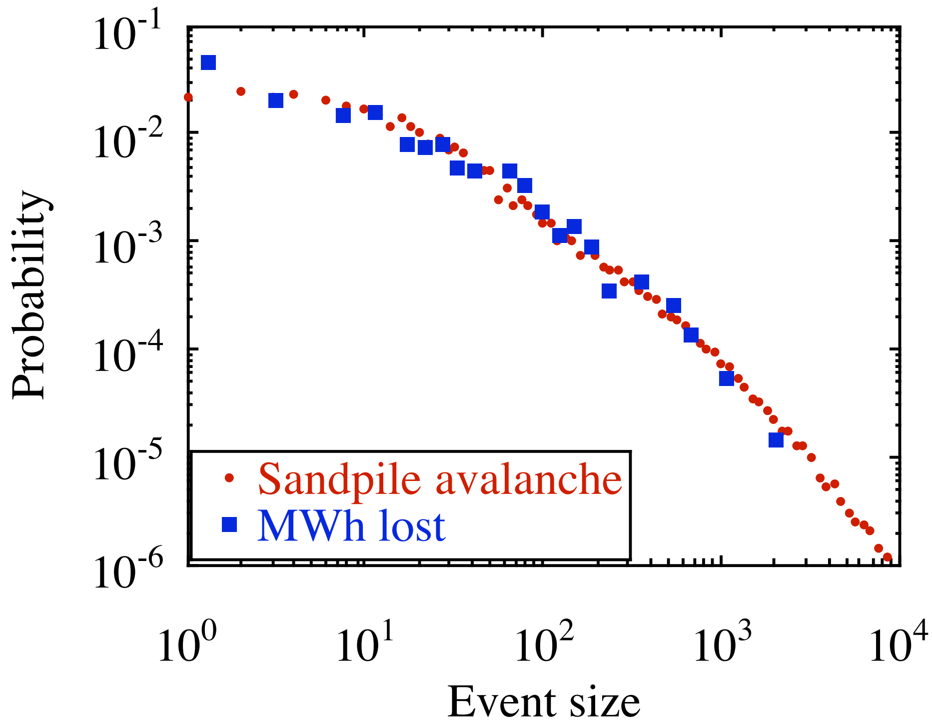

Figure 1. PDF of blackout sizes (MWh lost) compared to PDF of avalanche sizes from an SOC sand pile system.

2.3 SUMMARY OF MAIN IDEAS

This section summarizes some of the main ideas of the project that are foundations of the current work.

(1) Instead of looking at the details of particular blackouts, study the statistics, dynamics and risk of series of blackouts with approximate global models.

(2) 15 years of NERC blackout data yields a probability distribution of blackout sizes with a power tail. Thus large blackouts are much more likely than expected and, when costs are considered, their risk is comparable to the risk of small blackouts. The data also suggests North American grid operation near a critical point.

(3) Imagine increasing power system load from zero (independent failures and negligible chance of large blackout) to emergency loading of all components (certain cascading failure). We think there is a critical loading (phase transition) in between these extremes at which there is a sharply increased chance of cascading failure. Our models show power tails at this critical point.

(4) The practical implications of the critical loading are that we need ways to estimate the closeness to this critical loading in order to manage the risk of large blackouts by operating the power system with a suitably low risk of cascading failure. Therefore the current thrust of the project is to devise practical methods of monitoring or assessing criticality of the power system.

(5) There are a huge number of possible combinations of foreseeable and unforeseeable multiple contingencies that can lead to cascading failure. While it is definitely good practice to mitigate the most likely of the foreseeable contingencies, in this project we focus on the complementary problem of assessing the overall system stress that can cause failures to propagate after they are started.

(6) Load growth at 2% per year reduces power system margins of operation whereas the engineering responses to blackouts (caused by small margins) increase margins. These opposing forces could dynamically self-organize the system to the critical point. Mitigation of blackout risk should take care to account for counter-intuitive effects in complex self-organized critical systems. For example, suppressing small blackouts could lead the system to be operated closer to the edge and ultimately increase the risk of large blackouts.

3 PROJECT ACHIEVEMENTS

3.1 MAIN ACCOMPLISHMENTS

This section summarizes the main accomplishments of the project. A detailed technical account of these accomplishments can be found in the papers reprinted in section 6 and in the preprints that are available at http://eceserv0.ece.wisc.edu/~dobson/home.html. A summary of these accomplishments organized by project task can be found in section 3.2.

It is convenient to first list the three main models developed and used in the project so that they can be identified briefly in the sequel:

- **OPA model**. OPA is a software code to study the dynamics of power system blackouts. OPA models the cascading failures of the power system using DC load flow and LP dispatch and includes long term dynamics of load growth and power system improvement in response to blackouts. OPA was developed by ORNL, PSerc at Wisconsin and University of Alaska and was extensively developed in the previous CERTS project.

- **CASCADE model**. CASCADE is a simple analytically solvable model to study basic features of probabilistic cascading failure. CASCADE was developed from scratch by the previous CERTS project.

- **branching process model**. The branching process model is a simple analytically solvable model that approximates CASCADE.

The main accomplishments are:

- We analyzed the criticality condition yielding power tails in the distribution of the number of failures in the CASCADE model. This was done by approximating the CASCADE model as a branching process [see section 6.1]. The criticality parameter measures the propagation of failures during the cascade and the proximity of the system to a high risk of cascading failure. The approximation was generalized to the more realistic case of limited component interactions [see section 6.3]. The branching process approximation opens up possibilities for analyzing, quantifying and monitoring the risk of large cascading failures. In particular, the value of failure propagation λ can be linked to risk of blackouts of all sizes [see section 6.6].

- Progress was made in identifying and obtaining the criticality parameter using data from the OPA blackout simulation [see sections 6.2 and 6.5]. This allows the comparison of the OPA and CASCADE model and gives insights into both models, particularly the limitations in the CASCADE model that have to be addressed when applying a branching process perspective.

- Branching process models were proposed for the exponentially increasing portions of real blackouts and some initial methods of fitting the models to real blackout data were proposed and illustrated using data from WSCC blackouts [see section 6.10]. Further work will require access to summary data from the August 2003 blackout and this data has been requested from DOE. A start has been made on proposing and assessing the feasibility of real-time monitoring methods [see section 6.10], but much more exploration is needed to assess initial feasibility.

- Progress was also made in proposing ways of statistically estimating propagation of failures λ from general data. As well as the work described above [see sections 6.2, 6.3, 6.5] another possible statistic for λ was proposed and a method to find the criticality point by Monte Carlo simulation were outlined [see section 6.6]. These are all steps towards understanding criticality and developing methods to measure the criticality parameter from simulations and real data.

- The mathematical foundations of the CASCADE model and connections to models of branching processes, queues, random graphs, stochastic process fluctuations and epidemics were established and documented [DobsonPEIS05]. The generalized multinomial joint distribution of the number of failures in each stage was derived. The description and derivations of the CASCADE model were simplified and improved.

- The effect of grid upgrade strategies such as increasing component reliability and redundancy on the complex system dynamics of the transmission grid were studied [see section 6.7]. Some of the long-term effects on blackout risk were counter-intuitive, suggesting that care should be taken in planning upgrades in the light of complex system dynamics.

- Work on detecting criticality in a blackout simulation model that represents hidden failures in the protection system and exploring mitigation methods to shape the probability distribution of blackout sizes was completed and a journal paper is being published [Chen05]. This completes joint work with PSerc at Cornell University that was recently funded under CERTS.

- Joint work with University of Alaska was done on modeling cascading failure in interdependent infrastructures and on human factors in risk [see sections 6.8 and 6.9]. The models developed for the interdependent infrastructure generalize branching and complex systems models in the project in ways that are expected to be useful for blackout modeling. Moreover, a start on the human factors in risk is needed as an important but poorly modeled key factor in blackout risk and perception of blackout risk.

- Media interest in cascading failure blackouts and complex systems aspects after the August blackout led to quotes and background provided to over a dozen newspaper articles and appearances on NPR radio and ABC Nightline. Project research results were featured in Nature, National Post, Energia, and in lead articles in SIAM News and IEEE Spectrum. These articles may be accessed at the website http://eceserv0.ece.wisc.edu/~dobson/complexsystemsresearch.html

- The project work on electrical blackouts was recognized as one of DOE Office of Science Programs' Top Achievements in 2003.

- The project work on the distribution of blackout size as a result of complex systems effects has been identified as significant in assessing the risk of loss of offsite power for nuclear power plants [Raughley04]. Ben Carreras has been advising the U.S. Nuclear Regulatory Commission to assist this analysis.

- Substantial progress in establishing methods of cascading failure analysis and complex systems analysis were made. Four journal papers in electrical and systems engineering, physics, and probability journals were produced [CarrerasCAS04, CarrerasCHAOS04, DobsonPEIS05, Chen05] and many presentations were given at conferences and to industry. Collaboration with a consulting company was established and pursued and several proposals were made to industry and an ISO jointly with the consulting company. A session on cascading failure blackouts was organized at the PMAPS conference that brought together for the first time international researchers working on this topic. Lectures on the project material were presented to industry at the EEI Market Design & Transmission Pricing School, the Institute for Asset Management in Britain, and at a PSerc meeting. These activities are all intended to multiply the effectiveness, leverage, and impact of the project in a variety of industrial, academic, national and international contexts.

- Since the OPA model does not currently represent some of the factors that may be significant in cascading failure interactions, we established a collaboration with the University of Manchester to test their cascading failure model [Kirschen04, Rios02] for criticality. This collaboration has a paper in progress to be submitted to the 2005 PSCC conference. Researchers at Carnegie-Mellon also reported criticality phenomenon in their cascading failure model [LiaoCMU04] and we are also starting to collaborate with them and Iowa State under PSerc to investigate this. Strong interest has been expressed by PSerc industry members. Upgrade of OPA is planned for next year as outlined in section 3.6.3

- A web page to briefly explain the project results and give access to a selection of papers was set up at http://eceserv0.ece.wisc.edu/~dobson/complexsystemsresearch.html

3.2 ACCOMPLISHMENTS BY TASKS

This section summarizes the project accomplishments for the two years according to the planned tasks.

Task 1: Document explorations of blackout risk analysis and mitigation in complex system simulations

(a) Progress was made in identifying and obtaining the criticality parameter using data from the OPA blackout simulation. The criticality parameter determines how close the power system is to a significant risk of cascading failure and its determination from data could be used to monitor the risk of cascading failure. This work was documented in an initial conference paper [see section 6.2].

(b) Work on detecting criticality in a blackout simulation model that represents hidden failures in the protection system and exploring mitigation methods to shape the probability distribution of blackout sizes was completed and documented in the journal paper [Chen05].

(c) The media showed great interest in complex systems approach to blackout risk and mitigation. The CERTS team provided information about this research topic to reporters so that it could get public exposure and to contribute to the public information relevant to the August 2003 blackout. The December 2003 issue of SIAM news headlined an article on complex systems applied to blackouts that extensively described the project work in blackout risk analysis and mitigation [Robinson03]. (SIAM is the Society for Industrial and Applied Mathematics). The August 2004 issue of IEEE Spectrum lead article discussed the complex systems work of the project in some detail and contrasted the project work with other approaches [Fairley04].

(d) Extending and documenting the work on blackout risk mitigation using OPA is Task 4.

Task 2: Document the properties of a general cascading failure model

(a) The general cascading failure model CASCADE has been carefully stated and formulas for the probability distribution of the number of failures has been rigorously derived by two methods. The connections to known mathematics have been elucidated; it turns out that the cascading failure model is a new application and generalization of a quasibinomial distribution. The quasibinomial distribution has appeared in problems involving queues, epidemics and random mappings. Establishing the analysis and related applications of the cascading failure model is foundational for understanding the model and for effective further exploitation of the model. This work is documented in the journal paper [DobsonPEIS05].

(b) We approximated the CASCADE model as a branching process to give insight into the propagation of failures. The approximation and the implications for risk analysis of cascading failure were documented in an initial conference paper [see section 6.1]. The approximation was generalized to the more realistic case of limited component interactions and this was documented in another conference paper [see section 6.3].

(c) Analysis of the criticality conditions in the CASCADE model is task 5.

Task 3: Project first year report
The first year report was produced and is available in pdf format on the CERTS website.

Task 4: Document blackout risk mitigation using OPA
The effect of grid upgrade strategies such as increasing component reliability and redundancy on the complex system dynamics of the transmission grid were studied [see section 6.7]. Some of the long-term effects on blackout risk were counter-intuitive, suggesting that care should be taken in planning upgrades in the light of complex system dynamics. A journal paper submission on task 4 is planned but not yet completed. Work on blackout risk mitigation in another blackout simulation model that represents hidden failures in the protection system and exploring mitigation methods to shape the probability distribution of blackout sizes was completed and a journal paper is being published [Chen05]. This completes joint work with PSerc at Cornell University that was recently funded under CERTS.

Task 5: Analyze criticality conditions in CASCADE model
Much of the work on this task was directed towards approximating the CASCADE model with a branching process and analyzing the branching process. One of the criticality conditions for the CASCADE model shows up in the branching process approximation as the failure propagation parameter λ and several papers have explored ways to compute λ from CASCADE, OPA and real blackout data [see sections 6.2, 6.5, and 6.6]. Some work on an interpretation of the structure of criticality in CASCADE from the point of view of thermodynamics has been done.

Task 6: Understand criticality conditions in OPA model
Work further to that in Task 2(b) was done in relating the OPA criticality to CASCADE criticality [see section 6.5]. Also a straightforward method to find the criticality point by Monte Carlo simulation was outlined [see section 6.6]. Since the OPA model does not currently represent some of the factors that may be significant in cascading failure interactions, we established a collaboration with the University of Manchester to test their cascading failure model [Kirschen04, Rios02] for criticality. This collaboration has a paper in progress to be submitted to the 2005 PSCC conference. Researchers at Carnegie-Mellon also reported criticality phenomenon in their cascading failure model [LiaoCMU04] and we are also starting to collaborate with them and Iowa State under PSerc to investigate this. Strong interest has been expressed by PSerc industry members. Upgrade of OPA is planned for next year as outlined in section 3.6.3.

Task 7: Final report
This report is the final report.

3.3 PROJECT COORDINATION

The project is led by Ian Dobson and involves a team of researchers at PSerc at Wisconsin and ORNL. Close collaboration with Dr. David Newman at the Physics department in the University of Alaska-Fairbanks is ongoing. The project team has a substantial history of productive collaboration and is producing results in close collaboration and papers with joint authorship [BhattHICSS05, Carreras00, Carreras01a, Carreras01b, Carreras02, CarrerasCHAOS02, Carreras03, CarrerasCHAOS04, CarrerasCAS04, Carreras04, Dobson01, Dobson02, DobsonCHINA02, Dobson03, Dobson04, DobsonISCAS04, DobsonPEIS05, DobsonCMU04, DobsonHICSS05, NewmanCMU04, NewmanHICSS05].

Team communication is a judicious combination of email, phone, and face-to-face meetings. The team members meet for about three days about every four months.

3.4 PAPERS AND PRESENTATIONS

The following papers document in detail much of the technical progress on the project. The 2004 and January 2005 conference papers and 2004 journal papers are reprinted in section 6. The 2005 journal papers are not reprinted in this report. However, preprints of the 2005 journal papers are posted on the website at http://eceserv0.ece.wisc.edu/~dobson/home.html.

The following journal papers were produced:

Evidence for self-organized criticality in a time series of electric power system blackouts
B.A. Carreras, D.E. Newman, I. Dobson, A.B. Poole
IEEE Transactions on Circuits and Systems Part I
volume 51, no 9, September 2004, pp 1733-1740
(reprinted in section 6.11)

Abstract: We analyze a 15-year time series of North American electric power transmission system blackouts for evidence of self-organized criticality. The probability distribution functions of various measures of blackout size have a power tail and R/S analysis of the time series shows moderate long time correlations. Moreover, the same analysis applied to a time series from a sandpile model known to be self-organized critical gives results of the same form. Thus the blackout data seem consistent with self-organized criticality. A qualitative explanation of the complex dynamics observed in electric power system blackouts is suggested.

Complex dynamics of blackouts in power transmission systems
B.A. Carreras, V.E. Lynch, I. Dobson, D.E. Newman
Chaos: An Interdisciplinary Journal of Nonlinear Science
volume 14, no 3, September 2004, pp 643-652
(reprinted in section 6.12)

Abstract: A model has been developed to study the global complex dynamics of a series of blackouts in power transmission systems. This model includes a simple level of self-organization by incorporating the growth of power demand, the engineering response to system failures, and the upgrade of the generator capacity. Two types of blackouts have been identified with different dynamical properties. One type of blackout involves loss of load due to transmission lines reaching their load limits but no line outages. The second type of blackout is associated with multiple line outages. The dominance of one type of blackouts versus the other depends on operational conditions and the proximity of the system to one of its two critical points. The model shows a probability distribution of blackout sizes with power tails similar to that observed in real blackout data from North America.

Cascading dynamics and mitigation assessment in power system disturbances via a hidden failure model
J. Chen, J.S. Thorp, I. Dobson
to appear in
International Journal of Electrical Power and Energy Systems in 2005.
preprint available at http://eceserv0.ece.wisc.edu/~dobson/home.html

Abstract: A hidden failure embedded DC model of power transmission systems has been developed to study the observed power tails of North American blackout data. We investigate the impacts of several model parameters on the global dynamics and evaluate possible mitigation measures. The main parameters include system loading level, hidden failure probability, spinning reserve capacity and control strategy. The sensitivity of power-law behavior with respect to each of these parameters and the possible blackout mitigation are discussed and illustrated using simulation results from the WSCC 179-bus equivalent system and IEEE 118-bus test system. It is our intention that the study can provide guidance on when and how the suggested mitigation methods might be effective.

A loading-dependent model of probabilistic cascading failure
I. Dobson, B.A. Carreras, D.E. Newman
to appear in
Probability in the Engineering and Informational Sciences
vol. 19, no. 1, Jan 2005, pp. 15-32
preprint available at http://eceserv0.ece.wisc.edu/~dobson/home.html

Abstract: We propose an analytically tractable model of loading-dependent cascading failure that captures some of the salient features of large blackouts of electric power transmission systems. This leads to a new application and derivation of the quasibinomial distribution and its generalization to a saturating form with an extended parameter range. The saturating quasibinomial distribution of the number of failed components has a power law region at a critical loading and a significant probability of total failure at higher loadings.

The following conference papers were produced.

A branching process approximation to cascading load-dependent system failure
I. Dobson, B.A. Carreras, D.E. Newman
Thirty-seventh Hawaii International Conference on System Sciences, Hawaii, January 2004
(reprinted in section 6.1)

Abstract: Networked infrastructures operated under highly loaded conditions are vulnerable to catastrophic cascading failures. For example, electric power transmission systems must be designed and operated to reduce the risk of widespread blackouts caused by cascading failure. There is a need for analytically tractable models to understand and quantify the risks of cascading

failure. We study a probabilistic model of loading dependent cascading failure by approximating the propagation of failures as a Poisson branching process. This leads to a criticality condition for the failure propagation. At criticality there are power tails in the probability distribution of cascade sizes and consequently considerable risks of widespread catastrophic failure. Avoiding criticality or supercriticality is a key approach to reduce this risk. This approach of minimizing the propagation of failure after the cascade has started is complementary to the usual approach of minimizing the risk of the first few cascading failures. The analysis introduces a saturating form of the generalized Poisson distribution so that supercritical systems with a high probability of total failure can be considered.

Dynamical and probabilistic approaches to the study of blackout vulnerability of the power transmission grid
B.A. Carreras, V.E. Lynch, D.E. Newman, I. Dobson
Thirty-seventh Hawaii International Conference on System Sciences, Hawaii, January 2004
(reprinted in section 6.2)

Abstract: The CASCADE probabilistic model for cascading failures gives a simple characterization of the transition from an isolated failure to a system-wide collapse as system loading increases. Using the basic ideas of this model, the parameters that lead to a similar characterization for power transmission system blackouts are identified in the OPA dynamical model of series of blackouts. The comparison between the CASCADE and OPA models yields parameters that can be computed from the OPA model that indicate a threshold for cascading failure blackouts. This is a first step towards computing similar parameters for real power transmission systems.

Probabilistic load-dependent cascading failure with limited component interactions
I. Dobson, B.A. Carreras, D.E. Newman,
IEEE International Conference on Circuits & Systems, Vancouver, Canada, May 2004 (reprinted in section 6.3)

Abstract: We generalize an analytically solvable probabilistic model of cascading failure in which failing components interact with other components by increasing their load and hence their chance of failure. In the generalized model, instead of a failing component increasing the load of all components, it increases the load of a random sample of the components. The size of the sample describes the extent of component interactions within the system. The generalized model is approximated by a saturating branching process, and this leads to a criticality condition for cascading failure propagation that depends on the size of the sample. The criticality condition shows how the extent of component interactions controls the proximity to catastrophic cascading failure. Implications for the complexity of power transmission system design to avoid cascading blackouts are briefly discussed.

Complex systems analysis of series of blackouts: cascading failure, criticality, and self-organization

I. Dobson, B.A. Carreras, V.E. Lynch, D.E. Newman

IREP conference: Bulk Power System Dynamics and Control - VI, Cortina d'Ampezzo, Italy, August 2004

(reprinted in section 6.4)

Abstract: We give a comprehensive account of a complex systems approach to large blackouts caused by cascading failure. Instead of looking at the details of particular blackouts, we study the statistics, dynamics and risk of series of blackouts with approximate global models. North American blackout data suggests that the frequency of large blackouts is governed by a power law. This result is consistent with the power system being a complex system designed and operated near criticality. The power law makes the risk of large blackouts consequential and implies the need for nonstandard risk analysis.

Power system overall load relative to operating limits is a key factor affecting the risk of cascading failure. Blackout models and an abstract model of cascading failure show that there are critical transitions as load is increased. Power law behavior can be observed at these transitions.

The critical loads at which blackout risk sharply increase are identifiable thresholds for cascading failure and we discuss approaches to computing the proximity to cascading failure using these thresholds. Approximating cascading failure as a branching process suggests ways to compute and monitor criticality by quantifying how much failures propagate.

Inspired by concepts from self-organized criticality, we suggest that power system operating margins evolve slowly to near criticality and confirm this idea using a blackout model. Mitigation of blackout risk should take care to account for counter-intuitive effects in complex self-organized critical systems. For example, suppressing small blackouts could lead the system to be operated closer to the edge and ultimately increase the risk of large blackouts.

Estimating failure propagation in models of cascading blackouts

I. Dobson, B.A. Carreras, V.E. Lynch, B. Nkei, D.E. Newman

Eighth International Conference on Probability Methods Applied to Power Systems, Ames Iowa, September 2004

(reprinted in section 6.5)

Abstract: We compare and test statistical estimates of failure propagation in data from versions of a probabilistic model of loading-dependent cascading failure and a power systems blackout model of cascading transmission line overloads. The comparisons suggest mechanisms affecting failure propagation and are an initial step towards monitoring failure propagation in practical system data. Approximations to the probabilistic model describe the forms of probability distributions of cascade sizes.

A criticality approach to monitoring cascading failure risk and failure propagation in transmission systems

I. Dobson, B. A. Carreras, D. E. Newman

Electricity Transmission in Deregulated Markets, conference at Carnegie Mellon University, Pittsburgh PA USA, December 2004
(reprinted in section 6.6)

Abstract: We consider the risk of cascading failure of electric power transmission systems as overall loading is increased. There is evidence from both abstract and power systems models of cascading failure that there is a critical loading at which the risk of cascading failure sharply increases. Moreover, as expected in a phase transition, at the critical loading there is a power tail in the probability distribution of blackout size. (This power tail is consistent with the empirical distribution of North American blackout sizes.) The importance of the critical loading is that it gives a reference point for determining the risk of cascading failure. Indeed the risk of cascading failure can be quantified and monitored by finding the closeness to the critical loading. This paper suggests and outlines ways of detecting the closeness to criticality from data produced from a generic blackout model. The increasing expected blackout size at criticality can be detected by computing expected blackout size at various loadings. Another approach uses branching process models of cascading failure to interpret the closeness to the critical loading in terms of a failure propagation parameter λ. We suggest a statistic for λ that could be applied before saturation occurs. The paper concludes with suggestions for a wider research agenda for measuring the closeness to criticality of a fixed power transmission network and for studying the complex dynamics governing the slow evolution of a transmission network.

The Impact of Various Upgrade Strategies on the Long-Term Dynamics and Robustness of the Transmission Grid
D. E. Newman, B. A. Carreras, V. E. Lynch, I. Dobson
Electricity Transmission in Deregulated Markets, conference at Carnegie Mellon University, Pittsburgh PA USA, December 2004
(reprinted in section 6.7)

Abstract: We use the OPA global complex systems model of the power transmission system to investigate the effect of a series of different network upgrade scenarios on the long time dynamics and the probability of large cascading failures. The OPA model represents the power grid at the level of DC load flow and LP generation dispatch and represents blackouts caused by randomly triggered cascading line outages and overloads. This model represents the long-term, slow evolution of the transmission grid by incorporating the effects of increasing demand and engineering responses to blackouts such as upgrading transmission lines and generators. We examine the effect of increased component reliability on the long-term risks, the effect of changing operational margins and the effect of redundancy on those same long-term risks. The general result is that while increased reliability of the components decreases the probability of small blackouts, depending on the implementation, it actually can increase the probability of large blackouts. When we instead increase some types of redundancy of the system there is an overall decrease in the large blackouts with a concomitant increase of the smallest blackouts. As some of these results are counter intuitive these studies suggest that care must be taken when making what seem to be logical upgrade decisions.

Risk assessment in complex interacting infrastructure systems
D. E. Newman, B. Nkei, B. A. Carreras, I. Dobson, V. E. Lynch, P. Gradney
Thirty-eighth Hawaii International Conference on System Sciences, Hawaii, January 2005
(reprinted in section 6.8)

Abstract: Critical infrastructures have some of the characteristic properties of complex systems. They exhibit infrequent large failures events. These events, though infrequent, often obey a power law distribution in their probability versus size. This power law behavior suggests that ordinary risk analysis might not apply to these systems. It is thought that some of this behavior comes from different parts of the systems interacting with each other both in space and time. While these complex infrastructure systems can exhibit these characteristics on their own, in reality these individual infrastructure systems interact with each other in even more complex ways. This interaction can lead to increased or decreased risk of failure in the individual systems. To investigate this and to formulate appropriate risk assessment tools for such systems, a set of models are used to study to impact of coupling complex systems. A probabilistic model and a dynamical model that have been used to study blackout dynamics in the power transmission grid are used as paradigms. In this paper, we investigate changes in

the risk models based on the power law event probability distributions, when complex systems are coupled.

Understanding the effect of risk aversion on risk

U. Bhatt, D.E. Newman, B.A. Carreras, I. Dobson

Thirty-eighth Hawaii International Conference on System Sciences, Hawaii, January 2005

(reprinted in section 6.9)

Abstract: As we progress, society must intelligently address the following question: How much risk is acceptable? How we answer this question could have important consequences for the future state of our nation and the dynamics of its social structure. In this work, we will elucidate and demonstrate using a physically based model that the attempt to eliminate all thinkable risks in our society may be setting us up for even larger risks. The simplest example to illustrate this point is something with which we are all familiar and have known from the time we were very young. When children burn their finger on a hot item they learn the consequences of touching fire. This small risk has taught the child to avoid larger risks. In trying to avoid these small risks as well as larger risks, one runs the dual danger of not learning from the small ones and of having difficulty in differentiating between large and small risks. We will illustrate this problem with a series of social dynamics examples from the operation of NASA to network operation and then make an analogy to a complex system model for this type of dynamics. From these results, recommendations will be made for the types of risk responses that improve the situation versus those that worsen the situation. In order to progress, society has to recognize that accidents are unavoidable and therefore an intelligent risk management program must be implemented aimed toward avoiding or reducing major accidents. It is not possible to avoid all risk but it is better to avoid the greater risk situations for society.

Branching process models for the exponentially increasing portions of cascading failure blackouts

I. Dobson, B.A. Carreras, D.E. Newman

Thirty-eighth Hawaii International Conference on System Sciences, Hawaii, January 2005

(reprinted in section 6.10)

Abstract: We introduce branching process models in discrete and continuous time for the exponentially increasing phase of cascading blackouts. Cumulative line trips from real blackout data have portions consistent with these branching process models. Some initial calculations identifying parameters and using a branching process model to estimate blackout probabilities during and after the blackout are illustrated.

In addition to the conference papers listed above, which were all presented, the following presentations were made:

Blackout mitigation assessment in power transmission systems
B.A. Carreras, V.E. Lynch, D.E. Newman, I. Dobson
36th Hawaii International Conference on System Sciences, Hawaii, January 2003.

A probabilistic loading-dependent model of cascading failure and possible implications for blackouts
I. Dobson, B.A. Carreras, D.E. Newman
36th Hawaii International Conference on System Sciences, Hawaii, January 2003.

Cascading failure,
I. Dobson, B.A. Carreras, D.E. Newman
Talk at the University of Liege, Belgium March 2003

Cascading failure,
I. Dobson, B.A. Carreras, D.E. Newman
Talk at Imperial College, London England March 2003

Cascading failure,
I. Dobson
Brief presentation at press conference organized by Wisconsin Public Utility Institute, Madison WI, August 2003

Cascading failure and the risk of large blackouts,
I. Dobson, B.A. Carreras, D.E. Newman
Talk at UMIST, University of Manchester Institute for Science and Technology, Manchester, England, September 2003

Cascading failure and catastrophic risk in complex systems,
I. Dobson, B.A. Carreras, D.E. Newman
Invited talk at Institute for Asset Management Workshop, Birmingham, England, September 2003

Cascading failure and the risk of large blackouts,
I. Dobson, B.A. Carreras, D.E. Newman,
Talk to Wisconsin Public Service Commission, Madison WI, September 2003

Cascading failure and the risk of large blackouts,
I. Dobson, B.A. Carreras, D.E. Newman,
Talk to Graduate student seminar course, Electrical and Computer Engineering Department, University of Wisconsin, Madison WI, October 2003

Cascading failure, the risk of large blackouts, criticality and self-organization
I. Dobson, B.A. Carreras, D.E. Newman,
Talk to Plasma Physics seminar, University of Wisconsin, Madison WI, October 2003

Cascading failure, criticality and the risk of large blackouts,
I. Dobson, B.A. Carreras, D.E. Newman,
Talk to Systems group seminar, Electrical and Computer Engineering Department, University of Wisconsin, Madison WI, October 2003

Cascading failure, the risk of large blackouts, criticality and self-organization
I. Dobson, B.A. Carreras, D.E. Newman,
Talk to Chaos and Complex Systems seminar, University of Wisconsin, Madison WI, October 2003

Criticality and risk of large cascading blackouts
I. Dobson, B.A. Carreras,
Presentation at CERTS review meeting, Washington DC January 2004

Cascading failure analysis
I. Dobson, B.A. Carreras, D.E. Newman,
Presentation to L.R. Christensen Associates, Madison WI April 2004

Cascading failure analysis
I. Dobson, B.A. Carreras, D.E. Newman,
Presentation to Alliant Energy, Madison WI April 2004

Cascading failure analysis and criticality
R. Camfield, I. Dobson
Presentation to a major utility, May 2004.

Cascading failure propagation and branching processes
I. Dobson, B.A. Carreras, D.E. Newman,
Presentation to Silicon Graphics Inc and Hydro-Quebec TransEnergie, Madison WI June 2004

Cascading failure analysis
I. Dobson, B.A. Carreras, D.E. Newman,
Lecture at EEI Market Design & Transmission Pricing School
Madison, Wisconsin, July 2004

A preliminary coupled model of electricity markets and cascading line failures in power transmission systems
D. Berry,
Student Undergraduate Laboratory Internship poster session
Oak Ridge, Tennessee, August 2004

Criticality and risk of large cascading blackouts
I. Dobson, B.A. Carreras, D.E. Newman,
Presentation at PSerc Industry Advisory Board meeting, August 2004

The study of cascading failure in complex systems
B. Nkei, B.A. Carreras, V.E. Lynch,
2004 Virginia Tech Symposium for Undergraduate Research in Engineering
Blacksburg, Virginia, October 2004

Cascading failures in coupled systems
B. Nkei, V. E. Lynch, B. A. Carreras,
71st Annual Meeting of Southeastern Section of the American Physical Society,
Oak Ridge, Tennessee, November 2004

Blackouts
I. Dobson. 8 lectures (last quarter of the course) in Fall 2004 graduate course at University of Wisconsin: ECE 905 Special topics in Electric power system: operation, markets, reliability, and blackouts; applications of optimization, markets, reliability and self-organized criticality to electric power transmission networks. Students from electrical engineering and policy attended.
http://eceserv0.ece.wisc.edu/~dobson/ece905.html

I. Dobson was the organizer and chair of a **Special session on Probabilistic assessment of cascading events and blackouts** at the Eighth International Conference on Probability Methods Applied to Power Systems, Ames Iowa, Sept. 2004. This session brought together most of the international researchers in this emerging area.

3.5 NEWSPAPER AND MEDIA

There was considerable interest from the media in this project immediately following the August blackout. Considerable time was spent talking to the media, providing explanations, background and quotes. While some of the articles reflected general information, other articles (title in **bold** face) cited research results from the project. The articles and radio and TV contacts are listed below; most are available at http://eceserv0.ece.wisc.edu/~dobson/complexsystemsresearch.html.

Why the lights went out
Jonathan Kay, National Post, August 16 2003

"Last December, four U.S. scientists published a paper in the Journal Chaos entitled Critical points and transitions in an electric power transmission model for cascading failure blackouts. "Detailed analysis of large blackouts has shown that they involve cascading events in which a triggering failure produces a sequence of secondary failures that lead to blackout of a large area of the grid," the authors found. They presciently concluded that "large blackouts are much more likely than might be expected from [conventional statistical analysis]" and are "suggestive of a complex system operating close to a critical point."

At 4:10pm on Thursday, Ontario and seven states hit that "critical point." Within seconds, workers in New York City, Toronto and thousands of other communities found themselves staring at blank computer screens. Many were forced to walk home in sticky weather -- generally to dark, uncomfortably hot homes. Some are still without power as of Saturday morning. Their only consolation is that the biggest power outage in North American history evidently had nothing to do with terrorism."
…

How a butterfly's wing can bring down Goliath.
Chaos theories calculate the vulnerability of megasystems
Keay Davidson, San Francisco Chronicle, August 15 2003

This was a first world blackout
Chris Suellentrop, Slate magazine, August 15 2003

Wisconsin company believes blackout originated in Lansing, Mich.
Associated Press, Star Tribune, August 15 2003

David Newman appeared on NPR radio KUAC FM, August 27 2003

Ian Dobson appeared on ABC Nightline, August 18 2003

Energy scientist studies blackout triggers
Pat Daukantas, Government Computer News, August 22 2003

Blackout was no surprise to UAF professor
Ned Rozell, Anchorage Daily News, September 7 2003

The chaos behind the wall socket
Ned Rozell, Fairbanks Daily News-Miner, September 7 2003

Getting a grip on nation's grid grind
R. Cathey Daniels, Oak Ridger, September 16, 2003

Californians work to predict grid-crashing
Ian Hoffman, Oakland Tribune, August 25 2003

Set of rules too complex to be followed properly
James Glanz and Andrew Refkin, New York Times, August 19 2003

Elusive force may lie at root of blackout
Richard Perez-Pena and Eric Lipton, New York Times, September 23 2003

What’s Wrong with the Electric Grid?
Eric Lerner, Industrial Physicist, November 3 2003

Quick response is key in emergencies
Tom McGinty, NewsDay, November 9 2003

L'energia ha un punto critico
Donata Allegri, Ecplanet

The power grid: Fertile ground for math research
Sara Robinson, SIAM News, Volume 36, Number 8, October 2003

Black-out: cause e mezzi per prevenirli
Carlo Alberto Nucci e Alberto Borghetti, Rivista ENERGIA, n. 3, pp. 20-29, 2003

The power grid as complex system,
Sara Robinson, SIAM News, Volume 36, Number 10, December 2003

The unruly power grid,
Peter Fairley, IEEE Spectrum August 2004

Remember last year's big blackout? Get ready for another one
Stephen Strauss, The Globe and Mail, August 14, 2004

3.6 PLAN OF FUTURE WORK

This section presents a longer term plan of work that explains how the project is directed towards monitoring tools to be applied to the real power system.

3.6.1 Project Goal

Contribute to transmission system reliability by understanding large, cascading failure blackouts and providing tools for analyzing and monitoring their risk. In particular, the project will identify the threshold that leads to increased risk of cascading failure, express this threshold in terms of realistic power system parameters and develop monitoring tools and criteria to be applied in real power transmission systems.

3.6.2 Benefits

The main long-term benefit is to monitor and reduce the likelihood of large-scale blackouts in the United States by the use of operational criteria derived from the results of this project.

3.6.3 Technical approach

We will use a hierarchy of models that will include the CASCADE and OPA models and their extensions to be developed as needed. The CASCADE model is probabilistic model for cascading failures that gives a simple characterization of the transition from an isolated failure to a system-wide collapse as system loading increases. At the present funding level, this project will require funding for about three to four years. To reach this goal we need to achieve the following objectives:

1) Using the OPA model we must thoroughly understand the loading threshold that causes system-wide blackouts. We will compare the probabilistic CASCADE model, where this threshold is easy to identify, with the dynamical OPA model. This dynamical model incorporates the structure of a network, and a linear programming (LP) approach is used to find instantaneous solutions to the power demand. In such a model, the threshold to system-wide blackouts is not obvious, and its understanding is the first step in the path toward application to realistic systems. There are several potential ways of characterizing the threshold and we are investigating them. That is, we need to identify the key measurements to be carried out on the power system that will provide information on the closeness to the criticality threshold. In particular we need to test and refine metrics for monitoring closeness to criticality such as the branching process failure propagation parameter λ, average normalized total load transfer for a failing line, and the loading margin to critical loading.
2) Determine the secure operating conditions with respect to cascading failure. We will use both models in this study. We have to determine how close to the threshold it is possible to operate.

3) Studies of the impact of the slow time evolution and the self-organizing forces will be conducted on simpler models. They will provide guidance on the validity of the probabilistic criteria when translated to the self-organized system.
4) Based on the previous results we have to develop criteria and measurements that are applicable to real system.
5) We will explore the development of software tools to monitor and assess the security of the power system with respect to large cascading failures. First we will test these tools in simulated operation to assess their capabilities and limitations
6) Implement the criteria and tools developed so that is possible to monitor power system status and risk trade-offs and to be able to do "what-if" analysis. We will look for collaborations within CERTS in developing practical tools to carry out this task.

3.6.3 Plan for next step in OPA development

There are several power system cascading failure models with varying modeling emphases as summarized in the following table:

	OPA	hidden failure	Manchester	CMU	TRELSS
overloads	X	X	X	X	X
redispatch	X	X	X		X
hidden failure		X	X		
protection group					X
AC			X		X
Gen trip					X
voltage collapse			X		X
transient stability			X		
under freq load shed			X		
islanding	X		X		X
load increase and grid upgrade	X				
approx. max number of buses	400	300	1000	2500	13000
reference	[Carreras CHAOS02]	[Chen 05]	[Kirschen 04]	[LiaoCMU04]	[TRELSS]

Note that OPA is the only code that can currently address the load increase and grid upgrade complex dynamics.

The overall plan for the next step in development of OPA is to add to OPA the most straightforward and significant enhancements first and to seek to collaborate with the groups running the other models to gain quick access to features that would require substantial development resources. Further steps can be evaluated once this first step is undertaken and some sense of the importance of the various enhancements for cascading failure analysis has been gained.

The most promising enhancements to OPA to be first considered are then

- Representation of hidden failures in OPA
- Investigating the modeling of generator trips
- Improving the input to handle a standard format power system file

The AC load flow and the approximation of voltage and transient stability issues can be postponed in OPA in this first step and first pursued in collaboration with existing codes.

Significant progress has been made in pursuing and establishing the collaborations mentioned above. We have already successfully collaborated within CERTS with the hidden failure model developed at Cornell University [Chen05]. The collaboration with the University of Manchester has a paper in progress to be submitted to the 2005 PSCC conference. We are starting to collaborate with Carnegie-Mellon University (CMU) (and Iowa State) under PSerc to investigate their model. Strong interest has been expressed by PSerc industry members. Our collaboration with a consulting company has already approached a major utility to explore possibilities of running TRELSS.

4 DELIVERABLES AND BUDGET

4.1 DELIVERABLES

The deliverables for this project are a one-year report and this final report and the documented information in the conference and journal papers listed in section 3.4.

4.2 BUDGET

Benjamin A. Carreras (ORNL)
$65,000 for the year beginning Jan 1, 2003.
$60,000 for the year beginning Jan 1, 2004.

Ian Dobson (PSERC Wisconsin)
$55,000 for the year beginning Jan 1, 2003.
$60,000 for the year beginning Jan 1, 2004.

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6 TECHNICAL PAPERS

6.1 A branching process approximation to cascading load-dependent system failure

I. Dobson, B.A. Carreras, D.E. Newman

Thirty-seventh Hawaii International Conference on System Sciences
Hawaii, January 2004.

Thirty-seventh Hawaii International Conference on System Sciences, January 2004, Big Island, Hawaii. © 2004 IEEE.

A branching process approximation to cascading load-dependent system failure

Ian Dobson
ECE Department
University of Wisconsin
Madison WI 53706 USA
dobson@engr.wisc.edu

Benjamin A. Carreras
Oak Ridge National Laboratory
Oak Ridge TN 37831 USA
carrerasba@ornl.gov

David E. Newman
Physics Department
University of Alaska
Fairbanks AK 99775 USA
ffden@uaf.edu

Abstract

Networked infrastructures operated under highly loaded conditions are vulnerable to catastrophic cascading failures. For example, electric power transmission systems must be designed and operated to reduce the risk of widespread blackouts caused by cascading failure. There is a need for analytically tractable models to understand and quantify the risks of cascading failure. We study a probabilistic model of loading dependent cascading failure by approximating the propagation of failures as a Poisson branching process. This leads to a criticality condition for the failure propagation. At criticality there are power tails in the probability distribution of cascade sizes and consequently considerable risks of widespread catastrophic failure. Avoiding criticality or supercriticality is a key approach to reduce this risk. This approach of minimizing the propagation of failure after the cascade has started is complementary to the usual approach of minimizing the risk of the first few cascading failures. The analysis introduces a saturating form of the generalized Poisson distribution so that supercritical systems with a high probability of total failure can be considered.

1. Introduction

Networked infrastructures such as electric power transmission systems are vulnerable to widespread cascading failures when the systems are highly loaded. Since modern society depends on large infrastructures, catastrophes in which failures propagate to most or all of the system are of concern. For example, blackouts of substantial portions of the North American power system east or west of the Rocky Mountains have a huge cost to society, as demonstrated in 2003 and 1996 respectively. There is a need for analytically tractable models to understand and quantify the risks of cascading failure so that networked systems can be designed and operated to reduce the risk of catastrophic failure.

Analyses of 15 years of North American blackout data show an empirical probability distribution of blackout size which has heavy tails and evidence of power law dependence in these tails [24, 2, 11, 3, 6]. The exponent of the power tail is roughly estimated to be in the range –2 to –1. These data show that large blackouts are much more likely than might be expected from a distribution of blackout size in which the tails decay exponentially. Simulation models of cascading blackouts show similar power tails and the power tails have been attributed to the nature of the cascading process [19, 9, 7].

Because of protection and appropriate design and operational procedures, it is very rare for power transmission components to fail in the sense of the component breaking. However, it is routine for these components to be temporarily removed from service by protection equipment and the outaged or tripped component is then failed in the sense that it is temporarily not available to transmit power. Moreover there are sometimes misoperations or mistakes in protection, communication and control systems or operational procedures or sometimes the power system is operated under conditions that could not be anticipated in the original design settings or procedures. In the context of power transmission systems, the term “failure” as used in this paper should be understood in this broad and nuanced sense.

Notable general features of power transmission systems are the large number of components, the increased probability of component failure and interaction at high load, and the numerous, varied and widespread interactions between components. Large blackouts typically involve long sequences of component failures. Many of the interactions are rare, unanticipated or unusual, not least because of engineering efforts to design and operate the system so as to avoid the most common failures and interactions. Although we use electric power transmission system blackouts as the motivating example in this paper, these general features appear in other networked infrastructures so that it is likely that the ideas apply more generally.

One natural way to study cascading failure is to consider the failures propagating probabilistically according to a Galton-Watson-Bienaymé branching process [23]. For example, simple assumptions lead to a Poisson branching process that has the total number of components failed distributed according to the generalized Poisson distribution [17, 15].

On the other hand, the CASCADE model of probabilistic cascading failure [20] has the following general features:

1. Multiple identical components, each of which has a random initial load and an initial disturbance.

2. When a component overloads, it fails and transfers some load to the other components.

Property 2 can cause cascading failure: a failure additionally loads other components and some of these other components may also fail, leading to a cascade of failure. The components become progressively more loaded and the system becomes weaker as the cascade proceeds.

Both the Poisson branching process and CASCADE can exhibit criticality and power tails in the probability distribution of the number of failed components.

We begin the paper by reviewing standard results on branching processes and the generalized Poisson distribution and then consider the implications of these results for the risk of load-dependent cascading failure. A saturating form of the generalized Poisson distribution is introduced to allow study of the transition through criticality in a system with a large but finite number of components. We review the CASCADE model of cascading failure and then show how CASCADE can be approximated by the saturating generalized Poisson distribution. Then we discuss the implications of the approximation for analyzing CASCADE and understanding cascading failure in blackouts.

2. Review of branching processes

This section reviews standard material on Galton-Watson-Bienaymé branching processes [23] and generalized Poisson distributions [17, 15] as expressed in terms of cascading failures.

2.1. Generalities

We first consider an infinite number of system components. All components are initially unfailed. Component failures occur in stages with M_i the number of failures in stage i. We first assume an initial disturbance that causes one failure in stage zero so that $M_0 = 1$. This first failure is considered to cause a certain number of failures M_1 in stage 1. M_1 is determined according to a probability distribution with generating function $E[t^{M_1}] = f(t)$ and mean λ. In subsequent stages, each of the M_i failures in stage i independently causes a further number of failures in stage $i+1$ according to the same distribution $f(s)$. That is, the kth failure in stage i causes $M_{i+1}^{(k)}$ failures in stage $i+1$ and

$$M_{i+1} = M_{i+1}^{(1)} + M_{i+1}^{(2)} + \cdots + M_{i+1}^{(M_i)} \tag{1}$$

where $M_{i+1}^{(1)}, M_{i+1}^{(2)}, \cdots, M_{i+1}^{(M_i)}$ are independent. This independence is a plausible approximation in a system with many components and many component interactions so that series of failures propagating in parallel can be assumed not to interact. The generating function of M_k is

$$E[t^{M_k}] = f(f(f(...f(t)...))) = f^{(k)}(t) \tag{2}$$

and the mean $E[M_k] = \lambda^k$. If at any stage k, $M_k = 0$, then zero elements fail for all subsequent stages and the cascading process terminates.

There are three cases, depending on the mean λ of the number of failures caused by each failure in the previous stage. In the subcritical case $\lambda < 1$, a finite number of components will fail. In the supercritical case $\lambda > 1$, either a finite or infinite number of components can fail and the number of failures in each stage tends to zero or infinity respectively. The critical case is $\lambda = 1$.

We are most interested in the distribution of the total number of failures

$$M = \sum_{k=0}^{\infty} M_k \tag{3}$$

The generating function of M is $F(t) = E[t^M]$ and it satisfies the recursion $F(t) = tf(F(t))$.

2.2. Universality of the critical exponent

Under mild conditions on f, for the critical case $\lambda = 1$, $P[M = r] \sim r^{-\frac{3}{2}}$ as $r \to \infty$ [26, 23]. That is, the distribution of the total number of failures of a branching process at criticality has a universal property of a power tail with exponent $-\frac{3}{2}$. The details are in Otter's theorem [26]:

Theorem 1 *Suppose that $P[M_1 = 0] > 0$ and that there is a point a in the interior of the circle of convergence of f for which $f'(a) = f(a)/a$. (This is true, for example, if $1 < \lambda \leq \infty$ or if $f(s)$ is entire or if $f'(\rho) = \infty$, where ρ is the radius of convergence of f. The point $(a, f(a))$ is then the point where the graph of f, for real positive s, is tangent to a line through the origin. Let $\alpha = a/f(a)$ and let d be the largest integer such that $P[M_1 = r] \neq 0$ implies that r is a multiple of d, $r = 1, 2, ...$ If $r-1$ is not divisible by d, then $P[M = r] = 0$, while if $r-1$ is divisible by d, then*

$$P[M = r] = d\left(\frac{a}{2\pi\alpha f''(a)}\right)^{\frac{1}{2}} \alpha^{-r} r^{-\frac{3}{2}} + O\left(\alpha^{-r} r^{-\frac{5}{2}}\right) \qquad r \to \infty \tag{4}$$

Notice that $\alpha \geq 1$, the equality holding if and only if $\lambda = 1$. Also $d = 1$ when $P[M_1 = r] \neq 0$ for $r = 1, 2, ...$

2.3. Branching generated by a Poisson distribution

If, in addition to the independence assumptions above, the failures propagate in a large number of components so that each failure has a small uniform probability of independently causing each failure in a large number of other components, then the distribution of failures caused by each failure in the previous stage can be approximated as a Poisson distribution [17] so that

$$P[M_1 = m] = \frac{\lambda^m}{m!} e^{-\lambda} \quad , m = 0, 1, 2, ... \tag{5}$$

$$f(t) = e^{\lambda(t-1)} \tag{6}$$

The distribution of the total number of failures becomes

$$P[M = r] = (r\lambda)^{r-1} \frac{e^{-r\lambda}}{r!}, \quad 0 \leq \lambda \leq 1 \tag{7}$$

which is known as the Borel distribution.

2.4. A probabilistic initial disturbance and the generalized Poisson distribution

If we neglect the zero stage that has one failure, and consider the failures starting with stage 1, then (5) gives a distribution of initial failures according to a Poisson distribution with mean λ.

However, we distinguish the initial failures that are caused by some initial disturbance from the subsequent propagation of failures internal to the system. We want to represent the initial disturbance by its own probability distribution. This can be done by specifying a probability distribution for M_0, the number of failures in stage zero. If the initial failures are Poisson distributed with mean θ so that

$$P[M_0 = m] = \frac{\theta^m}{m!} e^{-\theta} \quad , m = 0, 1, 2, ... \tag{8}$$

$$f_0(t) = e^{\theta(t-1)} \tag{9}$$

then the generating function of M_k becomes $f_0(f^{(k)}(t)$ and the distribution of the total number of failures becomes

$$P[M = r] = \theta(r\lambda + \theta)^{r-1} \frac{e^{-r\lambda-\theta}}{r!} \quad , \theta \geq 0,\ 0 \leq \lambda \leq 1 \tag{10}$$

which is the generalized (or Lagrangian) Poisson distribution introduced by Consul and Jain [17, 12, 15]. The probability generating function of (10) is

$$E[s^M] = e^{\theta(t-1)} \text{ where } t \text{ is the function of } s \text{ satisfying } t = se^{\lambda(t-1)} \tag{11}$$

The mean of the generalized Poisson distribution (10) is

$$E[M] = \frac{\theta}{1-\lambda} \tag{12}$$

The generalized Poisson distribution is usually restricted to parameters such that $\lambda \leq 1$ to avoid the supercritical case in which there is a finite probability of M infinite.

3. Implications for risk of load-dependent cascading failure

The following sections show how a model of loading dependent cascading failure can be approximated as a branching process. To motivate this topic, this section supposes that cascading failure can be treated as a branching process and discusses some general implications of the branching results in Section 2 for risk analysis and mitigation of cascading failure.

Suppose that the system is at criticality ($\lambda = 1$) so that the probability distribution of the total number of failures M follows a power law with exponent $-\frac{3}{2}$. Since risk R is the product of probability and cost,

$$R(m) = P[M = m]C[m] \sim m^{-\frac{3}{2}} C[m] \tag{13}$$

First assume in (13) that the cost $C(m)$ is proportional to the total number of failures m. (This is a conservative estimate in applications such as blackouts; even if the direct costs are proportional to the blackout size and the total number of failures, the indirect costs can be very high for large blackouts [1].) Then $R(m) \sim m^{-\frac{3}{2}} m = m^{-\frac{1}{2}}$. This gives a weak decrease in risk as the number of failures increase, which means that the risk of cascading failure includes a strong contribution from large cascades. Moreover, if instead cost increases according to $C[m] \sim m^{\alpha}$ where $\alpha > \frac{3}{2}$, then (13) implies that the risk of large cascades exceeds that of small cascades, despite the large cascades being rarer.

Consider a general load dependence for component failure and interaction. We assume that system components are more likely to fail and more likely to cause other component failures when load increases. It is reasonable to assume that at zero load $\lambda < 1$, since a system design with a significant risk of cascading failure at zero load is unlikely to be feasible when operated at normal loads. Moreover, if the system is operated at an absurdly high load at which all components are at their limits, then failure of any component will on average cause many other components to fail and then $\lambda > 1$. We may also assume that λ is an increasing and continuous function of load. Then there is a critical load for which $\lambda = 1$ and the branching process is critical and the risk is governed by (13). The risk will be even higher for $\lambda > 1$.

Thus a simple criterion for avoiding the high risk of cascading failure associated with $\lambda \geq 1$ with some margin determined by a choice of $\lambda_{\max} < 1$ is

$$\boxed{\text{design and operate system so that } \lambda \leq \lambda_{\max} < 1} \tag{14}$$

Although this is a simple criterion, translating it to applicable design and operational criteria is a substantial task. Moreover, applying the criteria (14) generally requires the system to be operated with limited throughput. For example, in electric power transmission systems, the loading of transmission lines and other system components would be limited. Thus limiting the risk of cascading failure using (14) will have an economic cost. The dynamics and difficulties of managing this tradeoff should not be neglected.

One approach to limiting cascading failure is to describe the most likely sequences of cascading failures starting from the initiating failures and design and operate the system to reduce their probability. This standard approach is sensible and can reduce risk [22, 25, 10]. However, in large interconnected and interdependent systems there is a combinatorial explosion of possibilities. It is often impractical to envisage and to quantify and compute probabilities for all but the most likely or apparent of these cascading sequences. A large number of rare and hard to anticipate interactions may have to be neglected [27].

Criterion (14) suggests a different and complementary approach that focusses on limiting the average propagation of failures after a cascade is started. λ is the expected number of failures consequent upon a single failure. We suggest that estimation of average values of λ may be feasible using simulation [8] or otherwise and that the dependence of λ on load and system design could be determined to allow (14) to be implemented. Perhaps the simplifications in this approach could allow the contributions to λ from numerous but rare interactions to be accounted for more readily. There are a number of problems in establishing this approach. Two of these problems are

1. Branching processes usually assume an infinite number of components so that there can be an infinite number of failures in the supercritical case. This is not realistic when considering the transition from subcritical to supercritical.

2. Can loading dependent cascading failure be well approximated as a branching process?

Section 4 addresses problem 1 with a saturating branching process and the rest of this paper addresses problem 2 by showing how the CASCADE model of load-dependent cascading failure can be approximated by the saturating branching process.

4. Saturation due to finite system size

In our application we have a large but finite number n of components and we need to introduce a saturation or truncation of the Poisson branching process. Let

$$N = \min\{n-1, \text{integer part of } (n-\theta)/\lambda\} \tag{15}$$

Then the process evolves in the same way as the process with an infinite number of components when the total number of failures does not exceed N. If the total number of failures exceeds N, then it assumed that all n components fail and the process ends. If the parameters are such that $N < n-1$, this implies that it impossible for $N+1, N+2, ..., n-1$ components to fail. The saturation (15) is chosen so that the saturating model can be a good approximation to CASCADE and this is justified in subsections 6.1 and 6.2.

The standard result (10) above can be modified as follows to obtain the saturating model: The generating function $G(t)$ for the total number of failures remains valid to order N. Write $G^{[N]}(t)$ for the terms up to and including order N of $G(t)$. Then $G^{[N]}(t)$ generates the probabilities of the total number of failures r for $r \leq N$. However, the sum of the probabilities generated by $G^{[N]}(t)$ is $G^{[N]}(1)$ and $G^{[N]}(1) < 1$. The probability generating function $\hat{G}(t)$ for the saturating model can be obtained by making the probability of n failures equal to $1 - G^{[N]}(1)$:

$$\begin{aligned}\hat{G}(t) &= G^{[N]}(t) + (1 - G^{[N]}(1))t^n & (16)\\ &= \sum_{r=0}^{N} \theta(\theta + r\lambda)^{r-1} \frac{e^{-\theta - r\lambda}}{r!} t^r + (1 - G^{[N]}(1))t^n & (17)\end{aligned}$$

The corresponding probability distribution is:

Definition: $g(r, \theta, \lambda, n)$ is the probability that r components fail in the saturating generalized Poisson distribution model with initial disturbance mean failures θ, cascading mean failures λ, and n components. For $\theta < 0$,

$$\begin{aligned} g(r, \theta, \lambda, n) &= 1; \qquad r = 0 & (18)\\ g(r, \theta, \lambda, n) &= 0; \qquad r > 0 & (19)\end{aligned}$$

For $\theta \geq 0$,

$$\begin{aligned} g(r, \theta, \lambda, n) &= \theta(r\lambda + \theta)^{r-1} \frac{e^{-r\lambda - \theta}}{r!} \\ &\quad ; \qquad 0 \leq r \leq (n-\theta)/\lambda,\ r < n & (20)\\ g(r, \theta, \lambda, n) &= 0\ ; \quad (n-\theta)/\lambda < r < n,\ r \geq 0 & (21)\\ g(n, \theta, \lambda, n) &= 1 - \sum_{s=0}^{n-1} g(s, \theta, \lambda, n) & (22)\end{aligned}$$

The saturating form of the generalized Poisson distribution (20-22) limits the total number of failures to n even in the supercritical case and extends the range of parameters of the generalized Poisson distribution (10) to allow $\lambda > 1$.

There are other ways of normalizing or truncating the cascading process to avoid infinite quantities in the supercritical case. For example, one can normalize the number of failures M_k at stage k by their mean λ^k [23] or one can consider truncations motivated by not observing data in some ranges [17, 14]. However, these methods are not suited to our application.

The mean number of failures in the saturating generalized Poisson distribution is

$$E[M] = \sum_{r=0}^{N} r\theta(\theta + r\lambda)^{r-1}\frac{e^{-\theta-r\lambda}}{r!} + n(1 - G^{[N]}(1)) \tag{23}$$

5. Review of CASCADE

This section summarizes the CASCADE model of probabilistic load-dependent cascading failure and the saturating quasibinomial distribution from [20].

The CASCADE model has n identical components with random initial loads. For each component the minimum initial load is $L^{\min}$ and the maximum initial load is $L^{\max}$. For j=1,2,...,n, component j has initial load L_j that is a random variable uniformly distributed in $[L^{\min}, L^{\max}]$. $L_1, L_2, \cdots, L_n$ are independent.

Components fail when their load exceeds L^{fail}. When a component fails, a fixed amount of load P is transferred to each of the components.

To start the cascade, we assume an initial disturbance that loads each component by an additional amount D. Other components may then fail depending on their initial loads L_j and the failure of any of these components will distribute an additional load $P \geq 0$ that can cause further failures in a cascade.

Now we define the normalized CASCADE model. The normalized initial load ℓ_j is

$$\ell_j = \frac{L_j - L^{\min}}{L^{\max} - L^{\min}} \tag{24}$$

Then ℓ_j is a random variable uniformly distributed on $[0, 1]$. Let

$$p = \frac{P}{L^{\max} - L^{\min}}, \qquad d = \frac{D + L^{\max} - L^{\text{fail}}}{L^{\max} - L^{\min}} \tag{25}$$

Then the normalized load increment p is the amount of load increase on any component when one other component fails expressed as a fraction of the load range $L^{\max} - L^{\min}$. The normalized initial disturbance d is a shifted initial disturbance expressed as a fraction of the load range. Moreover, the failure load is $\ell_j = 1$

The saturating quasibinomial distribution is given by:

Definition: $f(r, d, p, n)$ is the probability that r components fail in the CASCADE model with normalized initial disturbance d, normalized load transfer amount p, and n components. For $d < 0$,

$$f(r, d, p, n) = 1; \qquad r = 0 \tag{26}$$

$$f(r, d, p, n) = 0; \qquad r > 0 \tag{27}$$

For $d \geq 0$,

$$f(r, d, p, n) = \binom{n}{r} d(rp + d)^{r-1}(1 - rp - d)^{n-r}$$
$$; \qquad 0 \leq r \leq (1 - d)/p, \ r < n \tag{28}$$

$$f(r, d, p, n) = 0 \, ; \quad (1 - d)/p < r < n, \ r \geq 0 \tag{29}$$

$$f(n, d, p, n) = 1 - \sum_{s=0}^{n-1} f(s, d, p, n) \tag{30}$$

If $np+d \leq 1$, (28) and (30) reduce to the quasibinomial distribution introduced as an urn model by Consul [13]. Thus (28–30) extend the quasibinomial distribution to parameters with $np + d > 1$. $np + d > 1$ corresponds to highly stressed systems with a significant probability of all components failing.

The distribution (26–30) can also be expressed using a saturation function ϕ as follows [21]:

$$f(r, d, p, n) = \begin{cases} \binom{n}{r} \phi(d)(d + rp)^{r-1}(\phi(1 - d - rp))^{n-r}, \\ \qquad\qquad r = 0, 1, ..., n - 1 \\ 1 - \displaystyle\sum_{s=0}^{n-1} f(s, d, p, n), \qquad r = n \end{cases} \tag{31}$$

where

$$\phi(x) = \begin{cases} 0 \ ; x < 0 \\ x \ ; 0 \leq x \leq 1 \\ 1 \ ; x > 1 \end{cases} \tag{32}$$

Note that (31) uses $0^0 \equiv 1$ and $0/0 \equiv 1$ when needed.

6. Approximating CASCADE as a branching process

We first approximate the distribution of the total number of failures in CASCADE by the distribution of total number of failures in a saturating Poisson branching process. Then we show how the cascading failures in CASCADE can be approximated stage by stage by a Poisson branching process.

6.1. Approximating the distribution of the total number of failures

The total number of failures in the CASCADE model is distributed according to the saturating quasibinomial distribution (26)-(30). We prove that the saturating quasibinomial distribution can be approximated by the saturating generalized Poisson distribution (18)-(22).

Let $n \to \infty$ and $p \to 0$ and $d \to 0$ in such a way that $\lambda = np$ and $\theta = nd$ are fixed. Then the appendix

[17] shows that the quasibinomial distribution tends to the generalized Poisson distribution. Hence for large n and for $0 \leq r \leq (1-d)/p = (n-\theta)/\lambda$, (28) may be approximated by (20). $(1-d)/p = (n-\theta)/\lambda$ also implies that (29) may be replaced by (21). Then the preceding results imply that (30) tends to (22).

6.2. Branching process obtained from CASCADE

This subsection informally shows how failures in CASCADE arise in stages approximately as stages of a saturating branching process. The CASCADE model produces failures in stages $i = 0, 1, 2, ...$ where M_i is the number of failures in stage i. The following is a normalized version of the algorithm for CASCADE that can be derived from [20].

Algorithm for normalized CASCADE model

0. All n components are initially unfailed and have initial loads $\ell_1, \ell_2, \cdots, \ell_n$ determined as independent random variables uniformly distributed in $[0, 1]$.
1. Add the initial disturbance d to the load of component j for each $j = 1, ..., n$. Initialize the stage counter i to zero.
2. Test each unfailed component for failure: For $j = 1, ..., n$, if component j is unfailed and its load > 1 then component j fails. Suppose that M_i components fail in this step.
3. If $M_i = 0$, stop; the cascading process ends.
4. If $M_i > 0$, then increment the component loads according to the number of failures M_i: Add $M_i p$ to the load of component j for $j = 1, ..., n$.
5. Increment the stage counter i and go to step 2

It is convenient throughout to restrict m_0, m_1,... to nonnegative integers and to write

$$s_i = m_0 + m_1 + ... + m_i \tag{33}$$

Consider the end of step 2 of stage $i \geq 1$ in the CASCADE algorithm. The failures that have occurred are $M_0 = m_0, M_1 = m_1, ..., M_i = m_i$, but the loads have not yet been incremented by $m_i p$ in the following step 4. Let

$$\alpha_{i+1} = \phi\left(\frac{m_i p}{1 - d - s_{i-1} p}\right) \tag{34}$$

where ϕ is the saturation function defined in (32).

Suppose that $d + s_{i-1} p \leq 1$. Then the loads of the $n - s_i$ unfailed components are uniformly distributed in $[d + s_{i-1} p, 1]$. This uniform distribution is conditioned on the $n - s_i$ components not yet having failed. In the following step 4, the probability that the load increment of $m_i p$ causes one of the unfailed components to fail is α_{i+1} and the probability of m_{i+1} failures in the $n - s_i$ unfailed components is

$$P[M_{i+1} = m_{i+1} | M_i = m_i, ..., M_0 = m_0] = \binom{n - s_i}{m_{i+1}} \alpha_{i+1}^{m_{i+1}} (1 - \alpha_{i+1})^{n - s_{i+1}} \quad , \; m_{i+1} = 0, 1, ..., n - s_i \tag{35}$$

and the generating function for (35) is

$$(1 + \alpha_{i+1}(t-1))^{n - s_i} \tag{36}$$

Suppose that $d + s_{i-1} p > 1$. Then all the components must have failed on a previous step and $P[M_{i+1} = m_{i+1} | M_i = m_i, ..., M_0 = m_0] = 1$ for $m_{i+1} = 0$ and vanishes otherwise. In this case $\alpha_{i+1} = 0$ and (35) and (36) are again verified.

Let $nd = \theta$ and $np = \lambda$. Then

$$\alpha_{i+1} = \phi\left(\frac{m_i \lambda}{n - \theta - s_{i-1} \lambda}\right) \tag{37}$$

There are three cases:

(1) $s_{i-1} > (n-\theta)/\lambda$. Then $\alpha_{i+1} = 0$, (36) evaluates to 1 and $P[M_{i+1} = 0 | M_i = m_i, ..., M_0 = m_0] = 1$. Case 1 is an already saturated case corresponding to all components failing in stage $i-1$ or previous stages.

(2) $s_{i-1} \leq (n-\theta)/\lambda$ and $s_i = m_i + s_{i-1} \geq (n-\theta)/\lambda$. Then $\alpha_{i+1} = 1$, (36) evaluates to t^{n-s_i} and $P[M_{i+1} = n - s_i | M_i = m_i, ..., M_0 = m_0] = 1$. Case 2 is a saturating case corresponding to all components failing in stage i.

(3) $s_i = m_i + s_{i-1} < (n-\theta)/\lambda$. Then

$$\alpha_{i+1} = \frac{m_i \lambda}{n - \theta - s_{i-1} \lambda}$$

Let $n \to \infty$ and $p \to 0$ so that $np = \lambda$. Since

$$(1 + \alpha_{i+1}(t-1))^{n - s_i} \to e^{m_i \lambda (t-1)} \text{ as } n \to \infty \tag{38}$$

we approximate (36) by

$$\left(e^{m_i \lambda (t-1)}\right)^{[n - s_i - 1]} + t^{n - s_i} \left(1 - \left(e^{m_i \lambda (t-1)}\right)^{[n - s_i - 1]} (1)\right) \tag{39}$$

That is, the approximation is

$$P[M_{i+1} = m_{i+1} | M_i = m_i, ..., M_0 = m_0] = \begin{cases} \dfrac{(m_i \lambda)^{m_{i+1}}}{m_{i+1}!} e^{-m_i \lambda} & , m_{i+1} = 0, 1, ..., n - s_i - 1 \\ 1 - \displaystyle\sum_{k=0}^{n - s_1 - 1} \frac{(m_i \lambda)^k}{k!} e^{-m_i \lambda} & , m_{i+1} = n - s_i. \end{cases} \tag{40}$$

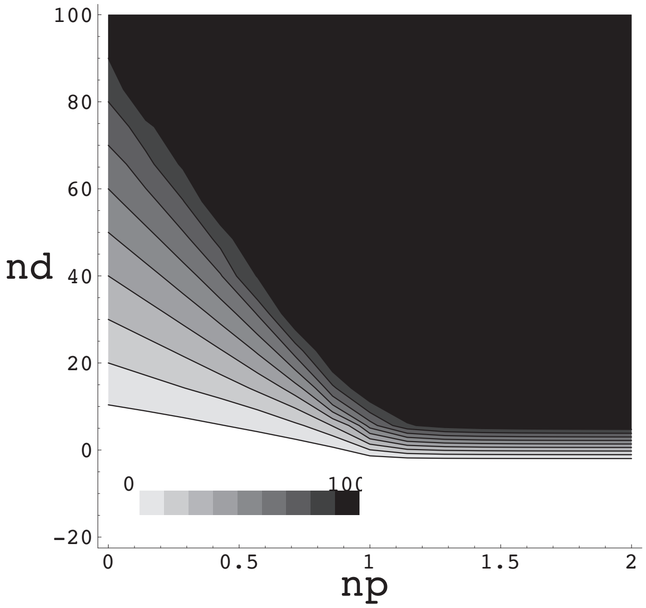

Figure 1. Average number $<r>$ of components failed in CASCADE as a function of np and nd for $n = 100$. Lines are contours of constant $<r>$. White indicates < 10 failures and black indicates > 90 failures.

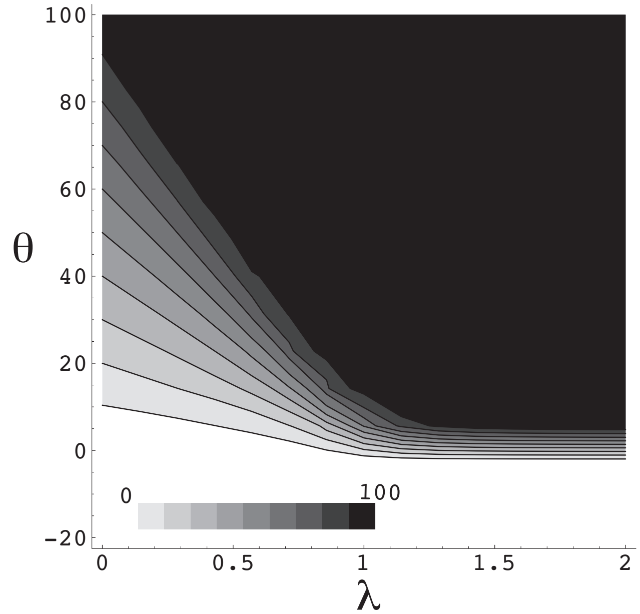

Figure 2. Average number $<r>$ of components failed in saturating generalized Poisson distribution as a function of λ and θ for $n = 100$. Lines are contours of constant $<r>$. White indicates < 10 failures and black indicates > 90 failures.

According to (38), for fixed r, the approximation (39) becomes exact as $n \to \infty$. That is, the coefficient of t^r in (39) tends to the coefficient of t^r in (36) as $n \to \infty$. However, the approximation (39) is inaccurate for the coefficient of t^r when $r = n - s_i$ or r is close to $n - s_i$.

Since $e^{m_i \lambda(s-1)} = \left(e^{\lambda(s-1)}\right)^{m_i}$, (39) or (40) is the distribution of the sum of m_i independent Poisson random variables with rate λ with saturation occurring when the total number of failures exceeds n. *Thus we can consider each failure as independently causing other failures in the next stage according to a saturating Poisson process.*

A similar approximation applies at stage zero. Suppose that in step 2 of stage zero in the CASCADE algorithm there are m_0 failures due to the initial disturbance d. The probability that the load increment of d causes one of the components to fail is $\phi(d)$ and the probability of m_0 failures in the n components is given by:

$$\binom{n}{m_0} \phi(d)^{m_0} \left(1 - \phi(d)\right)^{n-m_0} \tag{41}$$

Let $n \to \infty$ and $d \to 0$ so that $nd \to \theta$. Then we approximate (41) by the saturating Poisson distribution

$$P[M_0 = m_0] = \begin{cases} \dfrac{\theta^{m_0}}{m_0!} e^{-\theta} & , m_0 = 0, 1, ..., n-1 \\ 1 - \displaystyle\sum_{k=0}^{n-1} \dfrac{\theta^{m_0}}{m_0!} e^{-\theta} & , m_0 = n. \end{cases} \tag{42}$$

The approximations (40) and (42) show that the number of failures in each stage are, for large n and small p and d, governed by a saturating Poisson branching process with mean $\lambda = np$, except that on the first step the mean is $\theta = nd$. The approximation does not necessarily imply that concepts natural to the branching process translate directly to the CASCADE model. For example, each failure in CASCADE may be attributed to load increases caused by many previous failures, whereas it is natural to attribute each failure in a branching process to a single previous failure.

The mean number of failures in the CASCADE and the saturating generalized Poisson distribution as a function of θ and λ are compared in Figures 1 and 2. Scans corresponding to load increase with $d = p$ and $\theta = \lambda$ are compared in Figures 3 and 4. Note the closeness of the approximation for small and moderate r and the expected inaccuracy of the approximation near $r = n$.

7. Discussion

Large power system blackouts typically involve a cascading series of failures or outages in which the system becomes weaker or more stressed as the cascade proceeds. There are many ways in which failure or outage of a compo-

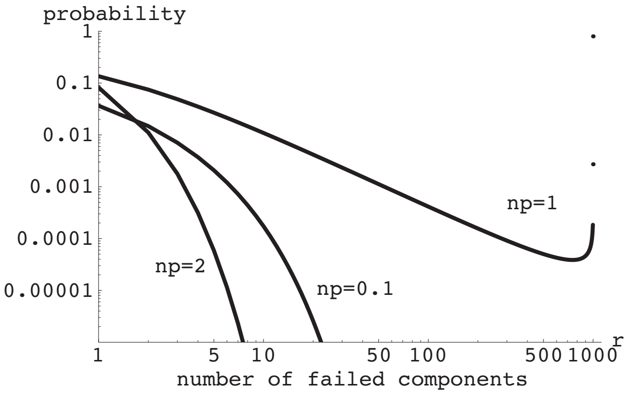

Figure 3. CASCADE probability distributions of total number of failures on log-log plot. $n = 1000$**. Note that the probability of 1000 components failing is 0.003 for** $np = 1$**, and 0.798 for** $np = 2$**.**

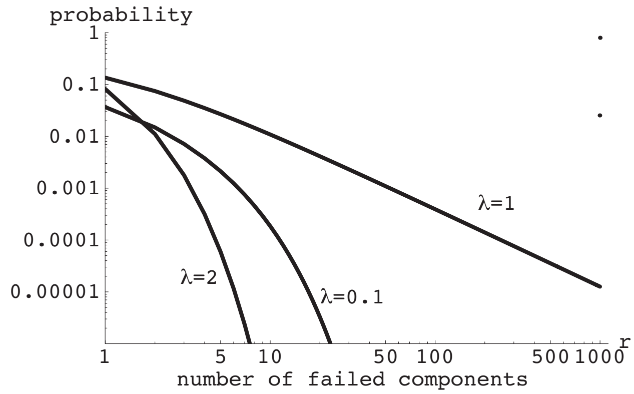

Figure 4. Saturating generalized Poisson probability distributions of total number of failures on log-log plot. $n = 1000$**. Note that the probability of 1000 components failing is 0.025 for** $\lambda = 1$**, and 0.797 for** $\lambda = 2$**.**

nent can adversely affect other components and make their failure more likely. For example, outage of a line can make more likely the failure of other components via redistribution of load, relay or control system misoperation [28], transient phenomena, or operator or planning error. Moreover, all these interactions generally become stronger as power system loading is increased and the significant interactions become more numerous. High loading tends to make interactions more nonlinear, harder to conceive of in advance and much more likely to cause further failures since margins are smaller. In the terminology of Perrow [27], highly loaded power systems are more complex and tightly coupled. The diversity of components and interactions in the power system is highly simplified in the CASCADE model to uniform components that interact in a uniform and simple way with all the other system components. The branching process model is even further abstracted in that component failures cause other failures by an unspecified mechanism. While this paper does claim to capture salient features of cascading blackouts in both of these simple models, it should be acknowledged that substantial work is needed to determine the detailed similarities and differences between these models and real blackouts via statistical measurements and simulations. Estimating λ from a simulation of cascading outages is considered in [8]. The consequences of nonuniform interactions between components or interactions limited to a subset of other components also needs to be examined in future work.

The CASCADE model captures the weakening of system as the cascade proceeds and reproduces some qualitative features of blackout size probability distributions observed in blackout data and simulations [19, 9, 7]. Since this paper shows that CASCADE is well approximated by a branching process and the saturating generalized Poisson distribution, the saturating generalized Poisson distribution also reproduces the same qualitative features of blackout size probabilities.

The approximation of CASCADE by the branching process allows the parameters of the two models to be related. Thus

$$\lambda = np \tag{43}$$

$$= \frac{nP}{L^{\max} - L^{\min}} \tag{44}$$

Recall that in CASCADE, p is the normalized load transfer amount and n is the number of components. (43) can be used to reinterpret $p = \lambda/n$ in the branching process as the probability that a component failure causes the failure of a specific other component. This is an important interpretation in contexts in which there is a cascading dependency between components that is not naturally expressed as an increment in loading.

The criterion (14) for minimizing cascading failure can be reexpressed using (43) as $np < \lambda_{\max}$. Then even if p is very small, large n can cause cascading failure. This suggests that numerous rare interactions can be equally influential in causing cascading failure as a smaller number of likely interactions. More generally, one can speculate that a design change that introduced a large number of unlikely failure interactions (plausibly similar to large n) could make cascading failure more likely, despite high reliability (low p). It is conceivable that coupling infrastructures together such as controlling the power system over an internet or certain types of global control schemes could make the system more vulnerable to cascading failure in this fashion. It is also interesting to note that many traditional power system controls are designed to reduce interactions by deliberate separation in distance, frequency, and time scale.

The criterion (14) for minimizing cascading failure can be reexpressed using (44) as

$$\lambda = \frac{nP}{L^{\max} - L^{\min}} < \lambda_{\max} \qquad (45)$$

There are several ways to represent system load increase in CASCADE [20]. One of these ways increases average component load by increasing $L_{\min}$. Then (45) shows how this form of load increase affects the criterion limiting the risk of cascading failure. The relation (45) between λ and $L_{\min}$ is nonlinear.

8. Conclusion

We introduce a saturating form of the generalized Poisson distribution and show that it approximates the distribution of total number of failures in the CASCADE model of load-dependent cascading failure. Moreover, successive failures in stages of CASCADE can be approximated by corresponding stages of a saturating Poisson branching process. The approximation of CASCADE as a branching process yields insights into the power tails and criticality observed in CASCADE. The branching process approximation is simpler and more analytically tractable than CASCADE while retaining qualitative features of load-dependent cascading failure. Moreover, at criticality the universality of the $-\frac{3}{2}$ power law in the probability distribution of the total number of failures in a branching process suggests that this is a signature for this type of cascading failure. The $-\frac{3}{2}$ power law is approximately consistent with North American blackout data and blackout simulation results.

Criticality or supercriticality in the branching process implies a high risk of catastrophic and widespread cascading failures. Maintaining sufficient subcriticality in the branching process according to a simple criterion (14) would limit the propagation of failures and reduce this risk. The approximation of CASCADE as a branching process allows the criterion to be expressed in terms of system loading (45). However, implementing the criterion to reduce the risk of catastrophic cascading failure would require limiting the system throughput and this is costly. Managing the tradeoff between the certain cost of limiting throughput and the rare but very costly widespread catastrophic cascading failure may be difficult. Indeed [18, 4, 5] maintain that for large blackouts, economic, engineering and societal forces may self-organize the system to criticality and that efforts to mitigate the risk should take account of these broader dynamics [6].

Our emphasis on limiting the propagation of system failures after they are initiated is complementary to more standard methods of mitigating the risk of cascading failure by reducing the risk of the first few likely failures caused by an initial disturbance as for example in [10].

The branching process approximation does capture some salient features of loading dependent cascading failure and suggests an approach to reducing the risk of large cascading failures by limiting the average propagation of failures. However, much work remains to establish the correspondence between these simplified global models and the complexities of cascading failure in real systems.

9. Acknowledgements

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A. Approximating quasibinomial distribution

The generalized Poisson distribution is [17, 16]

$$G(r,\theta,\lambda) = \theta(r\lambda+\theta)^{r-1}\frac{e^{-r\lambda-\theta}}{r!} \tag{46}$$

for $\lambda \leq 1$ and $\theta > 0$. We use Consul's derivation [16] that the quasibinomial distribution tends to the generalized Poisson distribution. The quasibinomial distribution is

$$\binom{n}{r} d(rp+d)^{r-1}(1-rp-d)^{n-r} \tag{47}$$

for $d + np \leq 1$ and $0 < d < 1$.

If $d \to 0$, $p \to 0$ and n increases without limit such that $nd = \theta$ and $np = \lambda$, then (47) can be written in the form

$$\frac{nd(rnp+nd)^{r-1}}{r!}\frac{n!}{(n-r)!\,n^r}\left[1-\frac{r\lambda+\theta}{n}\right]^{n-r} \tag{48}$$

which can be rewritten as

$$\begin{aligned}
&\theta(r\lambda+\theta)^{r-1}\frac{e^{-r\lambda-\theta}}{r!}\left[1-\frac{r\lambda+\theta}{n}\right]^{n} e^{r\lambda+\theta}\\
&\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right)\right]\left[1-\frac{r\lambda+\theta}{n}\right]^{-r}\\
=\;& G(r,\theta,\lambda)\left[1-\frac{(r\lambda+\theta)^2}{2n}\right]\\
&\left[1+\frac{2r(r\lambda+\theta)-r(r-1)}{2n}\right]+O(n^{-2})\\
=\;& G(r,\theta,\lambda)\left[1+\frac{2r(r\lambda+\theta)-r(r-1)-(r\lambda+\theta)^2}{2n}\right]\\
&+O(n^{-2})\\
=\;& G(r,\theta,\lambda)\left[1+\frac{r-(r(\lambda-1)+\theta)^2}{2n}\right]+O(n^{-2})
\end{aligned} \tag{49}$$

Hence the generalized Poisson distribution is the limit of the quasibinomial distribution.

Examination of the $1 + O(n^{-1})$ factor in (49) suggests that the approximation improves for $\lambda \approx 1$ and only slowly gets worse for larger r. For $\lambda \not\approx 1$, the $1 + O(n^{-1})$ factor suggests that the approximation gets worse for larger r.

6.2 Dynamical and probabilistic approaches to the study of blackout vulnerability of the power transmission grid

B.A. Carreras, V.E. Lynch, D.E. Newman, I. Dobson

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Dynamical and probabilistic approaches to the study of blackout vulnerability of the power transmission grid

B. A. Carreras
Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA
carrerasba@ornl.gov

V. E. Lynch
Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA
lynchve@ornl.gov

I. Dobson
ECE Department, University of Wisconsin, Madison, WI 53706 USA
dobson@engr.wisc.edu

D. E. Newman
Physics Department, University of Alaska, Fairbanks, AK 99775 USA
ffden@uaf.edu

Abstract

The CASCADE probabilistic model for cascading failures gives a simple characterization of the transition from an isolated failure to a system-wide collapse as system loading increases. Using the basic ideas of this model, the parameters that lead to a similar characterization for power transmission system blackouts are identified in the OPA dynamical model of series of blackouts. The comparison between the CASCADE and OPA models yields parameters that can be computed from the OPA model that indicate a threshold for cascading failure blackouts. This is a first step towards computing similar parameters for real power transmission systems.

1. Introduction

We have developed the ORNL-PSerc-Alaska (OPA) model to study blackout dynamics in the power transmission grid [1-3]. This model incorporates self-organization processes based on the engineering response to blackouts and the long-term economic response to customer load demand. It also incorporates the apparent critical nature of the transmission system. The combination of these mechanisms leads to blackouts that range in size from single load shedding to the blackout of the entire system. This model shows a probability distribution of blackout sizes with power tails [2] similar to that observed in real blackout data from North America.

In addition to the OPA model, we have constructed CASCADE, a probabilistic model that incorporates some general features of cascading failure. A detailed description of the CASCADE model is given in Refs. [4,8]. This model shows the existence of two critical thresholds. One is associated with the minimal load needed to start a disturbance. In a power transmission system, it can be interpreted as the load increase that will cause a line (or a few independent lines) to overload and fail. The second critical threshold is associated with the minimal load transfer throughout a cascading event that can lead to a total system blackout. This type of threshold is less evident in real systems, and the parameter or parameters controlling it are not easy to identify.

Those cascading events are similar to the "domino effect." In this case, the force needed to trip the first domino gives the first threshold. The second threshold is given by the ratio of the separation between dominos to their height; the threshold must be less than the critical value of one to cause all the dominos to fall. Of course, transmission systems are a great deal more complicated than dominos, but here we want to focus on identifying this second type of threshold.

To identify the type of threshold that causes system-wide blackouts, we compare the probabilistic model, where this threshold is easy to identify, with the dynamical model. This dynamical model incorporates the structure of a network, and a linear programming (LP) approach is used to find instantaneous solutions to the power demand. In such a model, the threshold to system-wide blackouts is not obvious, and its understanding may provide a path toward application to realistic systems.

2. Critical transitions in the CASCADE model

The CASCADE model has n identical components with random initial loads. The minimum initial load is $L_{\min}$, and the maximum initial load for each component is $L_{\max}$.. For j=1,2,...,n, component j has an initial load of L_j that is a random variable uniformly distributed in [$L_{\min}$, $L_{\max}$]. $L_1, L_2, \cdots, L_n$ are independent. Components fail when their load exceeds L_{fail}. When a component fails, a fixed amount of load P is transferred to each of the remaining components.

We assume an initial disturbance that starts the cascade by loading each component with an additional amount, D. Other components may then fail, depending on their initial loads, L_j, and the failure of any of these components will distribute an additional load, $P \geq 0$, that can cause further failures in a cascade. This model describes the cascading failure as an iterative process. In each iteration, loads fail as the transfer load, P, from other failures makes them reach the failure limit. The process stops when none of the remaining loads reaches the failure limit.

It is convenient to normalize all of the loads in the system so that they are distributed in the [0,1] interval. Thus, we normalize the initial load:

$$l_j = \frac{L_j - L_{\min}}{L_{\max} - L_{\min}}. \quad (1)$$

Then l_j is a random variable uniformly distributed on [0, 1]. Moreover, the failure load is $l_j = 1$. Let

$$p = \frac{P}{L_{\max} - L_{\min}}, \quad d = \frac{D + L_{\max} - L_{fail}}{L_{\max} - L_{\min}} \quad . \quad (2)$$

Then, p is the amount of load increase on any component when one other component fails when expressed as a fraction of the load range $L_{max} - L_{min}$. Similarly, d is the initial disturbance expressed as a fraction of the load range.

An analytic solution was found [4,8,9] for the probability, $f(r, d, p, n)$, of a cascade with r components failing:

$$f(r,d,p,n) = \begin{cases} \binom{n}{r} \phi(d)(rp+d)^{r-1}\left[\phi(1-rp-d)\right]^{n-r}, \\ \qquad r = 0,1,\ldots,n-1 \\ 1-\sum_{s=0}^{n-1} f(s,d,p,n), \quad r = n \end{cases} \tag{3}$$

where p is a positive quantity and the function ϕ is

$$\phi(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases} .$$

Equation (3) uses $0^0 \equiv 1$ and $0/0 \equiv 1$ where needed. If d ‡ 0 and $d + np \le 1$, then $\phi(x) = x$ and Eq. (3) reduces to the quasibinomial distribution introduced by Consul [10].

For a given system, there are two possible types of situations: (1) the system has no component failures or (2) some components in the system have failed. In the CASCADE model, there is clearly a transition from one situation to the other and the control parameter is d. The transition point is $d = 0$. The probability of failure is

$$P(d) = \begin{cases} 0, & d < 0 \\ 1 - f(0,p,d,n) = 1-(1-d)^n, & d > 0 \end{cases} . \tag{4}$$

Near the critical point, the transition probability scales as nd. For large systems, it is better to introduce $\theta \equiv nd$ as the control parameter for this transition. In this way, θ remains finite for $n \to \infty$. It is also useful to consider $\lambda \equiv np$, the total load transfer from a failing component, as the second parameter in this model. The use of the λ and θ is justified in Ref. [9] by approximating CASCADE as a branching process and identifying λ and θ as parameters of the branching process. The situation with no failures is rather simple, and there is a single point in configuration space with no ambiguity in its characterization. However, the failed system has multiple possible states, each characterized by the number r of failed loads. For a given set of values for θ and λ, there is a distribution of possible states, each characterized by a probability $p_b(r,\lambda,\theta,n)$,

$$p_b(r,\lambda,\theta,n) = \frac{f(r,\lambda,\theta,n)}{1-f(0,\lambda,\theta,n)} . \tag{5}$$

Because we are interested in system-wide collapses, an important quantity to consider is the probability of a full system cascade, $r = n$,

$$P_\infty = \frac{f(n,\lambda,\theta,n)}{1-f(0,\lambda,\theta,n)} . \tag{6}$$

This probability has the properties of the order parameter in a critical transition. As shown in Fig. 1, this expression is such as that $P = 0$ at $\lambda < \lambda_c$, where λ_c is the critical value of λ. However, above the critical value for λ, system-wide failures are possible. In the CASCADE model, which assumes a uniform random distribution of loads, the critical point is $\lambda_c = 1$. This is the second transition point that we discussed in the introduction. It separates the localized failures of the system from system-wide cascading failures. This type of transition is the one we want to also characterize for the OPA dynamical model.

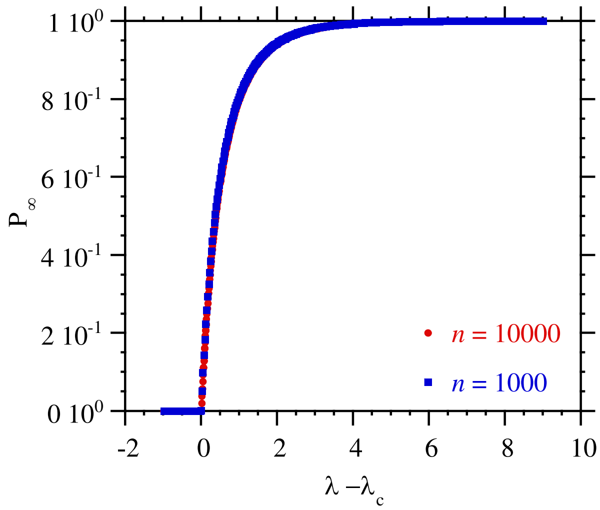

Figure 1: Probability of system-wide cascade events as a function of λ.

The parameter λ is a direct measure of the total load transferred by a failing component to the entire system. It also characterizes other properties of the system that are useful in giving a meaningful interpretation of λ for different systems. One of the approximate properties of the CASCADE model that applies when the model is not saturated due to finite size effects is that the average number of failures during the iteration k is

$$\langle r \rangle_k = \theta \lambda^k . \tag{7}$$

This is an important relationship that will be used in comparison with the dynamical model.

3. The dynamical OPA model and the cascading transition

We developed the OPA model to study the dynamics of a power transmission system [1-3]. In the OPA model, the dynamics involve two intrinsic time scales.

In the OPA model, there is a slow time scale of the order of days to years, over which load power demand slowly increases and the network is upgraded in response to the increased demand. The upgrades are done in two ways. Transmission lines are upgraded as engineering responses to blackouts and maximum generator power is increased in response to increasing demand. The transmission line upgrade is implemented as an increase in maximum power flow, $F_{ij}^{\max}$, for the lines that have overloaded during a blackout. That is, $F_{ij}^{\max}(t) = \mu F_{ij}^{\max}(t-1)$ if line ij overloads during a blackout. We take μ to be a constant. These slow, opposing forces of load increase and network upgrade self-organize the system into a dynamic equilibrium. As discussed elsewhere [3], this dynamical equilibrium is close to the critical points of the system [5, 6].

In the OPA model, there is also a fast time scale, of the order of minutes to hours, over which cascading overloads or outages may lead to a blackout. Cascading blackouts are modeled by overloads and outages of lines determined in the context of LP dispatch of a DC load flow model. Random line outages are triggered with a probability p_0. They simulate the consequence of intentional or accidental events. A cascading overload may also start if one or more lines are overloaded in the solution of the LP problem. In this situation, we assume that there is a probability, p_1, that an overloaded line will become an outage. When a solution is found, the overloaded lines of the solution are tested for possible outages. If there are one or more line outages, we reduce the maximum power flow allowed through this line by several orders of magnitude. In this way, there is practically no power flow through this line. Once the power flow through the lines is reduced, a new solution is then calculated. This process can lead to multiple iterations, and the process continues until a solution with no more line outages is found. The overall effect of the process is to generate a possible cascade of line outages that is consistent with the network constraints and the LP dispatch optimization.

The OPA model allows us to study the dynamics of blackouts in a power transmission system. This model shows dynamical behaviors characteristic of complex systems. It has a variety of transition points as power demand is increased [5, 6]. These transition points are related to a limitation in the generator power and/or single line overloads. These transition points correspond to single failures of the system and are the first type of transition discussed above. However, in contrast to the CASCADE model, there are multiple sources of single failure in this model.

Here, we study the critical point from the perspective of triggering system-wide blackouts as described in the previous section. The first thing to consider is the possible separation between regimes of single failures and regimes with cascading failures. For this model, calculation of the probability of a system collapse event is not possible. It would be necessary to carry out calculations for a very long time to obtain the necessary statistics. In particular, close to the transition, the required computational time is beyond our present capabilities. We need another approach.

In the OPA model, we find the separation between the two regimes as a function of two parameters, Γ and μ. Here, Γ is the ratio of minimal generator power margin, $(\Delta P/P)_c \equiv (P_G - P_0)/P_0$, to the root mean square of the fluctuation of the load demand $g \equiv \left\langle \left[(P_D - P_0)/P_0\right]^2 \right\rangle^{1/2}$.

$$\Gamma = (\Delta P/P)_c / g \quad . \tag{8}$$

P_G is the minimal generator power available, $P_0 = \hat{P}_0 e^{\hat{\lambda} t}$ is the mean load demand that increases at a constant rate $\hat{\lambda}$, and P_D is the actual load demand that fluctuates around the mean value.

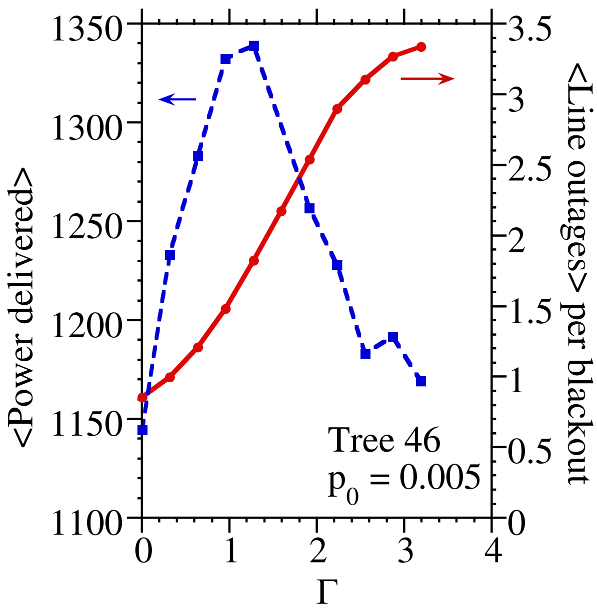

Figure 2: Averaged power delivered and number of line outages per blackout as a function of Γ.

Varying Γ and/or μ is not necessarily a realistic way of modeling the transmission system, but it allows us to understand its dynamics. For a 46-node tree network, we have done a sequence of calculations for different values of the minimal generator power margin $(\Delta P/P)_c$ at a constant g and μ. We have changed this margin from 0 to 100%. For each value of this parameter, we have carried out the calculations for more than 100,000 days in a steady-state regime. This number of days gives us reasonable statistics for the evaluations. One way of looking at the change of characteristic properties of the blackouts with Γ is by plotting the power delivered and the averaged number of line outages per blackout. These plots are shown in Fig. 2. We can see that at low and high values of Γ the power served is low. In the first case, because of limited generator power, the system cannot deliver enough power when there is a relatively large fluctuation in load demand. At high Γ, the power served is low because the number of line outages per blackout is large.

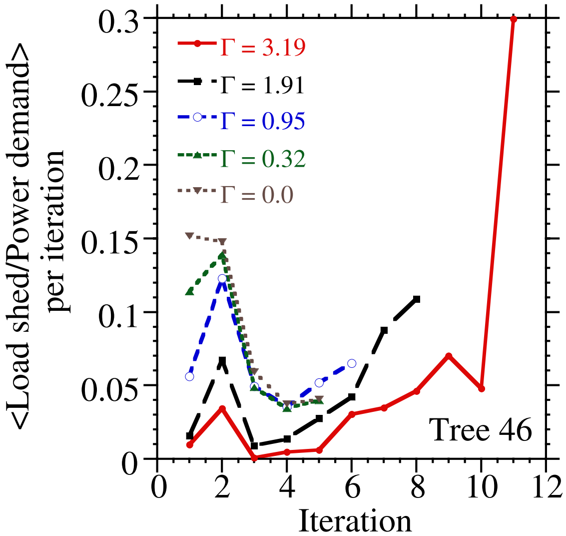

Figure 3: Averaged load shed per blackout normalized to the power demand as a function of iteration number for different values of Γ.

Looking at averaged quantities is not a good way of identifying the demarcation between single (or a few independent) failures and cascading events. To have a better sense of this demarcation, we have calculated the load shed per iteration, normalized to the total power demand, for all blackout events. In Fig. 3, we have plotted the averaged value over all the blackout events for five different values of Γ. We can see that at very low Γ the averaged event is limited to less than five iterations; most of the load is shed during the first couple of iterations. This is typical of isolated failures in a system. However, for large values of Γ, sufficient power is available in the first few iterations with very low load shed. The number of iterations of the cascade events increases and the load shed increases with the iteration number. These are the characteristic properties of large cascading events. At about Γ = 1.0, where the power served has a maximum (Fig. 2), there is the transition from one type of event to the other.

A similar study can be done keeping the parameter Γ fixed and varying the upgrading rate μ. In Fig. 4 and for the 94-node tree network, we show the distribution of the number of line outages for the worst blackouts in a year for different values of μ. We see that for a high upgrade rate, the number of line outages is rather small. However, as μ decreases, the worst blackouts involve a large part of the network.

The Γ and μ parameters have no direct connection to the parameter used in the probabilistic model to characterize the transition from a single failure to a cascading failure. Using the guidance of the CASCADE model, we will try to identify a parameter analogous to λ in the OPA model. To do so, we need to find a way of comparing both models.

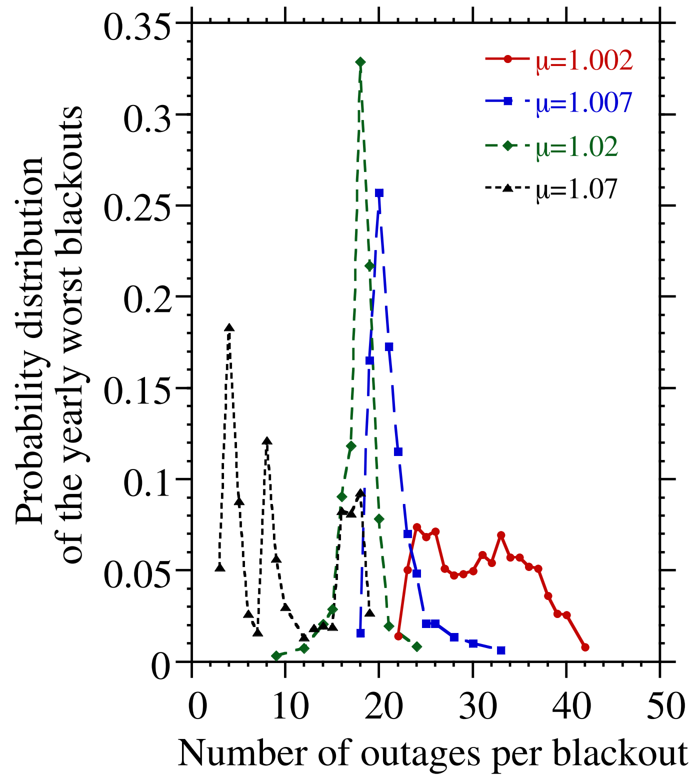

Figure 4: Probability distribution of the number of outages per blackout for the worst yearly blackouts. The calculation is for the 94-node tree network and Γ = 0.96.

4. Averaged number of line outages per iteration

In relating the OPA model to CASCADE, we will interpret the component failures in CASCADE as line outages in OPA. We can then associate the normalized loads, $l_i \in [0,1]$, in CASCADE to the fractional line overloads, M_i, in OPA. The fractional line overload for line i is defined as

$$M_i = \frac{F_i}{F_i^{\max}} \ , \tag{9}$$

where F_i is the power flow through line i and $F_i^{\max}$ is the maximum possible power flow through this line. For each network considered, the fraction of overloads M_i is also distributed in [0,1], but the distribution is not necessarily random. The average value of the M_i's as the average value of the l_i's in the CASCADE model gives no information on the criticality of the system. It only provides some information on the distribution of loads.

There are several ways of interpreting the parameter λ within the OPA model, and, of course, these different methods do not necessarily lead to the same value for λ. One way is to calculate the averaged number of line outages, $\langle N_{out}(j)\rangle$, per step j in cascading failures, and in analogy with Eq. (7) define

$$\lambda_{eff}(j) \equiv \langle N_{out}(j)\rangle^{1/j} \ . \tag{10}$$

A priori, there is no reason for λ_{eff} to be independent of *j* or to have any value similar to the critical value found in the CASCADE model.

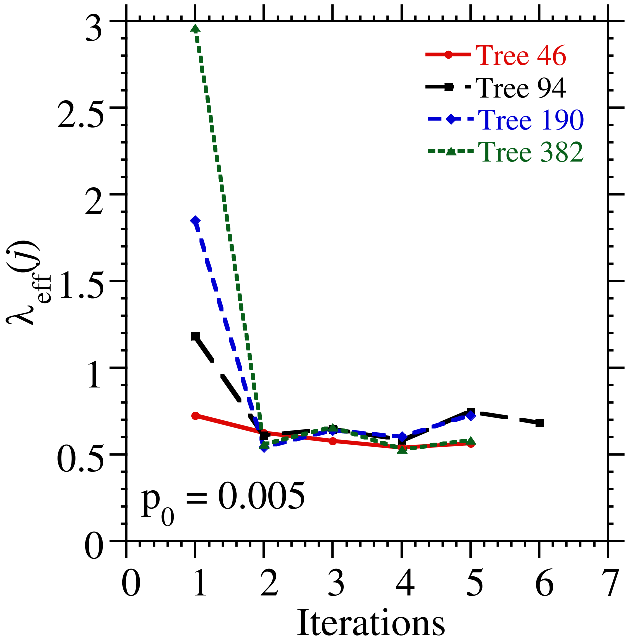

Figure 5: $\lambda_{eff}(j)$ as a function of iteration number for different tree networks.

In Fig. 5, we have plotted $\lambda_{eff}(j)$ as a function of *j* for four network configurations. These $\lambda_{eff}(j)$ networks have a tree-like structure with three line connections per node. These types of networks were discussed in Ref. [2]. The four networks considered here have 46, 94, 190, and 382 nodes.

The numerical results in Fig. 5 show that $\lambda_{eff}(j)$ is weakly varying with *j* for $j > 1$. For large values of *j*, the statistics are rather poor and the evaluation of λ_{eff} may have significant error bars. For the first iteration, we found strong variations of $\lambda_{eff}(1)$ with the size and conditions of the network. These variations are understandable because the calculations in Fig. 3 are done for a fixed probability, p_0, of the event being initiated by a line outage. As the number of lines increases, we can have more than one event simultaneously triggered by these random events. Changing the value of p_0 significantly changes $\lambda_{eff}(1)$. However, the change of p_0 has only a weak effect on $\lambda_{eff}(j)$ for $j > 1$. In Fig. 6, we show the effect of changing p_0 on $\lambda_{eff}(j)$.

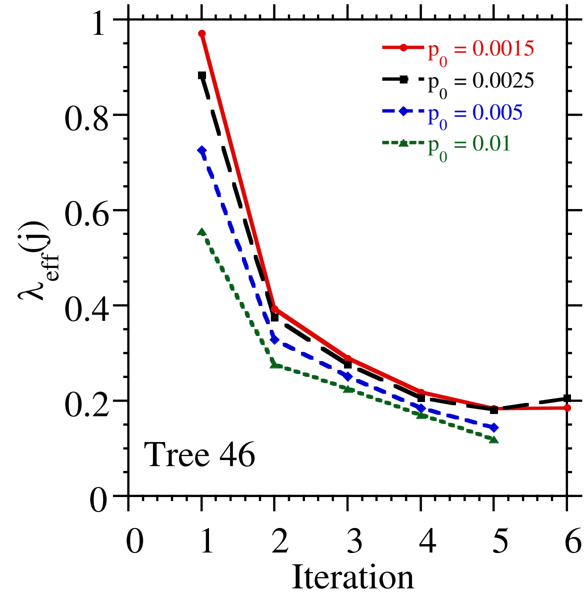

Figure 6: $\lambda_{eff}(j)$ as a function of iteration number for different values of the probability of an accidentally triggered line outage.

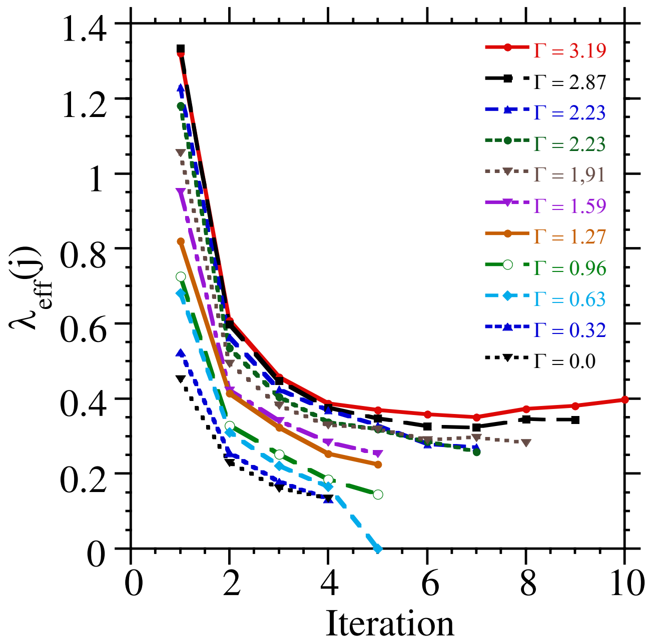

Figure 7: $\lambda_{eff}(j)$ as a function of iteration number for different values of Γ for the 46-node tree network.

Let us now consider the sequence of calculations in which Γ is varied for the 46-node tree network. We have seen that by varying Γ we can change the blackout events from a single failure to cascading events (Fig. 3). In Fig. 7, we have plotted $\lambda_{eff}(j)$ versus *j* for these different values of Γ. We can see that $\lambda_{eff}(j)$ increases uniformly with Γ. Also, the dependence on the iteration number, j > 1, becomes weaker. This may reflect the change in the dynamics going from blackouts dominated by generation limitations to blackouts that are dominated by line outages. The comparison with the

CASCADE model is relevant in the latter regime. The existence of a single λ describing the cascade process is one of the more significant results of these comparisons.

The dependence of λ_{eff} on j is not just a peculiarity of the structure of the ideal tree networks. In Fig. 8, we show the calculated $\lambda_{eff}(j)$ for the IEEE 118 bus network [7]. We can see that $\lambda_{eff}(j)$ is also weakly dependent on j for $j > 1$.

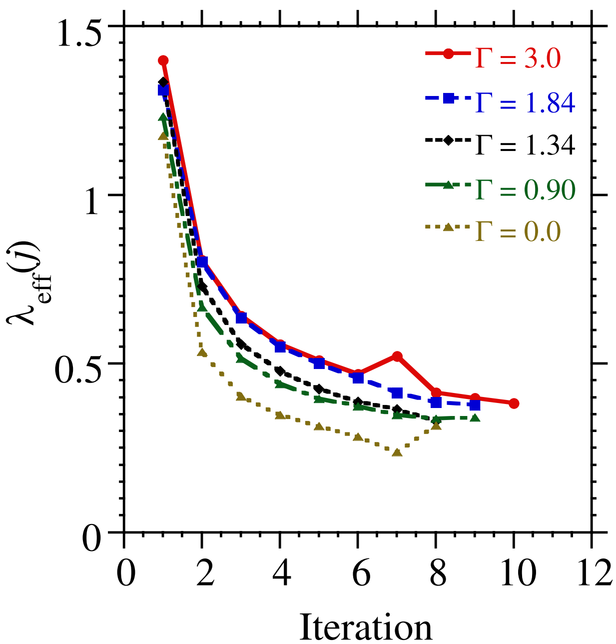

Figure 8: $\lambda_{eff}(j)$ as a function of iteration number for six values of Γ for the IEEE 118 bus network.

It is not surprising that $\lambda_{eff}(j)$ is larger for the first iteration than for the following ones. In the OPA model, unlike in the CASCADE model, there is power shed during each iteration. This power shed reduces the stress over the system and accordingly reduces the probability of line outages at high iterations. Therefore, we believe that the value of $\lambda_{eff}(j)$ for $j = 1$ is the most significant one to be compared with the parameters of the CASCADE model.

We can summarize the stability properties to cascading events of these networks by plotting in the Γ-μ plane the line $\lambda_{eff}(1) = 1$. This line gives the demarcation between the region with $\lambda_{eff}(1) > 1$, where cascading events are possible, and $\lambda_{eff}(1) < 1$, where the cascading events are suppressed. Such a plot is shown in Fig. 9 for the 46-node and 94-node tree networks and for the IEEE 118 bus network. The position of the line $\lambda_{eff}(1) = 1$ in the Γ-μ plane changes with the network configuration, but the three networks show a very similar structure.

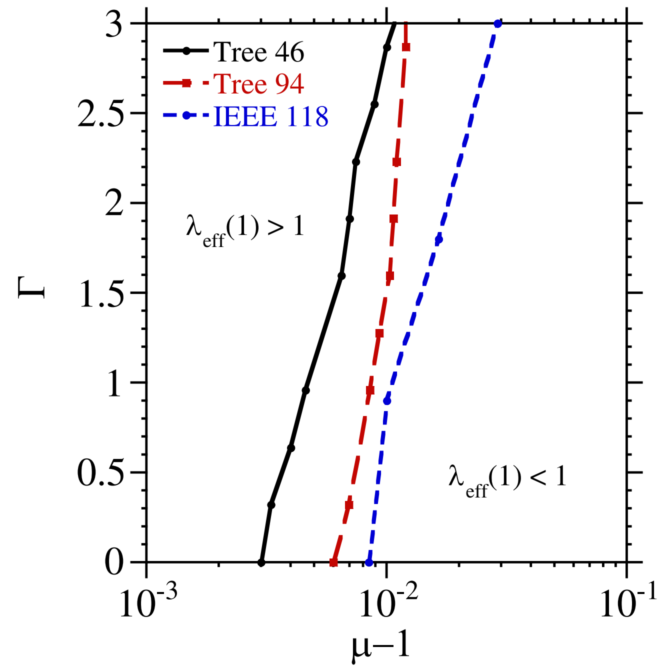

Figure 9: $\lambda_{eff}(1) = 1$ in the Γ−μ plane for the 46-node and 94-node tree networks and for the IEEE 118 bus network.

5. Load transfer during a cascading event

Another interpretation of the parameter λ in the CASCADE model is the total load transfer associated with a failing line. To calculate this transfer load, we use a tree network and we cause a single line outage at a time. We operate at very low power to prevent any of the M_i s from reaching 1 after the chosen line outage because that can cause a reorganization of the power that leads to a different solution. For each line outage, we calculate the effective λ_{0j} in the following way:

$$\lambda_{0j} = \frac{1}{M_j^0} \sum_{i=1}^{\bar{N}_L} \left(M_i^1 - M_i^0 \right) \quad , \tag{11}$$

where, $\bar{N}_L$ is the number of lines minus 1 because there is only one line outage. The superscript of the M_i's indicates step zero, the value of the M_i before the line outage, or step 1, after the line outage. The transfer load is normalized to M_j because we need the value of the transferred load when $M_j = 1$. This calculation is more elaborate than calculation of a standard line-outage power-distribution factor because the generation redispatches after the line outage.

We calculate λ_{0j} for each line j of the network and repeat the calculation n times for different random values of the loads. Then, we average λ_{0j} over the lines and over the calculated n samples. This gives us another determination of the effective λ, $\langle \lambda_0 \rangle$. We have done the calculation of $\langle \lambda_0 \rangle$ for the tree 46 configuration and several values of Γ. In Fig. 10, we compare these results to the $\lambda_{eff}(1)$ calculated in the previous section. We can see that the values are quite similar. This result is interesting because this method for determining $\langle \lambda_0 \rangle$ can be applied to a real power transmission network and this parameter can be used as an alternative way of determining how close a system is to the cascading threshold.

6. Conclusions

The CASCADE model gives a simple characterization for the transition from an isolated failure to a system-wide collapse. The characterization of this transition is very important, not only for power systems but for any large, man-made, networked system. The control parameter for this transition is directly related to the load transfer during cascading events. In real systems, perhaps more than one parameter can characterize this transition. Here, we have looked for ways of determining this control parameter for power transmission systems to quantify the way in which cascading failures propagate.

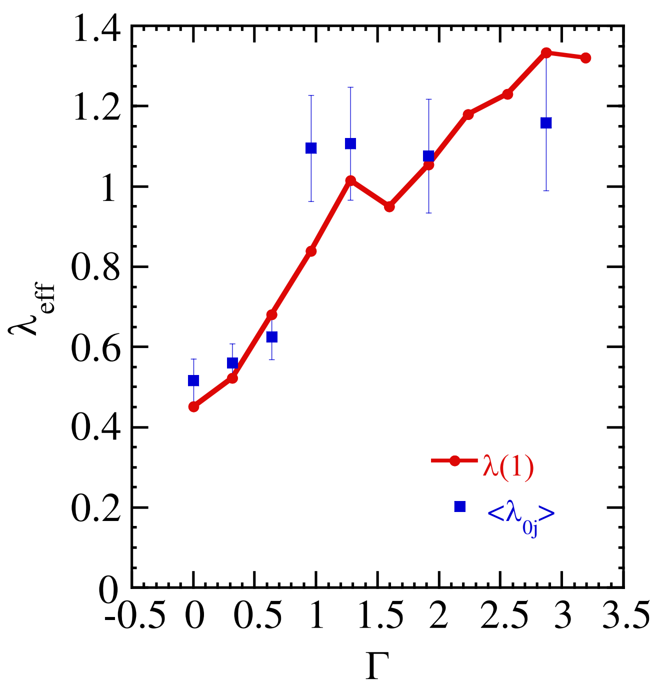

Figure 10: Comparison of the calculated $\lambda_{eff}(1)$ using the two methods discussed in this paper.

The OPA model gives a test bed to apply some of the concepts developed in the simpler probabilistic models. Using an analogy between the two types of models, we have been able to identify a similar transition from an isolated failure to a system-wide collapse in OPA. Furthermore, in defining the transition between these two operational regimes, we have been able to correlate the two parameters Γ and μ, which are related to the operation of the system, to $\langle \lambda_0 \rangle$, which can be determined for a real power transmission system. The relationship between those parameters and the threshold for cascading failure may lead to some practical criteria that will be applicable to the design and operation of power transmission systems.

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6.3 Probabilistic load-dependent cascading failure with limited component interactions

I. Dobson, B.A. Carreras, D.E. Newman

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PROBABILISTIC LOAD-DEPENDENT CASCADING FAILURE WITH LIMITED COMPONENT INTERACTIONS

Ian Dobson
ECE Department
University of Wisconsin
Madison WI 53706
dobson@engr.wisc.edu

Benjamin A. Carreras
Oak Ridge National Lab
Oak Ridge TN 37831
carrerasba@ornl.gov

David E. Newman
Physics Department
University of Alaska
Fairbanks AK 99775
ffden@uaf.edu

ABSTRACT

We generalize an analytically solvable probabilistic model of cascading failure in which failing components interact with other components by increasing their load and hence their chance of failure. In the generalized model, instead of a failing component increasing the load of all components, it increases the load of a random sample of the components. The size of the sample describes the extent of component interactions within the system. The generalized model is approximated by a saturating branching process and this leads to a criticality condition for cascading failure propagation that depends on the size of the sample. The criticality condition shows how the extent of component interactions controls the proximity to catastrophic cascading failure. Implications for the complexity of power transmission system design to avoid cascading blackouts are briefly discussed.

1. INTRODUCTION

Industrialized society depends heavily on complicated infrastructure systems with many interconnected components. These infrastructures can suffer widespread failures when stressed components fail successively, with each failure further stressing the system and making further failures more likely. For example, a long, intricate cascade of events caused the August 2003 blackout of a substantial portion of the electrical power system of Northeastern North America affecting fifty million people. The vital importance of the electrical power infrastructure to society motivates the study of models that capture salient features of cascading failure.

Previous work [3, 4, 5] introduced a probabilistic model of cascading failure with a large number of identical components called CASCADE. The components fail when their load exceeds a threshold, and become more loaded when any other component fails. The components initially have a random load and the cascade is started by an initial disturbance increasing the loading of all components. The number of components failed is a measure of the size of the cascade and it has an analytic probability distribution (a saturating form of the quasibinomial distribution). The CASCADE model can be approximated by a saturating Poisson branching process [6] and the relation of these models to cascading failure in simulated blackouts of power transmission systems is studied in [2, 3]. CASCADE is an abstract model of cascading failure and one of its purposes is helping to explain the results of power system models of cascading failure blackouts that represent the transmission network and generation redispatch [1, 3].

The CASCADE model (and its branching process approximation) show interesting behavior as the average initial component load is increased. In one scenario, as this loading is increased, the average number of failures sharply increases at a critical loading. Moreover, at this critical loading, the probability distribution of the number of failures has a power tail of exponent approximately −1.5. The critical loading marks a phase transition and an operational boundary with respect to cascading failure. That is, the risk of cascading failure becomes significant at or above the critical loading. Studying this criticality and finding ways to monitor and detect the corresponding criticality in more detailed simulation models or in real infrastructure systems is a promising new direction of research [6, 2].

One significant limitation of the CASCADE model is the assumption that all components interact. That is, when one component fails, the loading of *all* other components is increased. In applications such as blackouts, many thousands of components can interact by a variety of mechanisms and the interactions can sometimes span the entire system. However, it is more realistic to assume that when one component fails, it interacts with only a subset of the

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other components. This paper generalizes the CASCADE model to this limited interaction case and derives the new criticality condition from the branching process approximation to the generalized model. The result has implications for the interesting question of whether new system technologies that improve system performance by increased communication and coordination between system components introduce many unlikely failure modes that could increase the risk of catastrophic cascading failure [8].

2. CASCADE MODEL WITH k INTERACTIONS

This section summarizes the generalized CASCADE model. There are n identical components with random initial loads. For each component the minimum initial load is $L^{\min}$ and the maximum initial load is $L^{\max}$. Component j has initial load L_j that is a random variable uniformly distributed in $[L^{\min}, L^{\max}]$. $L_1, L_2, \cdots, L_n$ are independent.

Components fail when their load exceeds L^{fail}. When a component fails, a fixed amount of load P is transferred to k samples of the n components. The sampling is uniform so that the probability of choosing a particular component is $1/n$ and the components are sampled independently and with replacement. Moreover, the k samples are chosen independently for each failure.

To start the cascade, an initial disturbance loads k samples of the components by an additional amount D. Other components may then fail depending on their initial loads L_j and the failure of each of these components will distribute an additional load $P \geq 0$ that can cause further failures in a cascade.

It is useful to normalize the model so that $L^{\min}$ becomes zero and both $L^{\max}$ and L^{fail} become one [4, 5]. The normalized initial load $\ell_j = (L_j - L^{\min})/(L^{\max} - L^{\min})$ so that ℓ_j is a random variable uniformly distributed on $[0, 1]$. Let

$$p = \frac{P}{L^{\max} - L^{\min}}, \qquad d = \frac{D + L^{\max} - L^{\text{fail}}}{L^{\max} - L^{\min}} \tag{1}$$

Then p is the amount of load increase on any component when one other component fails expressed as a fraction of the load range $L^{\max} - L^{\min}$. d is the initial disturbance shifted by $L^{\max} - L^{\text{fail}}$ expressed as a fraction of the load range. (The shift ensures that the failure load is one [4, 5].)

The model produces failures in stages $i = 0, 1, 2, ...$ where M_i is the number of failures in stage i. It is convenient to state the normalized version of the algorithm. This can be obtained from [4] by adding the random sampling.

Algorithm for normalized CASCADE with k interactions

0. All n components are initially unfailed and have initial loads $\ell_1, \ell_2, \cdots, \ell_n$ determined as independent random variables uniformly distributed in $[0, 1]$.
1. Uniformly sample components k times independently with replacement and add the initial disturbance d to the load of a component each time it is sampled. Initialize the stage counter i to zero.
2. Test each unfailed component for failure: For $j = 1, ..., n$, if component j is unfailed and its load > 1 then component j fails. Suppose that M_i components fail in this step.
3. Independently for each of the M_i failures, uniformly sample components k times independently with replacement and add p to the load of a component each time it is sampled.
4. Increment the stage counter i and go to step 2.

3. BRANCHING PROCESS APPROXIMATION

In a Poisson branching process model of cascading failure, failures are produced in stages. Each failure at a given stage produces further next stage failures independently according to a Poisson distribution of rate λ. This section derives the Poisson branching process approximation of the generalized CASCADE model and shows that $\lambda = kp$. Thus $\lambda = kp$ governs the propagation of failures in the cascading process. The implications are discussed in section 4. Those readers interested in the details of the approximation in this section should read the simpler case in [6] first.

Consider the end of step 2 of stage $i \geq 1$ in the CASCADE algorithm. The failures that have occurred are $M_0 = m_0, M_1 = m_1, ..., M_i = m_i$, but component loads have not yet been incremented in the following step 3. Let T_{ji} be the number of times component j is sampled in the km_i samples of step 3 of stage i. Then the marginal distributions of $T_{ji}, j = 1, ..., n$ are binomial so that

$$P[T_{ji} = t \mid M_i = m_i] = \binom{km_i}{t}\left(\frac{1}{n}\right)^t \left(1 - \frac{1}{n}\right)^{km_i - t} \tag{2}$$

$$E[T_{ji} \mid M_i = m_i] = km_i/n \tag{3}$$

$$\text{Var}[T_{ji} \mid M_i = m_i] = (km_i/n)(1 - 1/n) \tag{4}$$

Write $\underline{T}_{ni} = (T_{1i}, T_{2i}, ..., T_{ni})$, $\underline{M}_i = (M_0, M_1, ..., M_i)$, $\underline{T}_i = (\underline{T}_{n0}, \underline{T}_{n1}, ..., \underline{T}_{ni})$, $S_i = M_0 + M_1 + ... + M_i$, $\Sigma_{ji} = T_{j1} + T_{j2} + ... + T_{ji}$, and use the corresponding lower case notation for the symbols $\underline{m}_i$, s_i, $\underline{t}_{ni}$, $\underline{t}_i$ and σ_{ji}. The complete history of the component sampling at step 3 of stage i is $\underline{T}_i = \underline{t}_i$.

Define α_{ji} and the saturation function ϕ as

$$\alpha_{ji} = \begin{cases} 0 & \text{; component } j \text{ failed before stage } i \\ \dfrac{pt_{ji}}{1 - dt_{j0} - p\sigma_{j(i-1)}} & \text{; component } j \text{ unfailed at beginning of stage } i \end{cases}$$

$$\phi(x) = \begin{cases} 0 \,; x < 0 \\ x \,; 0 \leq x \leq 1 \\ 1 \,; x > 1 \end{cases}$$

Consider unfailed component j and suppose its total stage i, step 2 additional load $dt_{j0} + p\sigma_{j(i-1)} < 1$. Then, when conditioned on $\underline{T}_{i-1} = \underline{t}_{i-1}$, the load of component j is uniformly distributed in $[dt_{j0} + p\sigma_{j(i-1)}, 1]$. In the following step 3, the probability that the load increment of pt_{ji} causes component j to fail is $\phi(\alpha_{ji})$. Now suppose that $dt_{j0} + p\sigma_{j(i-1)} \geq 1$. Then the probability that component j fails is $\phi(\alpha_{ji}) = 1$.

When conditioned on $\underline{T}_i = \underline{t}_i$, the component failures in step 2 of stage $i+1$ are independent and hence M_{i+1} has generating function

$$Ee^{z[M_{i+1}|\underline{T}_i]} = \prod_{j=1}^{n}(1 + (z-1)\phi(\alpha_{ji})) \tag{5}$$

Since $P[M_{i+1} = m_{i+1}|\underline{M}_i]$

$$= \sum_{\underline{t}_i} P[M_{i+1} = m_{i+1}|\underline{M}_i, \underline{T}_i = \underline{t}_i]P[\underline{T}_i = \underline{t}_i|\underline{M}_i]$$
$$= \sum_{\underline{t}_i} P[M_{i+1} = m_{i+1}|\underline{T}_i = \underline{t}_i]P[\underline{T}_i = \underline{t}_i|\underline{M}_i] \ ,$$

$$Ee^{z[M_{i+1}|\underline{M}_i]} = \sum_{\underline{t}_i} Ee^{z[M_{i+1}|\underline{T}_i]}P[\underline{T}_i = \underline{t}_i|\underline{M}_i]$$
$$= \sum_{\underline{t}_{i-1}} A_i P[\underline{T}_{i-1} = \underline{t}_{i-1}|\underline{M}_i] \tag{6}$$

where $A_i = \prod_{j=1}^{n}\sum_{\underline{t}_{ni}}(1 + (z-1)\phi(\alpha_{ji}))P[\underline{T}_{ni} = \underline{t}_{ni}|\underline{M}_i]$

$$= \prod_{j=1}^{n}\sum_{t_{ji}}(1 + (z-1)\phi(\alpha_{ji}))P[T_{ji} = t_{ji}|\underline{M}_i].$$

Define $X_{ji} = dt_{j0} + p\sigma_{ji}$. Then $X_{ji} \geq 1 \iff pt_{ji} \geq 1 - dt_{j0} - p\sigma_{j(i-1)} \iff \phi(\alpha_{ji}) = 1$. Using (3) and (4),

$$E[X_{ji} \mid \underline{M}_i] = \frac{kd + kps_i}{n} \tag{7}$$

$$\mathrm{Var}[X_{ji} \mid \underline{M}_i] = \frac{kd^2 + kp^2 s_i}{n}\Big(1 - \frac{1}{n}\Big) \tag{8}$$

It is convenient to renumber the components so that components $1, 2, ..., S_i$ are the S_i components that have failed in previous stages. Then $\alpha_{ji} = 0$ for $j = 1, 2, ..., S_i$. Moreover $A_i = \prod_{j=S_i+1}^{n} B_{ji}$ where

$$B_{ji} = \sum_{t_{ji}}\Big[(1 + (z-1)\alpha_{ji})P[T_{ji} = t_{ji}, X_{ji} < 1|\underline{M}_i]$$
$$+zP[T_{ji} = t_{ji}, X_{ji} \geq 1|\underline{M}_i]\Big]$$
$$= \left(1 + (z-1)\frac{pE[T_{ji}|X_{ji} < 1, \underline{M}_i]}{1 - dt_{j0} - p\sigma_{j(i-1)}}\right)$$
$$P[X_{ji} < 1|\underline{M}_i] + zP[X_{ji} \geq 1|\underline{M}_i] \tag{9}$$

Let $kp = \lambda$ and $kd = \theta$ and k/n be fixed and let $n, k \to \infty$ and $p, d \to 0$. If $E[X_{ji}] < 1$, using (7), (8) and (3),

$$P[X_{ji} \geq 1|\underline{M}_i] \leq P[|X_{ji} - E[X_{ji}]| \geq |1 - E[X_{ji}]| \,|\underline{M}_i]$$
$$\leq \frac{\mathrm{Var}[X_{ji}\,|\underline{M}_i]}{(1 - E[X_{ji}\,|\underline{M}_i])^2} \leq \frac{(n/k)(\theta^2 + \lambda^2 s_i)}{(n - \theta - \lambda s_i)^2} \to 0$$

and $E[T_{ji}|X_{ji} < 1, \underline{M}_i] \to E[T_{ji} \mid \underline{M}_i] = km_i/n$.
Similarly, if $E[X_{ji}] > 1$, $P[X_{ji} < 1 \mid \underline{M}_i] \to 0$.
Thus $P[X_{ji} < 1|\underline{M}_i] \to I[E[X_{ji}] < 1]$ and

$$B_{ji} \sim \Big(1 + \frac{z-1}{n}\lambda m_i\Big)I[E[X_{ji}] < 1] + zI[E[X_{ji}] > 1].$$

Now $E[X_{ji}] < 1 \iff kd/n + kps_i/n < 1 \iff \theta + \lambda s_i < n$.

If $s_i < (n-\theta)/\lambda$, since $(1+\frac{z-1}{n}\lambda m_i)^{n-s_i} \to e^{\lambda m_i(z-1)}$, $A_i \to e^{\lambda m_i(z-1)}$. Moreover, since the limit of A_i is independent of t_{i-1}, (6) implies that $Ee^{z[M_{i+1}|\underline{M}_i]} \to e^{\lambda m_i(z-1)}$. If $s_i > (n-\theta)/\lambda$, $A_i \to z^{n-s_i}$. Therefore, similarly to [6], we can approximate

$$Ee^{z[M_{i+1}|\underline{M}_i = \underline{m}_i]} \approx$$
$$\begin{cases} \left[e^{\lambda m_i(z-1)}\right]^{\dagger} + z^{n-s_i}\left(1 - \left[e^{m_i\lambda(z-1)}\right]^{\dagger}(1)\right); \\ \qquad\qquad\qquad s_i < (n-\theta)/\lambda, \\ z^{n-s_i}; \qquad s_i > (n-\theta)/\lambda. \end{cases} \tag{10}$$

where $[p(z)]^{\dagger}$ denotes terms of $p(z)$ of degree $\leq n - s_i - 1$.

Since $e^{m_i\lambda(s-1)} = \left(e^{\lambda(s-1)}\right)^{m_i}$, (10) is the distribution of the sum of m_i independent Poisson random variables with rate λ with saturation occurring when the total number of failures exceeds n [6]. Thus we can consider each failure as independently causing other failures in the next stage according to a saturating Poisson Galton-Watson branching process with rate $\lambda = kp$. (This result is the same for the original CASCADE model, except that in the original CASCADE model, $\lambda = np$ [6].)

The failures produced by the initial disturbance when $i = 0$ can also be approximated by a saturating Poisson distribution with rate θ.

4. CRITICALITY CONDITION & IMPLICATIONS

Galton-Watson branching processes proceed in stages to randomly generate an average of λ failures from each failure in the previous stage. It is well known [7] that the criticality condition for branching processes is $\lambda = 1$, and this conclusion also applies to saturating branching processes [6] and in particular to the saturating branching process derived in the previous section. λ governs the propagation of failures so that for $\lambda < 1$ the propagation of failures is likely to be limited, whereas for $\lambda > 1$ there is a high probability of propagation of failures to the entire system. Thus criticality

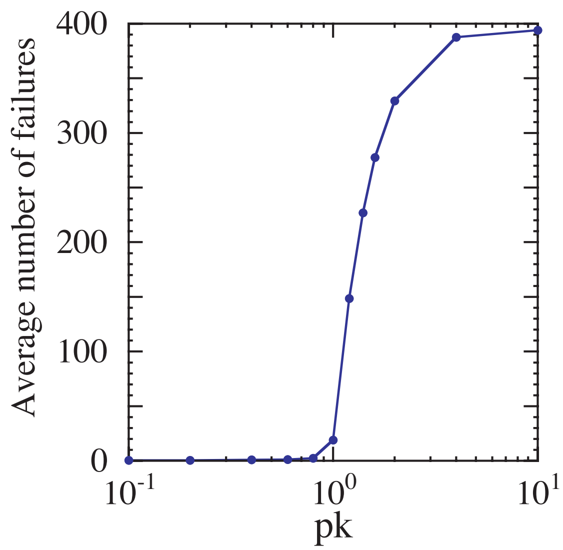

Fig. 1. Average number of failures versus pk as p is varied showing change in gradient at critical point pk=1. There are n=1000 components and the sample size k=100.

in the generalized CASCADE model occurs approximately at

$$\lambda = kp = 1 \tag{11}$$

Simulations of the generalized CASCADE model confirm (11). Figure 1 shows the sharp change at $kp = 1$ in the rate of increase of average number of components failed as initial average load is increased. (According to (1), fixing $L^{\max}$ and increasing average initial load $(L^{\max} + L^{\min})/2$ by increasing $L^{\min}$ increases p.) Figure 2 shows the power tail at criticality at $kp = 1$.

As explained in [6], the risk of cascading failure in these models can be minimized by fixing a design limit $\lambda_{\max} < 1$ and requiring $\lambda = kp < \lambda_{\max}$. Then, even if p is very small, large k can cause cascading failure. This suggests that numerous rare interactions between many components can be equally influential in causing cascading failure as a smaller number of likely interactions. Indeed, one can deduce that a design change that introduces a very large number of unlikely failure interactions, thus greatly increasing k, could greatly increase the risk of cascading failure, despite the rarity of the failures (low p). It is conceivable that coupling infrastructures together such as controlling the power transmission system over an internet or certain types of global control schemes could make the system more vulnerable to cascading failure in this fashion. Note that many traditional power system controls are designed to reduce interactions by deliberate separation in distance, frequency, and time scale. Thus the reliability concerns for the effect on cascading failure risk of complicated interconnecting solutions raised by (11) may be consistent with traditional power engineering practice. Our analysis of cascading failure risk is indeed highly approximate and global in nature, but it starts to quantify trade-offs of complexity versus reliability in engineering large networked systems.

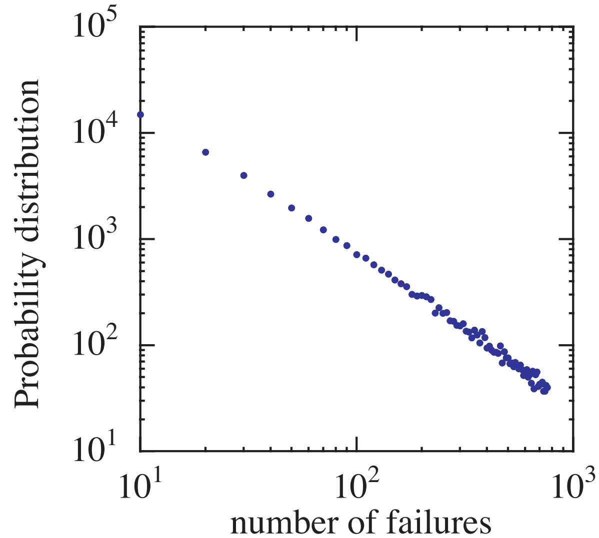

Fig. 2. Probability distribution of number of failures on log-log plot at criticality kp=1. There are n=1000 components.

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6.4 Complex systems analysis of series of blackouts: cascading failure, criticality, and self-organization

I. Dobson, B.A. Carreras, V.E. Lynch

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Complex Systems Analysis of Series of Blackouts: Cascading Failure, Criticality, and Self-organization

Ian Dobson
ECE Department
University of Wisconsin
Madison, WI 53706 USA
dobson@engr.wisc.edu

Benjamin A. Carreras   Vickie E. Lynch
Oak Ridge
National Laboratory
Oak Ridge TN 37831 USA
carreras@fed.ornl.gov

David E. Newman
Physics Department
University of Alaska
Fairbanks AK 99775 USA
ffden@uaf.edu

Abstract

We give a comprehensive account of a complex systems approach to large blackouts caused by cascading failure. Instead of looking at the details of particular blackouts, we study the statistics, dynamics and risk of series of blackouts with approximate global models. North American blackout data suggests that the frequency of large blackouts is governed by a power law. This result is consistent with the power system being a complex system designed and operated near criticality. The power law makes the risk of large blackouts consequential and implies the need for nonstandard risk analysis.

Power system overall load relative to operating limits is a key factor affecting the risk of cascading failure. Blackout models and an abstract model of cascading failure show that there are critical transitions as load is increased. Power law behavior can be observed at these transitions.

The critical loads at which blackout risk sharply increases are identifiable thresholds for cascading failure and we discuss approaches to computing the proximity to cascading failure using these thresholds. Approximating cascading failure as a branching process suggests ways to compute and monitor criticality by quantifying how much failures propagate.

Inspired by concepts from self-organized criticality, we suggest that power system operating margins evolve slowly to near criticality and confirm this idea using a blackout model. Mitigation of blackout risk should take care to account for counter-intuitive effects in complex self-organized critical systems. For example, suppressing small blackouts could lead the system to be operated closer to the edge and ultimately increase the risk of large blackouts.

1 Introduction

Cascading failure is the usual mechanism for large blackouts of electric power transmission systems. For example, long, intricate cascades of events caused the August 1996 blackout in Northwestern America (NERC [44]) that disconnected 30,390 MW of power to 7.5 million customers [41, 57]). An even more spectacular example is the August 2003 blackout in Northeastern America that disconnected 61,800 MW of power to an area spanning 8 states and 2 provinces and containing 50 million people [56]. The vital importance of the electrical infrastructure to society motivates the understanding and analysis of large blackouts.

Here are some substantial challenges:

- North American power transmission system data appears to show power tails in the probability distribution of blackout sizes, making the risk of large blackouts consequential. What is the origin and the implications of this distribution of blackout sizes? Can this probability distribution be changed within economic and engineering constraints to minimize the risk of blackouts of all sizes?
- Large blackouts are typically caused by long, intricate cascading sequences of rare events. Dependencies between the first few events can be assessed for a subset of the most likely or anticipated events and this type of analysis is certainly useful in addressing part of the problem (e.g. [48]). However, this combinatorial analysis gets overwhelmed and becomes infeasible for long sequences of events or for the huge number of all possible rare events and interactions, many of which are unanticipated, that cascade to cause large blackouts. How does one do risk analysis of rare, cascading, catastrophic events? Can one monitor or mitigate the risk of these cascading failures at a more global level without working out all the details?
- Much of the effort in avoiding cascading failure has focussed on reducing the chances of the start of a cascading failure. How do we determine whether power system design and operation is such that cascades will tend to propagate after they have started? That is, where is the "edge" for propagation of cascading failure?

The aim of global complex systems analysis of power system blackouts is to provide new insights and approaches that could address these challenges. We focus on global bulk properties of series of blackouts rather than on the details of a particular blackout. Concepts from complex systems, statistical physics, probability and risk analysis are combined with power system modeling to study blackouts from a top-down perspective.

1.1 Literature review

We briefly review some other approaches to complex systems and cascading failure in power system blackouts.

Chen and Thorp [17] and Chen, Thorp, and Dobson [18] model power system blackouts using the DC load flow approximation and standard linear programming optimization of the generation dispatch and represent in detail hidden failures of the protection system. The expected blackout size is obtained using importance sampling and it shows some indications of a critical point as loading is increased. The distribution of power system blackout size is obtained by rare event sampling and blackout risk assessment and mitigation methods are studied. Rios, Kirschen, Jawayeera, Nedic, and Allan [51] evaluate expected blackout cost using Monte Carlo simulation of a power system model that represents the effects of cascading line overloads, hidden failures of the protection system, power system dynamic instabilities, and the operator responses to these phenomena. Kirschen, Jawayeera, Nedic, and Allan [40] then apply correlated sampling and their Monte Carlo simulation to develop a calibrated reference scale of system stress that relates system loading to blackout size and test it on a 1000 bus power system. Hardiman, Kumbale, and Makarov [35] simulate and analyze cascading failure using the TRELSS software. In its "simulation approach" mode, TRELSS represents cascading outages of lines, transformers and generators due to overloads and voltage violations in large AC networks (up to 13000 buses). Protection control groups and islanding are modeled in detail. The cascading outages are ranked in severity and the results have been applied in industry to evaluate transmission expansion plans. Other modes of operation are available in TRELSS that can rank the worst contingencies and take into account remedial actions and compute reliability indices.

Ni, McCalley, Vittal, and Tayyib [48] evaluate expected contingency severities based on real time predictions of the power system state to quantify the risk of operational conditions. The computations account for current and voltage limits, cascading line overloads, and voltage instability. Zima and Andersson [59] study the transition into subsequent failures after an initial failure and suggest mitigating this transition with a wide-area measurement system.

Roy, Asavathiratham, Lesieutre, and Verghese [52] construct randomly generated tree networks that abstractly represent influences between idealized components. Components can be failed or operational according to a Markov model that represent both internal component failure and repair processes and influences between components that cause failure propagation. The effects of the network degree and the inter-component influences on the failure size and duration are studied. Pepyne, Panayiotou, Cassandras, and Ho [50] also use a Markov model for discrete state power system nodal components, but propagate failures along the transmission lines of a power systems network with a fixed probability. They study the effect of the propagation probability and maintenance policies that reduce the probability of hidden failures.

The challenging problem of determining cascading failure due to dynamic transients in hybrid nonlinear differential equation models is addressed by DeMarco [24] using Lyapunov methods applied to a smoothed model and by Parrilo, Lall, Paganini, Verghese, Lesieutre, and Marsden [49] using Karhunen-Loeve and Galerkin model reduction. Watts [58] describes a general model of cascading failure in which failures propagate through the edges of a random network. Network nodes have a random threshold and fail when this threshold is exceeded by a sufficient fraction of failed nodes one edge away. Phase transitions causing large cascades can occur when the network becomes critically connected by having sufficient average degree or when a highly connected network has sufficiently low average degree so that the effect of a single failure is not swamped by a high connectivity to unfailed nodes. Lindley and Singpurwalla [42] describe some foundations for causal and cascading failure in infrastructures and model cascading failure as an increase in a component failure rate within a time interval after another component fails.

Chen and McCalley [19] fit the empirical probability distribution of 20 years of North American multiple line failures with a cluster distribution derived from a negative binomial probability model. Carlson and Doyle have introduced a theory of highly optimized tolerance (HOT) that describes power law behavior in a number of engineered or otherwise optimized applications [6]. Stubna and Fowler [55] published an alternative view based on HOT of the origin of the power law in the NERC data. To apply HOT to the power system, it is assumed that blackouts propagate one dimensionally [55] and that this propagation is limited by finite resources that are engineered to be optimally distributed to act as barriers to the propagation [6]. The one dimensional assumption implies that the blackout size in a local region is inversely proportional to the local resources. Minimizing a blackout cost proportional to blackout size subject to a fixed sum of resources leads to a probability distribution of blackout sizes with an asymptotic power tail and two free parameters. The asymptotic power tail exponent is exactly –1 and this value follows from the one dimensional assumption. The free parameters can be varied to fit the NERC data for both MW lost and customers disconnected. However [55] shows that a better fit to both these data sets can be achieved by modifying HOT to allow some misallocation of resources.

The historically high reliability of power transmission systems in developed countries is largely due to estimating the transmission system capability and designing and operating the system with margins with respect to a chosen subset of likely and serious contingencies. The analysis is usually either deterministic analysis of estimated worst cases or Monte Carlo simulation of moderately detailed probabilistic models that capture steady state interactions [4]. Combinations of likely contingencies and some dependencies between events such as common mode or common cause are sometimes considered. The analyses address the first few likely and anticipated failures rather than the propagation of many rare or unanticipated failures in a cascade.

1.2 Blackout mechanisms

We review cascading failure mechanisms of large blackouts to provide context for the cascading failure modeling. Bulk electrical power transmission systems are complex networks of large numbers of components that interact in diverse ways. When component operating limits are exceeded protection acts and the component "fails" in the sense of not being available to transmit power. Components can also fail in the sense of misoperation or damage due to aging, fire, weather, poor maintenance or incorrect design or operating settings. In any case, the failure causes a transient and causes the power flow in the component to be redistributed to other components according to circuit laws, and subsequently redistributed according to automatic and manual control actions. The transients and readjustments of the system can be local in effect or can involve components far away, so that a component disconnection or failure can effectively increase the loading of many other components throughout the network. In particular, the propagation of failures is not limited to adjacent network components. The interactions involved are diverse and include deviations in power flows, frequency, and voltage as well as operation or misoperation of protection devices, controls, operator procedures and monitoring and alarm systems. However, all the interactions between component failures tend to be stronger when components are highly loaded. For example, if a more highly loaded transmission line fails, it produces a larger transient, there is a larger amount of power to redistribute to other components, and failures in nearby protection devices are more likely. Moreover, if the overall system is more highly loaded, components have smaller margins so they can tolerate smaller increases in load before failure, the system nonlinearities and dynamical couplings increase, and the system operators have fewer options and more stress.

A typical large blackout has an initial disturbance or trigger events followed by a sequence of cascading events. Each event further weakens and stresses the system and makes subsequent events more likely. Examples of an initial disturbance are short circuits of transmission lines through untrimmed trees, protection device misoperation, and bad weather. The blackout events and interactions are often rare, unusual, or unanticipated because the likely and anticipated failures are already routinely accounted for in power system design and operation. The complexity is such that it can take months after a large blackout to sift through the records, establish the events occurring and reproduce with computer simulations and hindsight a causal sequence of events.

2 Blackout data and risk

2.1 Power tails in North American blackout data

We consider the statistics of series of blackouts. The North American Electrical Reliability Council (NERC) has a documented list summarizing major blackouts of the North American power transmission system from 1984 to 1998 [45]. It is apparent that large blackouts are rarer than small blackouts, but how much rarer are they? One might expect a probability distribution of blackout sizes to fall off at most exponentially as the blackout size increases. However, analyses of the NERC data show that the probability distribution of the blackout sizes does not decrease exponentially with the size of the blackout, but rather has a power law tail [15, 7, 8, 16].

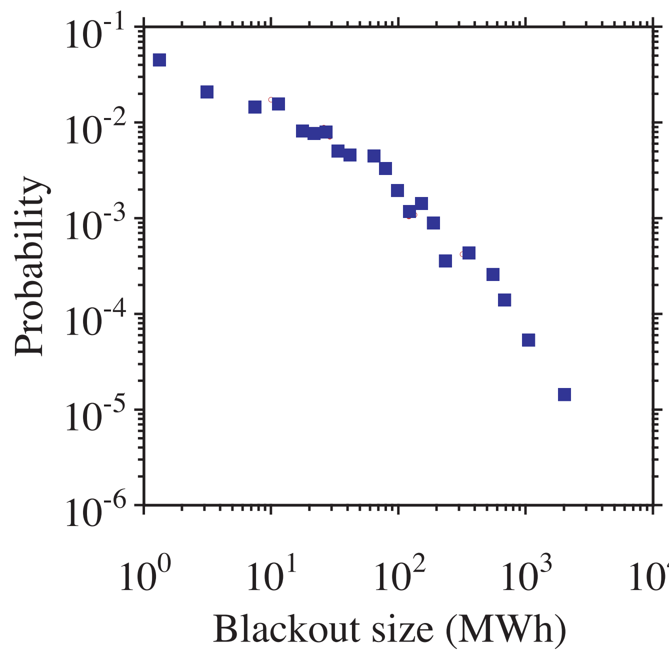

Fig. 1: Log-log plot of scaled PDF of energy unserved during North American blackouts 1984 to 1998.

For example, Fig. 1 plots on a log-log scale the empirical probability distribution of energy unserved in the North American blackouts. The fall-off with blackout size is close to a power dependence with an exponent between -1 and -2. (A power dependence with exponent -1 implies that doubling the blackout size only halves the probability and appears on a log-log plot as a straight line of slope -1). Thus the NERC data suggests that large blackouts are much more likely than might be expected. The power tails are of course limited in extent in a practical power system by a finite cutoff near system size corresponding to the largest possible blackout.

2.2 Blackout risk with respect to blackout size

Blackout risk is the product of blackout probability and blackout cost. Here we assume that blackout cost is roughly proportional to blackout size, although larger blackouts may well have costs (especially indirect costs) that increase faster than linearly. In the case of the exponential tail, large blackouts become rarer much faster than blackout costs increase so that the risk of large blackouts is negligible. However, in the case of a power law tail, the larger blackouts can become rarer at a similar rate as costs increase, and then the risk of large blackouts is comparable to, or even exceeding, the risk of small blackouts [12]. Thus power laws in blackout size distributions significantly affect the risk of large blackouts. Standard probabilistic techniques that assume independence between events imply exponential tails and are not applicable to systems that exhibit power tails.

Consideration of the probability distribution of blackout sizes leads naturally to a more sophisticated framing of the problem of avoiding blackouts. Instead of seeking only to limit blackouts in general, one can seek to manipulate the probability distribution of blackouts to jointly limit the frequency of small, medium and large blackouts. This elaboration is important because measures taken to limit the frequency of small blackouts may inadvertently increase the frequency of large blackouts when the complex dynamics governing transmission expansion are considered as discussed in section 8.

The strength of our conclusions is naturally somewhat limited by the short time period (15 years) of the available blackout data and the consequent limited resolution of the statistics. To further understand the mechanisms governing the complex dynamics of power system blackouts, modeling of the power system is indicated. We consider both abstract models of cascading failure and a power system blackout model in the following section.

3 Three models of cascading failure

This section summarizes three models of cascading failure that are used to explore aspects of blackouts. The first two models aim to represent some of the salient features of cascading failure in blackouts with an analytically tractable probabilistic model and the third model represents a power transmission system.

1. The CASCADE model is an abstract probabilistic model of cascading failure that captures the weakening of the system as the cascade proceeds [27, 32].
2. The branching process model is a useful approximation to the CASCADE model [28].
3. The OPA model for a fixed network is a power systems model that represents cascading line overloads and outages at the level of DC load flow and LP dispatch of generation [11].

While our main motivation is large blackouts, the abstract CASCADE and branching process models are sufficiently simple and general that they could be applied to cascading failure of other large, interconnected infrastructures [47].

3.1 CASCADE model

The features that the CASCADE model abstracts from the formidable complexities of large blackouts are the large but finite number of components, components that fail when their load exceeds a threshold, an initial disturbance loading the system, and the additional loading of components by the failure of other components. The initial overall system stress is represented by upper and lower bounds on a range of initial component loadings. The model neglects the length of times between events and the diversity of power system components and interactions. Of course, an analytically tractable model is necessarily much too simple to represent with realism all the aspects of cascading failure in blackouts; the objective is rather to help understand some global systems effects that arise in blackouts and in more detailed models of blackouts.

3.1.1 Description of CASCADE model

The CASCADE model [27, 32] has n identical components with random initial loads. For each component the minimum initial load is $L^{\min}$ and the maximum initial load is $L^{\max}$. For j=1,2,...,n, component j has initial load L_j that is a random variable uniformly distributed in $[L^{\min}, L^{\max}]$. $L_1, L_2, \cdots, L_n$ are independent.

Components fail when their load exceeds L^{fail}. When a component fails, a fixed amount of load P is transferred to each of the components.

To start the cascade, we assume an initial disturbance that loads each component by an additional amount D. Other components may then fail depending on their initial loads L_j and the failure of any of these components will distribute an additional load $P \geq 0$ that can cause further failures in a cascade.

Now we define the normalized CASCADE model. The normalized initial load ℓ_j is

$$\ell_j = \frac{L_j - L^{\min}}{L^{\max} - L^{\min}} \tag{1}$$

Then ℓ_j is a random variable uniformly distributed on $[0, 1]$.

Let

$$p = \frac{P}{L^{\max} - L^{\min}}, \qquad d = \frac{D + L^{\max} - L^{\text{fail}}}{L^{\max} - L^{\min}} \tag{2}$$

Then the normalized load increment p is the amount of load increase on any component when one other component fails expressed as a fraction of the load range $L^{\max} - L^{\min}$. The normalized initial disturbance d is a shifted initial disturbance expressed as a fraction of the load range. Moreover, the failure load is $\ell_j = 1$.

3.1.2 Distribution of the number of failures

The distribution of the total number of component failures S is

$$P[S = r] = \begin{cases} \binom{n}{r} \phi(d)(d + rp)^{r-1}(\phi(1 - d - rp))^{n-r}, \\ \qquad\qquad\qquad r = 0, 1, ..., n-1, \\ 1 - \sum_{s=0}^{n-1} P(S = s), \qquad r = n, \end{cases} \tag{3}$$

where $p \geq 0$ and the saturation function is

$$\phi(x) = \begin{cases} 0 \ ; x < 0, \\ x \ ; 0 \leq x \leq 1, \\ 1 \ ; x > 1. \end{cases} \tag{4}$$

When using (3) it is assumed that $0^0 \equiv 1$ and $0/0 \equiv 1$.

If $d \geq 0$ and $d + np \leq 1$, then there is no saturation ($\phi(x) = x$) and (3) reduces to the quasibinomial distribution

$$P[S = r] = \binom{n}{r} d(d + rp)^{r-1}(1 - d - rp)^{n-r}. \tag{5}$$

The quasibinomial distribution was introduced by Consul [21] to model an urn problem in which a player makes strategic decisions and further studied by Burtin [5], Islam, O'Shaughnessy, and Smith [37], and Jaworski [38]. The saturation in (3) extends the parameter range of the quasibinomial distribution and the saturated distribution can represent highly stressed systems with a high probability of all components failing.

3.2 Branching process

The branching process approximation to the CASCADE model gives a way to quantify the propagation of cascading failures with a parameter λ and further simplifies the mathematical modeling [28].

In a Galton-Watson branching process [36, 1] the failures are regarded as produced in stages. The failures in each stage independently produce further failures in the next stage according to a probability distribution with mean λ. An exception is that the first stage assumes a probability distribution with mean θ to represent the initial disturbance. We assume in this section that each failure produces 0,1,2,3,... further failures according to a Poisson distribution. Thus, after the initial disturbance, each failure in each stage independently produces further failures in the next stage according to a Poisson distribution of mean λ.

The branching process is a transient discrete time Markov process and its behavior is governed by the parameter λ. The mean number of failures in stage k is $\theta\lambda^{k-1}$. In the subcritical case of $\lambda < 1$, the failures will die out (i.e., reach and remain at zero failures at some stage) and the mean number of failures in each stage decreases geometrically. In the supercritical case of $\lambda > 1$, although it possible for the process to die out, often the failures increase without bound. Of course, there are a large but finite number of components that can fail in a blackout and in the CASCADE model, so it is also necessary to account for the branching process saturating with all components failed.

The stages of the CASCADE model can be approximated by the stages of a saturating branching process by letting the number of components n become large while p and d become small in such a way that $\lambda = np$ and $\theta = nd$ remain constant. The number S of components failed in the saturating branching process is a saturating form of the generalized Poisson distribution:

For $\theta \geq 0$,

$$P[S = r] = \theta(r\lambda + \theta)^{r-1} \frac{e^{-r\lambda - \theta}}{r!} \quad ; \quad 0 \leq r \leq (n - \theta)/\lambda, \ r < n \tag{6}$$

$$P[S = r] = 0 \ ; \quad (n - \theta)/\lambda < r < n, \ r \geq 0 \tag{7}$$

$$P[S = r] = 1 - \sum_{s=0}^{n-1} g(s, \theta, \lambda, n) \tag{8}$$

In the subcritical or critical case $\lambda \leq 1$, there is no saturation and (6)-(8) reduce to

$$P[S = r] = \theta(r\lambda + \theta)^{r-1} \frac{e^{-r\lambda - \theta}}{r!} \tag{9}$$

which is the generalized (or Lagrangian) Poisson distribution introduced by Consul and Jain [23, 20, 22].

Further approximation of (6)-(8) yields [30]

$$P[S = r] \approx \frac{\theta \, e^{(1-\lambda)\frac{\theta}{\lambda}}}{\lambda\sqrt{2\pi}} r^{-1.5} e^{-r/r_0} \tag{10}$$
$$; 1 \ll r < r_1 = \min\{n/\lambda, n\}, \quad \theta/\lambda \sim 1$$
$$\text{where} \qquad r_0 = (\lambda - 1 - \ln\lambda)^{-1}$$

In the approximation (10), the term $r^{-1.5}$ dominates for $r \lesssim r_0$ and the exponential term e^{-r/r_0} dominates for $r \gtrsim r_0$. Thus (10) reveals that the distribution of the number of failures has an approximate power law region of exponent -1.5 for $1 \ll r \lesssim r_0$ and an exponential tail for $r_0 \lesssim r < r_1$. Note that near criticality, $\lambda \approx 1$ and r_0 becomes large.

For a very general class of branching processes (not necessarily assuming that each failure produces further failures with a Poisson distribution), at criticality, the probability distribution of the total number of failures has a power law form with exponent -1.5. That is, as one doubles the number of failures the probability of that number of failures is divided by $2^{1.5}$. The universality of the -1.5 power law at criticality in the probability distribution of the total number of failures in a branching process suggests that this is a signature for this type of cascading failure. In particular, the generalized Poisson distributions (6)-(8) and (9) have a -1.5 power law at $\lambda = 1$.

The approximation of CASCADE by a branching process implies that the CASCADE model has approximately a -1.5 power law at $np = 1$. Moreover, the -1.5 power law is approximately consistent with the North American blackout data described in section 2.1.

Criticality or supercriticality in the branching process implies a high risk of catastrophic and widespread cascading failures. Maintaining sufficient subcriticality in the branching process according to a simple criterion such as requiring $\lambda < \lambda_{max} < 1$ would limit the propagation of failures and reduce this risk. The approximation of CASCADE as a branching process allows the criterion to be expressed in terms of system loading. However, implementing the criterion to reduce the risk of catastrophic cascading failure would require limiting the system throughput and this is costly. Managing the tradeoff between the certain cost of limiting throughput and the rare but very costly widespread catastrophic cascading failure may be difficult. Indeed we maintain in section 6 that for large blackouts, economic, engineering and societal forces may self-organize the system to criticality and that efforts to mitigate the risk should take account of these broader dynamics [12].

Our emphasis on limiting the propagation of system failures after they are initiated is complementary to more standard methods of mitigating the risk of cascading failure by reducing the risk of the first few likely failures caused by an initial disturbance as for example in [48].

The branching process approximation does capture some salient features of loading dependent cascading failure and suggests an approach to reducing the risk of large cascading failures by limiting the average propagation of failures. However, much work remains to establish the correspondence between these simplified global models and the complexities of cascading failure in real systems.

3.3 OPA blackout model for a fixed network

This section summarizes the OPA blackout model when the network is assumed to be fixed [11]. This model represents blackouts caused by probabilistic cascading line overloads and outages and is used to produce blackout statistics. Lines fail probabilistically and the consequent redistribution of power flows is calculated using the DC load flow approximation and a standard LP dispatch of generation. Cascading line outages leading to blackout are modeled. There is also a version of OPA that additionally represents the complex dynamics as the network evolves and this is discussed in section 6.2.

Cascading failure can happen at any time but tends to be more likely and more widespread at peak load when the network is most stressed. For simplicity, the daily peak load is chosen as representative of the loading during each day and the cascade is computed based on that peak load. Each day has the possibility of one cascade. The lines involved in the cascade are represented but the timing of events is neglected.

The OPA model represents transmission lines, loads and generators with the usual DC load flow assumptions. Starting from a solved base case, blackouts are initiated by random line outages. Whenever a line is outaged, the generation and load is redispatched using standard linear programming methods. The cost function is weighted to ensure that load shedding is avoided where possible. If any lines were overloaded during the optimization, then these lines are outaged with probability p_1. The process of redispatch and testing for outages is iterated until there are no more outages.

The OPA model does not attempt to capture the intricate details of particular blackouts, which may have a large variety of complicated interacting processes also involving, for example, protection systems, dynamics and human factors. However, the OPA model does represent in a simplified way a dynamical process of cascading overloads and outages that is consistent with some basic network and operational constraints.

4 Critical loading

As load increases, it is clear that cascading failure becomes more likely, but exactly how does it become more likely? Our results show that the cascading failure does not gradually and uniformly become more likely; instead there is a point of criticality or phase transition at which the cascading failure becomes more likely.

In complex systems and statistical physics, criticality is associated with power tails in probability distributions. Other indicators of criticality are changes in gradient (for a type 2 phase transition) or a discontinuity (for a type 1 phase transition) in some measured quantity as system passes through the critical point.

The importance of the critical loading is that it defines a reference point for increasing risk of cascading failure. The terminology of "criticality" comes from statistical physics and it is of course extremely useful to use the standard scientific terminology. However, while the power tails at critical loading indicate a substantial risk of large blackouts, it is premature at this stage of risk analysis to automatically presume that operation at criticality is bad because it entails some substantial risks. There is also economic gain from an increased loading of the power transmission system. Indeed, one of the objectives in pursuing the risk analysis of cascading blackouts is to determine and quantify the tradeoffs involved so that sensible decisions about optimal design and operation and blackout mitigation can be made.

4.1 *Qualitative effect of load increase on distribution of blackout size*

Consider cascading failure in a power transmission system in the impractically extreme cases of very low and very high loading. At very low loading, any failures that occur have minimal impact on other components and these other components have large operating margins. Multiple failures are possible, but they are approximately independent so that the probability of multiple failures is approximately the product of the probabilities of each of the failures. Since the blackout size is roughly proportional to the number of failures, the probability distribution of blackout size will have a tail bounded by an exponential. The probability distribution of blackout size is different if the power system were to be operated recklessly at a very high loading in which every component was close to its loading limit. Then any initial disturbance would necessarily cause a cascade of failures leading to total or near total blackout. It is clear that the probability distribution of blackout size must somehow change continuously from the exponential tail form to the certain total blackout form as loading increases from a very low to a very high loading. We are interested in the nature of the transition between these two extremes. Our results presented below suggest that the transition occurs via a critical loading at which there is a power tail in the probability distribution of blackout size. This concept is shown in Figure 2.

4.2 *Critical transitions as load increases in CASCADE*

This subsection describes one way to represent a load increase in the CASCADE model and how this leads to a parameterization of the normalized model. Then the effect of the load increase on the distribution of the number of components failed is described.

We assume for convenience that the system has $n = 1000$ components. Suppose that the system is operated so that the initial component loadings vary from $L^{\min}$ to $L^{\max} = L^{\text{fail}} = 1$. Then the average initial component loading $L = (L^{\min} + 1)/2$ may be increased by increasing $L^{\min}$. The initial disturbance $D = 0.0004$ is assumed to be the same

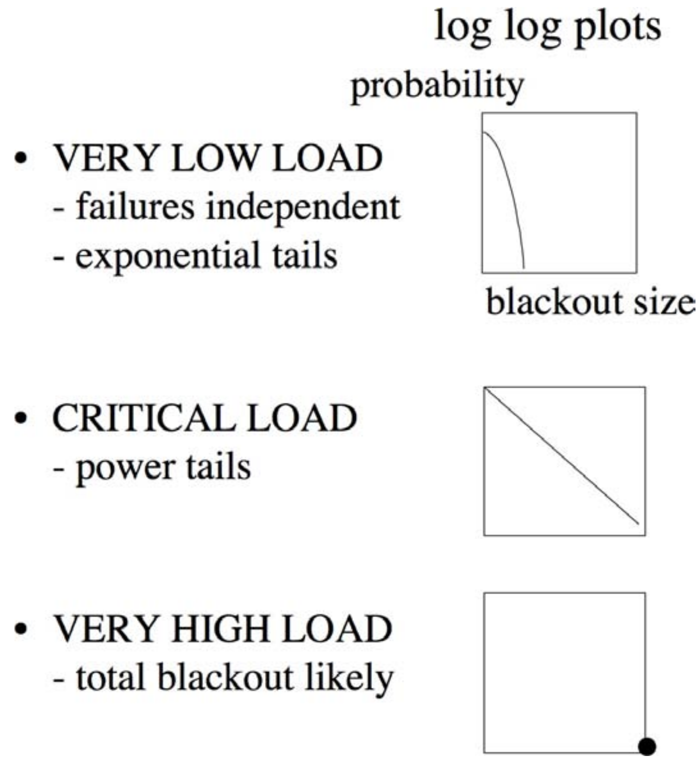

Fig. 2: Log-log plots sketching idealized blackout size probability distributions for very low, critical, and very high power system loadings.

as the load transfer amount $P = 0.0004$. These modeling choices for component load lead via the normalization (2) to the parameterization

$$p = d = \frac{0.0004}{2 - 2L}\,, \qquad 0.5 \le L < 1. \tag{11}$$

The increase in the normalized power transfer p with increased L may be thought of as strengthening the component interactions that cause cascading failure.

The distribution for the subcritical and nonsaturating case $L = 0.6$ has an approximately exponential tail as shown in Figure 3. The tail becomes heavier as L increases and the distribution for the critical case $L = 0.8$, $np = 1$ has an approximate power law region over a range of S. The power law region has an exponent of approximately -1.4 and this compares to the exponent of -1.5 obtained by the analytic approximation discussed in subsection 3.2. The distribution for the supercritical and saturated case $L = 0.9$ has an approximately exponential tail for small r, zero probability of intermediate r, and a probability of 0.80 of all 1000 components failing. If an intermediate number of components fail in a saturated case, then the cascade always proceeds to all 1000 components failing.

The increase in the mean number of failures as the average initial component loading L is increased is shown in Figure 4. The sharp change in gradient at the critical loading $L = 0.8$ corresponds to the saturation of (3) and the consequent increasing probability of all components failing. Indeed, at $L = 0.8$, the change in gradient in Figure 4 together with the power law region in the distribution of S in Figure 3 suggest a type two phase transition in the system. If we interpret the number of components failed as corresponding to blackout size, the power law region is consistent with the North American blackout data discussed in section 2. In particular, North American blackout data suggest an empirical distribution of blackout size with a power tail with exponent between –1 and –2. This power tail indicates a significant risk of large blackouts that is not present when the distribution of blackout sizes has an exponential tail.

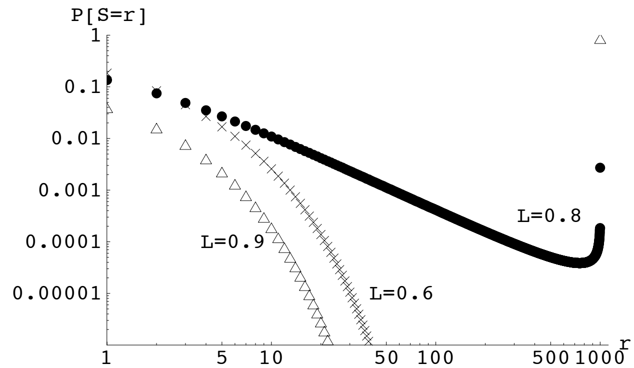

Fig. 3: Log-log plot of distribution of number of components failed S for three values of average initial load L. Note the power law region for the critical loading $L = 0.8$. $L = 0.9$ has an isolated point at $(1000, 0.80)$ indicating probability 0.80 of all 1000 components failed. Probability of no failures is 0.61 for $L = 0.6$, 0.37 for $L = 0.8$, and 0.14 for $L = 0.9$.

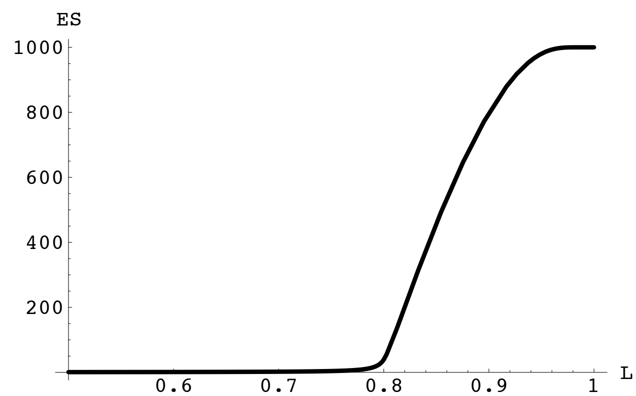

Fig. 4: Mean number of components failed ES as a function of average initial component loading L. Note the change in gradient at the critical loading $L = 0.8$. There are $n = 1000$ components and ES becomes 1000 at the highest loadings.

The model results show how system loading can influence the risk of cascading failure. At low loading there is an approximately exponential tail in the distribution of number of components failed and a low risk of large cascading failure. There is a critical loading at which there is a power law region in the distribution of number of components failed and a sharp increase in the gradient of the mean number of components failed. As loading is increased past the critical loading, the distribution of number of components failed saturates, there is an increasingly significant probability of all components failing, and there is a significant risk of large cascading failure.

4.3 Critical transitions as load increases in OPA

Criticality can be observed in the fast dynamics OPA model as load power demand is slowly increased as shown in Fig. 5. (Random fluctuations in the pattern of load are superimposed on the load increase in order to provide statistical data.) At a critical loading, the gradient of the expected blackout size sharply increases. Moreover, the PDF of blackout size shows power tails at the critical loading as shown in Fig. 6. OPA can also display complicated critical point behavior corresponding to both generation and transmission line limits [11].

As noted in section 1.1, the cascading hidden failure model of Chen and Thorp also shows some indications of criticality as load is increased [17, 18].

5 Quantifying proximity to criticality

At criticality there is a power tail, a sharp increase in mean blackout size, and an increased risk of cascading failure. Thus criticality gives a reference point or a power system operational limit with respect to cascading failure. That is, we are suggesting adding an increased risk of cascading failure limit to the established power system operating limits such as thermal, voltage, and transient stability. How does one practically monitor or measure margin to criticality?

One approach is to increase loading in a blackout simulation incorporating cascading failure mechanisms until criticality is detected by a sharp increase in mean blackout size. The mean blackout size is calculated at each loading level by running the simulation repeatedly with some random variation in the system initial conditions so that a variety of cascading outages are simulated. This approach is straightforward and likely to be useful, but it is not fast and it seems that it would be difficult or impossible to apply to real system data. Also it could be challenging to describe and model a good sample of the diverse interactions involved in cascading failure in a fast enough simulation. This approach, together with checks on the power law behavior of the distribution of blackout size, was used to find criticality in several power system and abstract models of cascading failure [11, 17, 18, 32, 28]. Confirming criticality in this way in a range of power system models incorporating more detailed or different cascading failure mechanisms

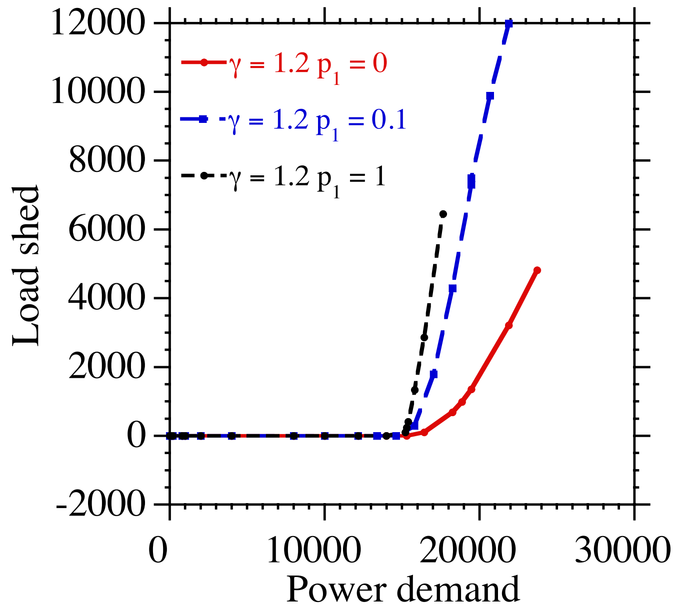

Fig. 5: Average blackout size in OPA as loading increases. Critical loading occurs at kink in curves where average blackout size sharply increases.

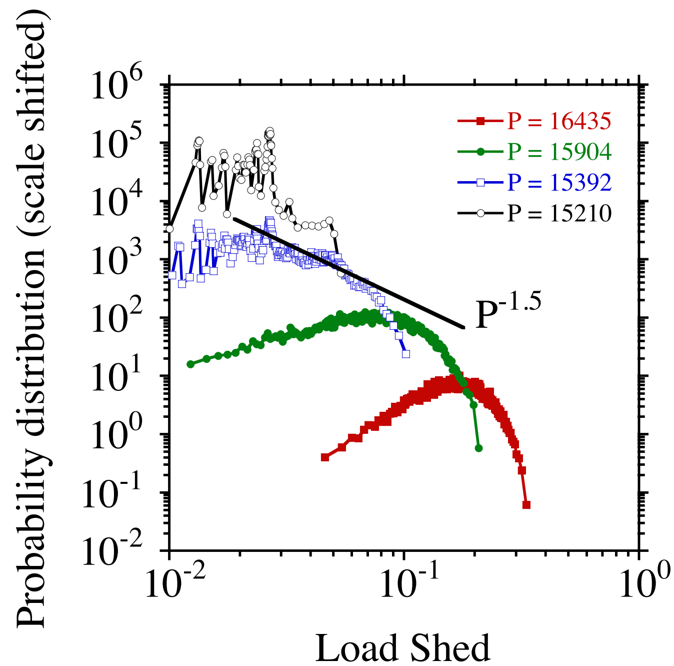

Fig. 6: Blackout size PDF at critical loading P=15392 and other loadings.

would help to establish further the key role that criticality plays in cascading failure.

Another approach that is currently being developed [13, 30, 31] is to monitor or measure from real or simulated data how much failures propagate after they are initiated. Branching process models such as the Galton-Watson process described in section 3.2 have a parameter λ that measures both the average failure propagation and proximity to criticality. In branching process models, the average number of failures is multiplied by λ at each stage of the branching process. Although there is statistical variation about the mean behavior, it is known [1] that for subcritical systems with $\lambda < 1$, the failures will die out and that for supercritical systems with $\lambda > 1$, the number of failures can exponentially increase. (The exponential increase will in practice be limited by the system size and any blackout inhibition mechanisms; current research seeks to understand and model the blackout inhibition mechanisms.)

The idea is to statistically estimate λ from simulated or real failure data. Essentially this approach seeks to approximate and fit the data with a branching process model. The ability to estimate λ and any other parameters of the branching process model would allow the computation of the corresponding distribution of blackout size probability and hence estimates of the blackout risk.

Note that the cascading failure limit measures overall system stress in terms of how failures propagate once started; it is complementary to measures to limit cascading failure by inhibiting the start of cascade such as the n-1 criterion.

6 Self-organization and slow dynamics of network evolution

6.1 *Qualitative description of self-organization*

We qualitatively describe how the forces shaping the evolution of the power network could give rise to self-organizing dynamics. The power system contains many components such as generators, transmission lines, transformers and substations. Each component experiences a certain loading each day and when all the components are considered together they experience some pattern or vector of loadings. The pattern of component loadings is determined by the power system operating policy and is driven by the aggregated customer loads at substations. The power system operating policy includes short term actions such as generator dispatch as well as longer term actions such as improvements in procedures and planned outages for maintenance. The operating policy seeks to satisfy the customer loads at least cost. The aggregated customer load has daily and seasonal cycles and a slow secular increase of about 2% per year.

The probability of component failure generally increases with component loading. Each failure is a limiting or zeroing of load in a component and causes a redistribution of power flow in the network and hence a discrete increase in the loading of other system components. Thus failures can cascade. If a cascade of events includes limiting or zeroing the load at substations, it is a blackout. A stressed power system experiencing an event must either redistribute load satisfactorily or shed some load at substations in a blackout. A cascade of events leading to blackout usually occurs on a time scale of minutes to hours and is completed in less than one day.

It is customary for utility engineers to make prodigious efforts to avoid blackouts and especially to avoid repeated blackouts with similar causes. These engineering responses to a blackout occur on a range of time scales longer than one day. Responses include repair of damaged equipment, more frequent maintenance, changes in operating policy away from the specific conditions causing the blackout, installing new equipment to increase system capacity, and adjusting or adding system alarms or controls. The responses reduce the probability of events in components related to the blackout, either by lowering their probabilities directly or by reducing component loading by increasing component capacity or by transferring some of the loading to other components. The responses are directed towards the components involved in causing the blackout. Thus the probability of a similar blackout occurring is reduced, at least until load growth degrades the improvements made. There are similar, but less intense responses to unrealized threats to system security such as near misses and simulated blackouts.

The pattern or vector of component loadings may be thought of as a system state. Maximum component loadings are driven up by the slow increase in customer loads via the operating policy. High loadings increase the chances of cascading events and blackouts. The loadings of components involved in the blackout are reduced or relaxed by the engineering responses to security threats and blackouts. However, the loadings of some components not involved in the blackout may increase. These opposing forces driving the component loadings up and relaxing the component loadings are a reflection of the standard tradeoff between satisfying customer loads economically and security. The opposing forces apply over a range of time scales. We suggest that the opposing forces, together with the underlying growth in customer load and diversity give rise to a dynamic equilibrium.

These ideas of complex dynamics by which the network evolves are inspired by corresponding concepts of self-organized criticality (SOC) in statistical physics. As a brief introduction to the concept, a self-organized critical system is one in which the nonlinear dynamics in the presence of perturbations organize the overall average system state near to, but not at, the state that is marginal to major disruptions. Self-organized critical systems are characterized by a spectrum of spatial and temporal scales of the disruptions that exist in remarkably similar forms in a wide variety of physical systems [2, 3, 39]. In these systems, the probability of occurrence of large disruptive events decreases as a power function of the event size. This is in contrast to many conventional systems in which this probability decays exponentially with event size.

6.2 OPA blackout model for a slowly evolving network

The OPA blackout model [14, 25, 9, 10] represents the essentials of slow load growth, cascading line outages, and the increases in system capacity caused by the engineering responses to blackouts. Cascading line outages leading to blackout are regarded as fast dynamics and are modeled as described in section 3.3 and the lines involved in a blackout are predicted. The slow dynamics model the growth of the load demand and the engineering response to the blackout by upgrades to the grid transmission capability. The slow dynamics represents the complex dynamics outlined in section 6.1. The slow dynamics is carried out by the following small changes applied at each day: All loads are multiplied by a fixed parameter that represents the daily rate of increase in electricity demand. If a blackout occurs, then the lines involved in the blackout have their line flow limits increased slightly. The generation is increased at randomly selected generators subject to coordination with the limits of nearby lines when the generator capacity margin falls below a threshold. The OPA model is "top-down" and represents the processes in greatly simplified forms, although the interactions between these processes still yield complex (and complicated!) behaviors. The simple representation of the processes is desirable both to study only the main interactions governing the complex dynamics and for pragmatic reasons of model tractability and simulation run time.

6.3 Self-Organization

We propose one way to understand the origin of the dynamics and distribution of power system blackouts. Indeed, we suggest that the slow, opposing forces of load increase and network upgrade in response to blackouts shape the system operating margins so that cascading blackouts occur with a frequency governed approximately by a power law relationship between blackout probability and blackout size. That is, these forces drive the system to a dynamic equilibrium just below and near criticality.

The load increase is a force weakening the power system (reducing operating margin) and the system upgrades are a force strengthening the system (increasing operating margin). If the power system is weak, then there will be more blackouts and hence more upgrades of the lines involved in the blackout and this will eventually strengthen the power system. If the power system is strong, then there will be fewer blackouts and fewer line upgrades, and the load increase will weaken the system. Thus the opposing forces drive the system to a dynamic equilibrium that keeps the system near a certain pattern of operating margins relative to the load. This process is observed in OPA results. Note that engineering improvements and load growth are driven by strong, underlying economic and societal forces that are not easily modified.

Moreover, when the generator upgrade process is suitably coordinated with the line upgrades and load increase, OPA results show power tails in the PDF of blackout sizes. For example, OPA results for the IEEE 118 bus network and an artificial 382 bus tree-like network are shown in Figure 7. Both the power law region of the PDF and the consistency with the NERC blackout data are evident. We emphasize that this criticality was achieved by the internal dynamics modeled in the system and is in this sense self-organizing to criticality.

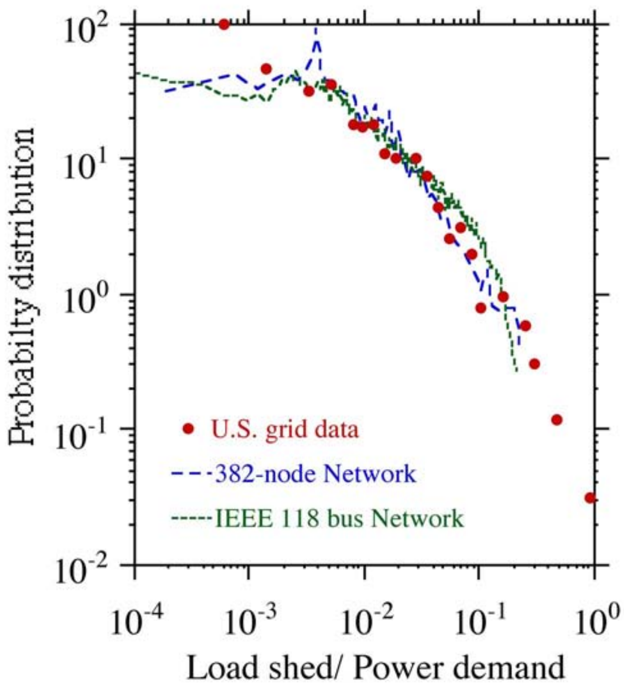

Fig. 7: Blackout size PDF resulting from self-organization showing OPA results on 2 networks. The NERC blackout data is also shown for comparison.

6.4 Blackout mitigation

While much remains to be learned about these complex dynamics, it is clear that these global dynamics have important implications for power system control and operation and for efforts to reduce the risk of blackouts.

The success of mitigation efforts in self-organized critical systems is strongly influenced by the dynamics of the system. Unless the mitigation efforts alter the self-organization forces driving the system, the system will be pushed to criticality. To alter those forces with mitigation efforts may be quite difficult because the forces are an intrinsic part of our society. Then the mitigation efforts can move the system to a new dynamic equilibrium while remaining near criticality and preserving the power tails. Thus, while the absolute frequency of disruptions of all sizes may be reduced, the underlying forces can still cause the relative frequency of large disruptions to small disruptions to remain the same.

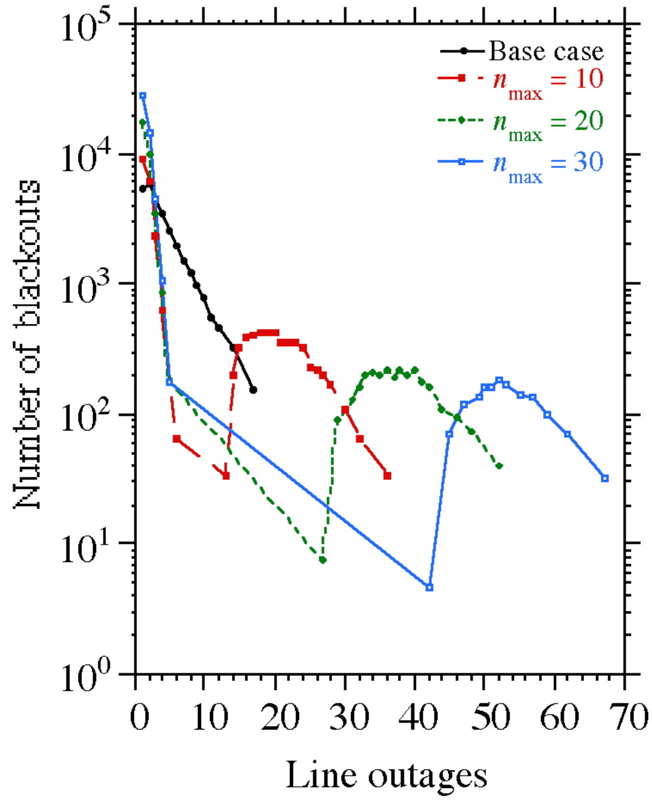

Fig. 8: Number of blackouts as number of line outages varies for differing inhibition of line outages ($n_{\max}$ is the maximum number of line overloads for which outages are inhibited). Results are obtained using OPA model on the IEEE 118 bus system.

Indeed apparently sensible efforts to reduce the risk of smaller blackouts can sometimes increase the risk of large blackouts. This occurs because the large and small blackouts are not independent but are strongly coupled by the dynamics. For example the longer term response to small blackouts can influence the frequency of large blackouts in such a way that measures to reduce the frequency of small blackouts can eventually reposition the system to have an increased risk of large blackouts. The possibility of an overall adverse effect on risk from apparently sensible mitigation efforts shows the importance of accounting for complex system dynamics when devising mitigation schemes [12]. For example [12], Figure 8 shows the results of inhibiting small numbers of line outages using the OPA model with self-organization on the IEEE 118 bus system. One of the causes of line outages in OPA is the outage of lines with a probability p_1 when the line is overloaded. The results show the effect of inhibiting these outages when the number of overloaded lines is less than $n_{\max}$. The inhibition corresponds to more effective system operation to resolve these overloads. Blackout size is measured by number of line overloads. The inhibition is, as expected, successful in reducing the smaller numbers of line outages, but eventually, after the system has repositioned to its dynamic equilibrium, the number of larger blackouts has increased. The results shown in Figure 8 are distributions of blackouts in the self-organized dynamic equilibrium and reflect the long-term effects of the inhibition of line outages. It is an interesting open question to what extent power transmission systems are near their dynamic equilibrium, but operation near dynamic equilibrium is the simplest assumption at the present stage of knowledge of these complex dynamics.

Similar effects are familiar and intuitive in other complex systems. For example, more effectively fighting small forest fires allows the forest system to readjust with increased brush levels and closer tree spacing so that when a forest fire does happen by some chance to progress to a larger fire, a huge forest fire is more likely [12].

7 Conclusions

We have summarized and explained an approach to series of cascading failure blackouts at a global systems level. This way of studying blackouts is complementary to existing detailed analyses of particular blackouts and offers some new insights into blackout risk, the nature of cascading failure, the occurrence of criticality, and the complex system dynamics of blackouts.

The power law region in the distribution of blackout sizes in North American blackout data [15, 16] has been reproduced by power system blackout models [11, 14, 18] and some abstract models of cascading failure [32, 28] and engineering design [55]. The power law profoundly affects the risk of large blackouts, making this risk comparable to, or even exceeding the risk of small blackouts. The power law also precludes many conventional statistical models with exponential-tailed distributions and new approaches need to be developed such as [32, 28, 31, 19].

We think that the power law in the distribution of blackout sizes arises from cascading failure when the power system is loaded near a critical loading. Several power system blackout models [11, 18] and abstract models of cascading failure [32, 28] show evidence of a critical loading at which the probability of cascading failure sharply increases. We suggest that determining the proximity to critical loading from power system simulations or data is an important problem. It seems that Monte-Carlo simulation methods will be able to usefully compute the proximity to critical loading [11, 18, 40]. Moreover, branching process models of cascading failure provide ways of quantifying with a parameter λ the extent to which failures propagate after they are started. We are pursuing practical methods of estimating λ from real or simulated failure data [28, 30, 31].

A novel and much larger view of the power system dynamics considers the opposing forces of growing load and the upgrade of the transmission network in response to real or simulated blackouts. Our simulation results show that these complex dynamics can self-organize the system to be near criticality [14]. These complex dynamics are driven by strong societal and economic forces and the difficulties or tradeoffs in achieving long-term displacement of the power system away from the complex systems equilibrium caused by these forces should not be underestimated. Indeed we have simulated a simple example of a blackout mitigation method that successfully limits the frequency of small blackouts, but in the long term increases the frequency of large blackouts as the transmission system readjusts to its complex systems equilibrium [12]. In the light of this example, we suggest that the blackout mitigation problem be reframed as jointly mitigating small and large blackouts.

There are good prospects for extracting engineering and scientific value from the further development of models, simulations and computations and we hope that this paper encourages further developments and practical applications in this emerging and exciting area of research. There is an opportunity for systems research to make a substantial contribution to understanding and managing the risk of cascading failure blackouts.

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6.5 Estimating failure propagation in models of cascading blackouts

I. Dobson, B.A. Carreras, V.E. Lynch, B. Nkei, D.E. Newman

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Estimating Failure Propagation in Models of Cascading Blackouts

Ian Dobson Benjamin A. Carreras Vickie E. Lynch Bertrand Nkei David E. Newman

***Abstract*— We compare and test statistical estimates of failure propagation in data from versions of a probabilistic model of loading-dependent cascading failure and a power systems blackout model of cascading transmission line overloads. The comparisons suggest mechanisms affecting failure propagation and are an initial step towards monitoring failure propagation from practical system data. Approximations to the probabilistic model describe the forms of probability distributions of cascade sizes.**

I. Introduction

Large blackouts of electric power transmission systems are typically caused by cascading failure of loaded system components. For example, long, intricate cascades of events caused the Western North American blackout of 30,390 MW in August 1996 [18] and the Eastern North America blackout of 61,800 MW in August 2003 [21]. The vital importance of the electrical infrastructure to society motivates the analysis and monitoring of the risks of cascading failure. In particular, in addition to limiting the start of outages that cascade, it is useful to be able to monitor the tendency of cascading failures to propagate after they are started [14], [5].

CASCADE is a probabilistic model of loading-dependent cascading failure that is simple enough to be analytically tractable [13], [16], [15]. CASCADE contains no power system modeling, but does seem to approximately capture some of the salient features of cascading failure in large blackouts. The CASCADE model has many identical components randomly loaded. An initial disturbance adds load to each component and causes some components to fail by exceeding their loading limit. Failure of a component causes a fixed load increase for other components. As components fail, the system becomes more loaded and cascading failure of further components becomes likely.

The CASCADE model can be well approximated by a Galton-Watson branching process in which failures occur in stages and each failure in each stage causes further failures in the next stage according to a Poisson distribution [14]. The average number of failures in the initial disturbance is θ and the subsequent stochastic propagation of the failures is controlled by the parameter λ, which is the average number of failures caused by each failure in the previous stage.

OPA is a power system blackout model that represents probabilistic cascading line outages and overloads [3]. The network is conventionally modeled using DC load flow and LP dispatch of the generation. The initial disturbance is generated by random line outages and load variations. Overloaded lines outage with a given probability and the subsequent power flow redistribution and generator redispatch can overload further lines, which can then probabilistically outage in a cascading fashion. There is no attempt to represent all the diverse interactions that can occur during a blackout. However, the modeling does represent a feasible cascading blackout consistent with some basic network and operational constraints. OPA can also model the slow evolution of the network as load grows and the network is upgraded in response to blackouts [1], [12], [2], [4], but in this paper the network is assumed to be fixed and these complex systems dynamics are neglected.

Other authors have constructed power system blackout models involving cascading failure emphasizing different aspects of the problem. Chen and Thorp [6], [7] model hidden failures and compute vulnerability of key lines using importance sampling and examine criticality and blackout mitigation. Ni, McCalley, Vittal, and Tayyib [19] show how to monitor the risk of a variety of system limits being exceeded; minimizing this risk would have the effect of limiting the risk of cascading events starting. Chen, Zhu, and McCalley [8] show how to evaluate the risk of the first few likely cascading failures. Rios, Kirschen, Jawayeera, Nedic, and Allan [20] use Monte Carlo simulation to estimate the cost of security taking account of hidden failures, cascading outages and transient instability.

Our ultimate goal is to understand cascading failure in large blackouts from a global systems point of view, identify the main parameters governing the cascading process, and suggest ways to estimate these parameters from real or simulated outage data. These metrics will allow monitoring of the risk of cascading failure and quantifying of the tradeoffs involved in blackout mitigation. In this paper, we take a step towards this goal by comparing the abstract cascading failure model CASCADE with the power system blackout model OPA. The comparison reveals which features of the OPA blackouts are captured by the CASCADE model. In particular, we seek to characterize in OPA and measure from OPA results the parameter λ governing the propagation of failures after the

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I. Dobson is with the ECE Dept., University of Wisconsin, Madison WI 53706 USA (email: dobson@engr.wisc.edu). B.A. Carreras, V. Lynch, and B. Nkei are with Oak Ridge National Laboratory, Oak Ridge TN 37831 USA (email: carrerasba@ornl.gov). D.E. Newman is with the Physics Dept., University of Alaska, Fairbanks AK 99775 USA (email: ffden@uaf.edu).

start of the cascade. Resolving problems in measuring λ from OPA results is a first step towards measuring the degree to which failures propagate in power systems. If the overall system stress is such that failures propagate minimally, then any failures that occur are likely to be a single failure or a short sequences of failures that cause small blackouts or no blackout. However, if the overall system stress is such that failures propagate readily, then there is a substantial risk of cascading failure leading to large blackouts and it is in the national interest to quantify this risk and examine the economics and engineering of mitigating this risk.

II. CASCADE MODEL AND BRANCHING PROCESS PARAMETERS

This section summarizes the CASCADE model of probabilistic load-dependent cascading failure and its branching process approximation [13], [16], [15], [14]. (Here the normalized version of CASCADE is summarized; for many purposes, the unnormalized version is more useful and flexible [13], [16].)

The CASCADE model has n identical components with random initial loads. For each component the minimum initial load is 0 and the maximum initial load is 1. For j=1,2,...,n, component j has initial load ℓ_j that is a random variable uniformly distributed in $[0, 1]$. $\ell_1, \ell_2, \cdots, \ell_n$ are independent.

Components fail when their load exceeds 1. When a component fails, a fixed amount of load $p \geq 0$ is transferred to each of k components. The k components to which load is transferred are chosen randomly each time a component fails [15].

To start the cascade, we assume an initial disturbance that loads each component by an additional amount d. Other components may then fail depending on their initial loads ℓ_j and the failure of any of these components will distribute the additional load p that can cause further failures in a cascade. The cascade proceeds in stages with M_1 failures due to the initial disturbance, M_2 failures due to load increments from the M_1 failures, M_3 failures due to load increments from the M_2 failures, and so on. The size of the cascading failure is measured by the total number of components failed S.

For the case $k = n$ in which load is transferred to all the system components when each failure occurs, the distribution of S is a saturating quasibinomial distribution [16], [13], [9]:

$$P[S=r] = \begin{cases} \binom{n}{r} \phi(d)(d+rp)^{r-1}(\phi(1-d-rp))^{n-r}, \\ \qquad\qquad\qquad\qquad r = 0, 1, ..., n-1 \\ 1 - \sum_{s=0}^{n-1} P[S=s], \qquad r = n, \end{cases} \tag{1}$$

where the saturation function ϕ is

$$\phi(x) = \begin{cases} 0 & ; x < 0, \\ x & ; 0 \leq x \leq 1, \\ 1 & ; x > 1. \end{cases} \tag{2}$$

Note that (1) uses $0^0 \equiv 1$ and $0/0 \equiv 1$ when needed.

In the case $k < n$, no analytic formula such as (1) is currently available, but it can be shown that the following approximation (4) remains valid [15].

Define

$$\lambda = kp \quad \text{and} \quad \theta = nd \tag{3}$$

λ may be interpreted as the total amount of load increment associated with any failure and is a measure of how much the components interact. θ may be interpreted as the average number of failures due to the initial disturbance.

Now we approximate the CASCADE model [14], [15]. Let $n \to \infty$, $k \to \infty$ and $p \to 0$, $d \to 0$ in such a way that $\lambda = kp$ and $\theta = nd$ are fixed. For $\theta \geq 0$,

$$P[S=r] \approx \begin{cases} \theta(r\lambda+\theta)^{r-1}\frac{e^{-r\lambda-\theta}}{r!} & ; 0 \leq r \leq (n-\theta)/\lambda,\ r < n \\ 0 & ; (n-\theta)/\lambda < r < n,\ r \geq 0 \\ 1 - \sum_{s=0}^{n-1} P[S=s] & ; r = n \end{cases} \tag{4}$$

The approximate distribution (4) is a saturating form of the generalized Poisson distribution [11], [10]. Moreover, under the same approximation, the stages of the CASCADE model become stages of a Galton-Watson branching process [14], [17]. In particular, the initial failures are produced by a Poisson distribution with parameter θ. Each initial failure independently produces more failures according to a Poisson distribution with parameter λ, and each of those failures independently produces more failures according to a Poisson distribution with parameter λ, and so on. This branching process leads to another interpretation of λ as the average number of failures per failure in the previous stage. λ is a measure of the average propagation of the failures [14].

The expected number of failures in stage j of the branching process is given by

$$EM_j = \theta\lambda^{j-1} \tag{5}$$

until saturation due to the system size occurs. Formula (5) is exact for the branching process before saturation and an approximation for the expected number of failures in each stage of CASCADE.

Further approximation is useful. Using Stirling's formula and a limiting expression for an exponential for $r \gg 1$, (4) becomes

$$P[S=r] \approx \frac{\theta}{\lambda\sqrt{2\pi}} \exp[(1-\lambda)\tfrac{\theta}{\lambda}] \frac{\exp[-r(\lambda-1-\ln\lambda)]}{(r+\frac{\theta}{\lambda})\sqrt{r}} \qquad ; 1 \ll r < r_1 = \min\{n/\lambda, n\} \tag{6}$$

and if $\theta/\lambda \sim 1$ so that also $r \gg \theta/\lambda$,

$$P[S=r] \approx \frac{\theta \exp[(1-\lambda)\frac{\theta}{\lambda}]}{\lambda\sqrt{2\pi}} r^{-\frac{3}{2}} \exp[-r(\lambda-1-\ln\lambda)] \qquad ; 1 \ll r < r_1 = \min\{n/\lambda, n\} \tag{7}$$

Let

$$r_0 = (\lambda - 1 - \ln\lambda)^{-1} \tag{8}$$

In the approximation (7), the term $r^{-\frac{3}{2}}$ dominates for $r \lesssim r_0$ and the exponential term dominates for $r \gtrsim r_0$. Thus (7) reveals that the distribution of the number of failures has an approximate power law region of exponent -1.5 for $1 \ll r \lesssim r_0$ and an exponential tail for $r_0 \lesssim r < r_1$. The approximation

(7) implies that r_0 is only a function of λ and does not depend on θ or the system size n.

We discuss some of the implications of the approximation for the form of the distribution of S.

1) For $\lambda = 1$, (6) becomes $P[S = r] \approx \frac{\theta}{\sqrt{2\pi}} r^{-\frac{3}{2}}$, r_0 becomes infinite and the power law region extends up to the system size.
2) $r_0^{-1} = \lambda - 1 - \ln\lambda$ is a nonnegative function of λ with a quadratic minimum of zero at $\lambda = 1$. Therefore, for a range of λ near 1, $r_0 = (\lambda - 1 - \ln\lambda)^{-1}$ is large and the power law region extends to large r.
3) The risk of large cascades as compared with the risk of small cascades is approximately determined by λ. Small λ gives a distribution with an exponential tail past a small number of failures and a negligible probability of large cascades. λ near 1 gives a power tail with a significant probability of large cascades and $\lambda > 1$ gives a significant probability of all components failing.
4) If $\lambda < 1$, then $P[S = r]$ increases with θ. The increase is linear for small θ and exponential with large θ. Thus reducing the probability of failures by decreasing the size of the initial disturbance is most effective when λ is less than and bounded away from 1 and θ is large.

III. OPA BLACKOUT MODEL

In the OPA model [3], there is a fast time scale of the order of minutes to hours, over which cascading overloads or outages may lead to a blackout. Cascading blackouts are modeled by overloads and outages of lines determined in the context of LP dispatch of a DC load flow model. To start the cascade, random line outages are triggered with a probability p_0. A cascading overload may also start if one or more lines are overloaded in the solution of the LP optimization. In this situation, we assume that there is a probability p_1 that an overloaded line will outage. When a solution is found, the overloaded lines of the solution are tested for possible outages. Outaged lines are in effect removed from the network and then a new solution is calculated. This process can lead to multiple iterations, and the process continues until a solution with no more line outages is found. We regard each iteration as one stage of the cascading blackout process. The overall effect of the process is to generate a possible cascade of line outages that is consistent with the network constraints and the LP dispatch optimization.

The parameters p_0 and p_1 determine the initial disturbance. The level of stress on the system is determined by a multiplier on the loads in the power system.

IV. ESTIMATING θ AND λ FROM DATA

This section proposes methods of estimating θ and λ from the data produced by CASCADE or OPA. Both CASCADE and OPA produce a stochastic sequence of failures in stages with M_1 failures due to the initial disturbance and subsequent numbers of failures M_2, M_3,.... In the case of OPA the failures are transmission line outages. If at any stage (including the first stage), there are zero failures, then the cascade of failures ends.

For $d > 0$, the probability of a nontrivial cascade in the CASCADE model is easily obtained from (1) as

$$\begin{aligned} P[S>0] &= 1 - P[S=0] \\ &= 1 - (1-d)^n = 1 - (1-\theta/n)^n \end{aligned} \tag{9}$$

Let the observed frequency of nontrivial cascades be

$$f = \frac{\#\text{ cascades with } S > 0}{\#\text{ samples}} \tag{10}$$

Then (9) suggests the following estimator for θ:

$$\hat{\theta} = n - n(1-f)^{\frac{1}{n}} \tag{11}$$

Let the sample mean of the number of failures in stage j of the cascade be

$$m_j = \frac{1}{\#\text{ samples}} \sum_{\text{samples}} \#\text{ failures in stage } j \tag{12}$$

Then (5) suggests the following estimator for λ based on the data from stage j of the cascade:

$$\hat{\lambda}_j = (m_j/\hat{\theta})^{\frac{1}{j-1}} \tag{13}$$

The naive estimators in (11) and (13) have been tested on data produced by CASCADE and they appear empirically to be useful statistics. For example, for $\lambda < 1.3$, Fig. 1 shows the estimated $\hat{\lambda}_j$ as constant with respect to the stage j as expected. (For $\lambda > 1.3$, the estimated $\hat{\lambda}_j$ decreases with the stage j because at higher λ and at higher j there are more cascades with all 190 components failed and this saturation effect reduces $\hat{\lambda}_j$. Recall that the formula (5) used to derive (13) assumes no saturation.)

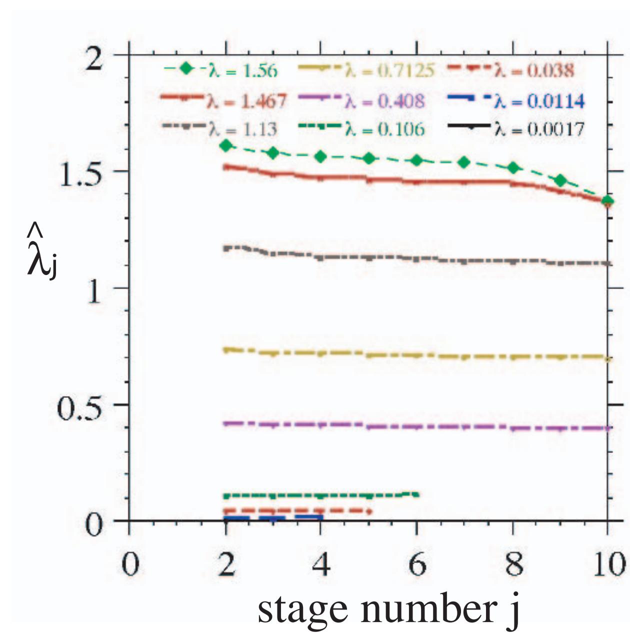

Fig. 1. $\hat{\lambda}_j$ as a function of stage number j from CASCADE model for $\lambda = 0.0017, 0.0114, 0.038, 0.106, 0.408, 0.7125, 1.13, 1.467$, and 1.56. There are $n = 190$ components and $\theta = 0.095$.

V. Results

A. Comparing CASCADE and OPA

The OPA model on a 190 node tree-like network [2] was used to produce line outage data. The load multiplier parameter was varied to vary the system stress. The $\hat{\lambda}_j$ computed from the OPA results is plotted in Fig. 2. We can see that at high load $\hat{\lambda}_j$ is a decreasing function of the stage j while for low loads $\hat{\lambda}_j$ is an increasing function of the stage j. This functional form is not seen in the CASCADE model results in Fig. 1.

The probability distributions for number of lines outaged in OPA corresponding to Fig. 2 are shown in Fig. 3. We can attempt to match these probability distributions with CASCADE by using $\hat{\theta}$ from the OPA results as an estimate of θ and using $\hat{\lambda}_1$ from the OPA results as an estimate of λ. The resulting CASCADE probability distributions are shown in Fig. 4. Although there is reasonable qualitative agreement between the probability distributions from OPA and CASCADE for smaller λ, the OPA probability distributions for larger λ contain a peak not present in the CASCADE probability distributions. We consider a modification to CASCADE to explain this peak in subsection V-B.

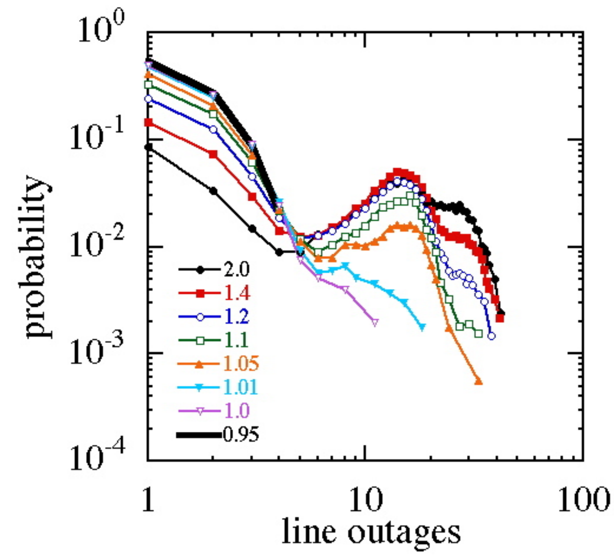

Fig. 3. Probability distributions of number of line outages from OPA model for various values of loading multiplier.

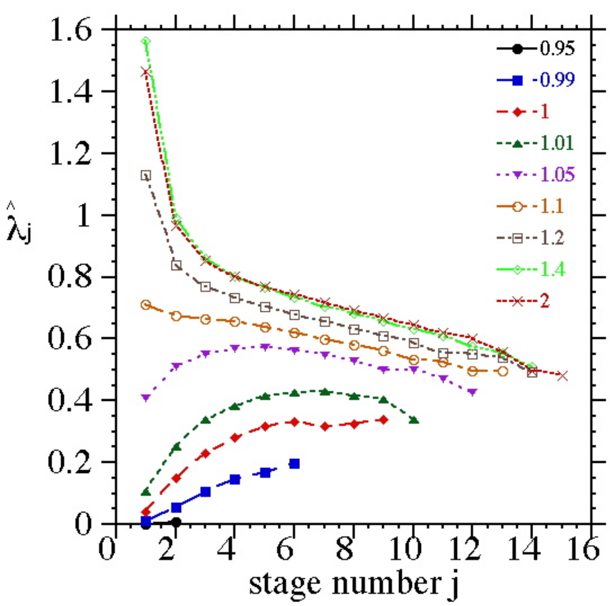

Fig. 2. $\hat{\lambda}_j$ as a function of stage number j from OPA model for various values of loading multiplier.

B. Blackout inhibition modification to CASCADE

In a blackout, there is not only an effect by which line outages further load the system and tend to cause further outages. There is also an effect by which sufficient line outages will cause load to be shed and this load shedding reduces the load on the system. (It is also possible, but perhaps less common, for load shedding to introduce large disturbances and imbalances that further stress portions of the system.) Moreover sufficient line outages will tend to island the system and this can have the effect of limiting further outages. That is, sufficiently many line outages can have an inhibitory effect on further cascading outages.

We attribute the peak in the OPA probability distributions for larger λ to this inhibitory effect. One can argue that for small λ, it is not likely that the cascade will include enough line outages to encounter the inhibitory effect. Moreover, the inhibitory effect could result in the decrease in $\hat{\lambda}_j$ as the stage j increases observed for larger λ in Fig. 2.

CASCADE does not model the inhibitory effect and one way to test these explanations is to modify CASCADE to model the inhibitory effect. A crude modeling of the inhibitory effect in CASCADE is to halt the cascading process after a fixed number of components $r_{\max}$ have failed. That is, when $r_{\max}$ components have failed, the current stage of the cascade is completed, thus allowing more than $r_{\max}$ components to fail, but the next stage of the cascade is suppressed.

The results of the modified CASCADE model with $r_{\max} = 10$ are shown in Figs. 5 and 6. The decrease in $\hat{\lambda}_j$ with j for larger λ is evident in Fig. 5 and the peak in the probability distribution for larger λ is evident in Fig. 6. These qualitative dependencies in the modified CASCADE results are similar to the OPA results in Figs. 3 and 4. However, Fig. 5 does not show the increase in $\hat{\lambda}_j$ with j for smaller λ observed in Fig. 2 and a further modification to CASCADE to examine this is considered in subsection V-C.

We comment further on the modified CASCADE results in Fig. 5. The value $\hat{\lambda}_1$ in the first stage agrees with the input λ. That is, the inhibition does not seem to affect the initial propagation of the cascade. Also $\hat{\lambda}_j$ appears to decrease to a limiting value $\hat{\lambda}_*$ for values of $\lambda > \hat{\lambda}_*$. For $\lambda < \hat{\lambda}_*$, $\hat{\lambda}_j$ is independent of the stage j.

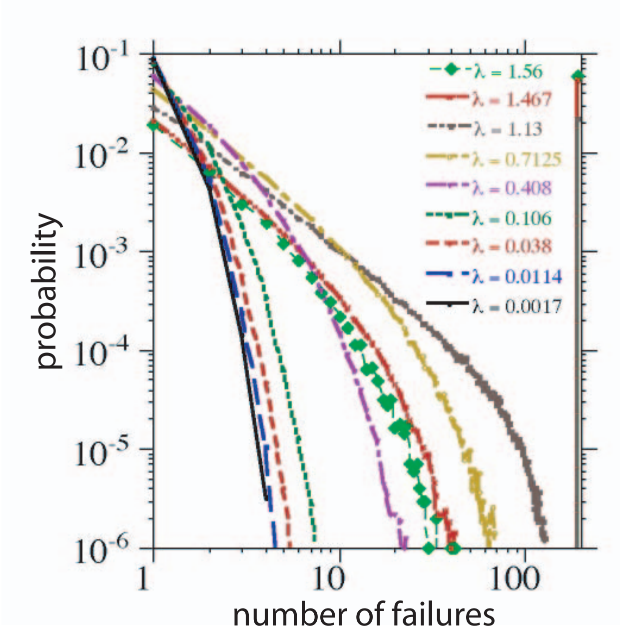

Fig. 4. Probability distributions of number of failures from CASCADE model using the values of $\hat{\lambda}_1$ from Fig. 2 and $\hat{\theta} = 0.095$. There are $n = 190$ components. Results for $\lambda > 1$ show a significant probability of all 190 components failing.

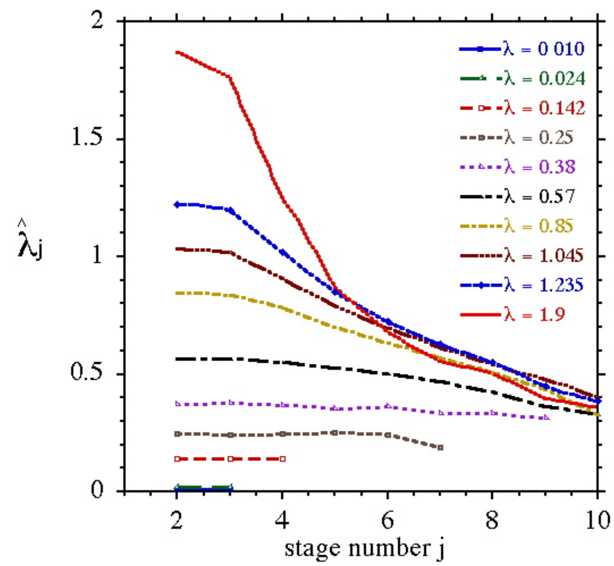

Fig. 5. $\hat{\lambda}_j$ as a function of stage j from CASCADE model with inhibition of line outages for various values of λ.

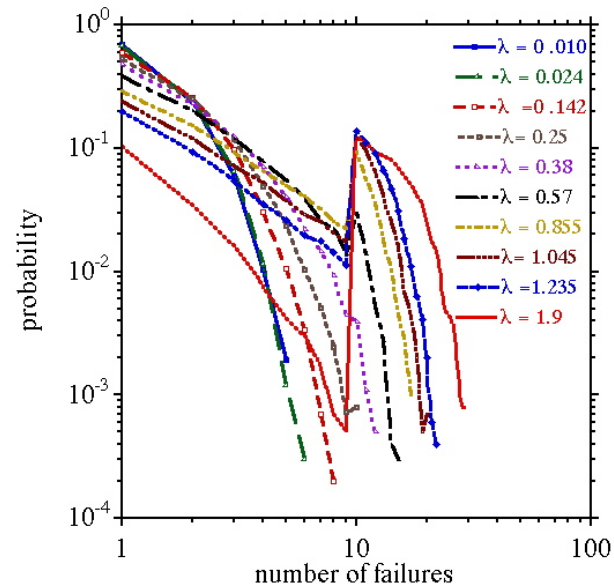

Fig. 6. Probability distributions of number of failures from CASCADE model with inhibition of line outages for various values of λ.

C. *Random line failure modification to CASCADE*

One effect present in OPA but not present in CASCADE is that overloaded lines do not always fail, but rather fail with probability p_1. Implementing this additional modification in CASCADE for various values of p_1 gives $\hat{\lambda}_j$ values as shown in Fig. 7. Some similar results for OPA are shown in Fig. 8 and there is now some qualitative similarity between OPA and the further modified version of CASCADE. In particular, for lower values of p_1, $\hat{\lambda}_j$ increases with stage j.

VI. Conclusion

We have used the CASCADE probabilistic model of cascading failure and its approximations to define an estimator $\hat{\lambda}_j$ of the propagation of failures at stage j of the cascade. The approximations to CASCADE also describe the extent of the region of power law behavior in probability distributions of cascade size. Testing the estimator $\hat{\lambda}_j$ on data produced by the cascading blackout model OPA suggests that, while $\hat{\lambda}_1$ appears to reflect the initial propagation of line outages, $\hat{\lambda}_j$ may decrease or increase with j. Modifications to the CASCADE model that also produce the decrease or increase of $\hat{\lambda}_j$ with j suggest explanations of these effects. For example, the decrease in $\hat{\lambda}_j$ for larger λ may be attributed to the inhibition of line outages by load shedding after a sufficient number of lines are outaged.

These initial results show that the interplay between the CASCADE and OPA models is useful for understanding the propagation of failures in cascading blackouts and in particular will be helpful in devising and testing statistical estimators to quantify this propagation.

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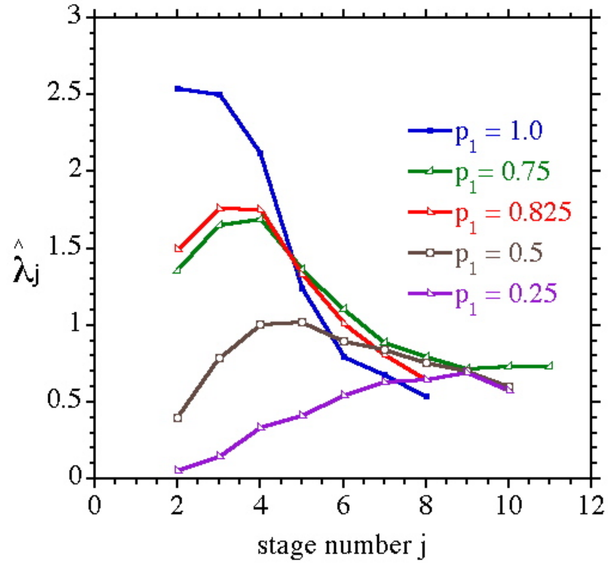

Fig. 7. $\hat{\lambda}_j$ as a function of stage j from CASCADE model with inhibition of line outages and overloaded lines outaged with probability p_1 for various values of p_1. The parameters are $p = 0.25$, $k = 10$, $r_{\max} = 15$, $n = 200$.

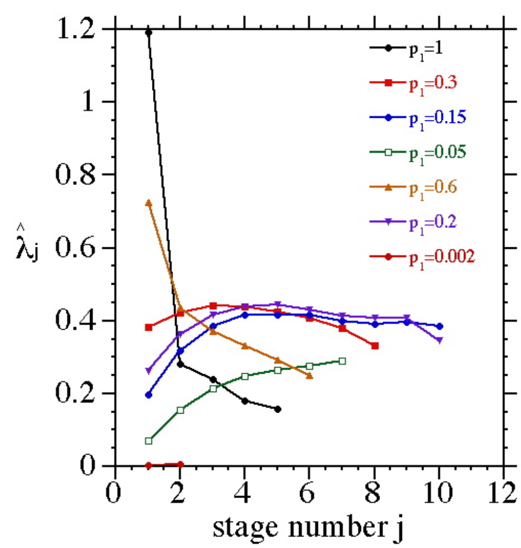

Fig. 8. $\hat{\lambda}_j$ as a function of stage j from OPA model for various values of p_1 on a 94 node tree-like network.

6.6 A criticality approach to monitoring cascading failure risk and failure propagation in transmission systems

I. Dobson, B.A. Carreras, D.E. Newman

Electricity Transmission in Deregulated Markets
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A criticality approach to monitoring cascading failure risk and failure propagation in transmission systems

Ian Dobson, Benjamin A. Carreras, David E. Newman

Abstract— **We consider the risk of cascading failure of electric power transmission systems as overall loading is increased. There is evidence from both abstract and power systems models of cascading failure that there is a critical loading at which the risk of cascading failure sharply increases. Moreover, as expected in a phase transition, at the critical loading there is a power tail in the probability distribution of blackout size. (This power tail is consistent with the empirical distribution of North American blackout sizes.) The importance of the critical loading is that it gives a reference point for determining the risk of cascading failure. Indeed the risk of cascading failure can be quantified and monitored by finding the closeness to the critical loading. This paper suggests and outlines ways of detecting the closeness to criticality from data produced from a generic blackout model. The increasing expected blackout size at criticality can be detected by computing expected blackout size at various loadings. Another approach uses branching process models of cascading failure to interpret the closeness to the critical loading in terms of a failure propagation parameter λ. We suggest a statistic for λ that could be applied before saturation occurs. The paper concludes with suggestions for a wider research agenda for measuring the closeness to criticality of a fixed power transmission network and for studying the complex dynamics governing the slow evolution of a transmission network.**

Index Terms— **blackouts, power system security, stochastic processes, branching process, cascading failure, reliability, risk analysis, complex system, phase transition.**

I. Introduction

Cascading failure is the usual mechanism for large blackouts of electric power transmission systems. For example, long, intricate cascades of events caused the August 1996 blackout in Northwestern America that disconnected 30,390 MW to 7.5 million customers [29], [28], [39]) and the August 2003 blackout in Northeastern America that disconnected 61,800 MW to an area containing 50 million people [38]. The vital importance of the electrical infrastructure to society motivates the understanding and analysis of large blackouts.

Electric power transmission systems are complex networks of large numbers of components that interact in diverse ways. When component operating limits are exceeded, protection acts and the component "fails" in the sense of not being available to transmit power. Components can also fail in the sense of misoperation or damage due to aging, fire, weather, poor maintenance or incorrect settings. In any case, the failure causes a transient and causes the power flow in the component to be redistributed to other components according to circuit laws, and subsequently redistributed according to automatic and manual control actions. The transients and readjustments of the system can be local in effect or can involve components far away, so that a component disconnection or failure can effectively increase the loading of many other components throughout the network. In particular, the propagation of failures is not limited to adjacent network components. The interactions involved are diverse and include deviations in power flows, frequency, and voltage as well as operation or misoperation of protection devices, controls, operator procedures and monitoring and alarm systems. However, all the interactions between component failures tend to be stronger when components are highly loaded. For example, if a more highly loaded transmission line fails, it produces a larger transient, there is a larger amount of power to redistribute to other components, and failures in nearby protection devices are more likely. Moreover, if the overall system is more highly loaded, components have smaller margins so they can tolerate smaller increases in load before failure, the system nonlinearities and dynamical couplings increase, and the system operators have fewer options and more stress.

A typical large blackout has an initial disturbance or trigger events followed by a sequence of cascading events. Each event further weakens and stresses the system and makes subsequent events more likely. Examples of an initial disturbance are short circuits of transmission lines through untrimmed trees, protection device misoperation, and bad weather. The blackout events and interactions are often rare, unusual, or unanticipated because the likely and anticipated failures are already routinely accounted for in power system design and operation.

Blackouts are traditionally analyzed after the blackout by a thorough investigation of the details of the particular sequence of failures. This is extremely useful for finding areas of weakness in the power system and is good engineering practice for strengthening the transmission system [29], [38], [28], [39].

I. Dobson is with the ECE department, University of Wisconsin, Madison WI 53706 USA; email dobson@engr.wisc.edu. B.A. Carreras is with Oak Ridge National Laboratory, Oak Ridge TN 37831 USA; email: carrerasba@ornl.gov. D.E. Newman is with the Physics department, University of Alaska, Fairbanks AK 99775 USA; email ffden@uaf.edu. Part of this research was coordinated by the Consortium for Electric Reliability Technology Solutions and was funded by the Assistant Secretary for Energy Efficiency and Renewable Energy, Office of Power Technologies, Transmission Reliability Program of the U.S. Department of Energy under contract number DE-AC05-00OR22725 and DE-A1099EE35075 with the National Science Foundation. Part of this research has been carried out at Oak Ridge National Laboratory, managed by UT-Battelle, LLC, for the U.S. Department of Energy under contract number DE-AC05-00OR22725. I. Dobson and D.E. Newman gratefully acknowledge support in part from National Science Foundation grants ECS-0214369 and ECS-0216053.

We take a different and complementary approach and seek to determine the risk of series of blackouts from a global, top-down perspective. That is, we are not concerned with the deterministic details of a particular blackout, but rather the overall probability and risk of blackouts from a bulk systems perspective. Our overall approach draws from probability and statistics, power systems engineering, statistical physics, risk analysis, and modeling and simulation.

There are two measures of blackout size that immediately present themselves as useful for blackouts. Utilities are interested in number of failures such as transmission line failures because these are operational data that can be monitored in a control center and can sometimes be prevented or mitigated. Customers, industry, regulators and politicians are interested in quantities that directly affect them such as load shed or energy not served.

For an extensive listing and short description of previous work by other authors in cascading failure blackouts we refer the reader to [18] (particularly for cascading failure in power systems) and [22] (cascading failure in general). Much of the authors' previous work in cascading failure blackouts ([8], [4], [7], [22], [6], [17]) is summarized in [18].

We now briefly summarize the most immediate technical background for this paper. Branching processes [26], [2], [24] are shown to approximate an abstract model of cascading failure called CASCADE in [17]. CASCADE is compared to a power systems model of cascading line outages in order to estimate failure propagation in [6], [20]. Initial work fitting supercritical branching processes in discrete and continuous time to observed blackout data is in [21].

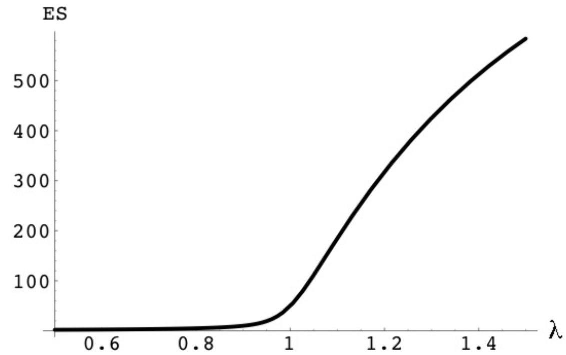

Fig. 1. Average number of failures in branching process model with $n = 1000$ as λ increases. Critical loading occurs at kink in curve at $\lambda = 1$ where the average number of failures sharply increases.

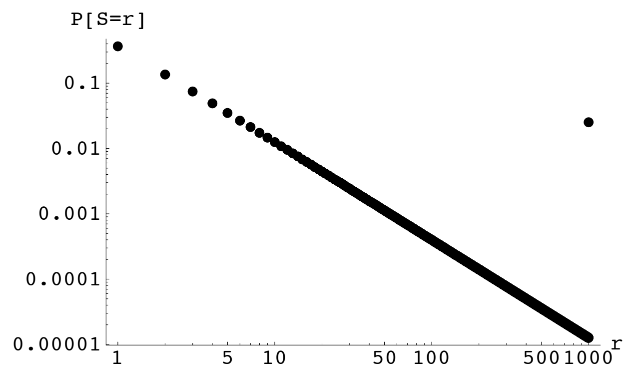

Fig. 2. Log-log plot of PDF of total number of failures in branching process model at criticality.

II. Criticality and blackout risk

As load increases, it is clear that cascading failure becomes more likely, but exactly how does it become more likely? Our previous work shows that the cascading failure does not gradually and uniformly become more likely; instead there is a transition point at which the cascading failure becomes increasingly more likely. This transition point has some of the properties of a critical transition or a phase transition.

In complex systems and statistical physics, a critical point for a type 2 phase transition is characterized by a discontinuity of the gradient in some measured quantity. At this point fluctuations of this quantity can be of any size and their correlation length becomes of the order of the system size. As a consequence, the probability distribution of the fluctuations has a power tail. Figures 1 and 2 show the criticality phenomenon in the branching process cascading failure model that is introduced in section III. At criticality Figure 2 shows a power dependence with exponent -1.5 before saturation. (A power dependence with exponent -1 implies that doubling the blackout size only halves the probability and appears on a log-log plot as a straight line of slope -1. An exponent of -1.5 as shown by the slope -1.5 in the log-log plot of Figure 2 implies that doubling the blackout size divides the probability by $2^{1.5}$.)

A similar form of critical transition has been observed in blackout simulations [4], [11] and abstract models of cascading failure [22], [17]. A power law distribution of blackout size with exponent between -1 and -2 is also consistent with the empirical probability distribution of energy unserved in North American blackouts from 1984 to 1998 [8], [9]. This suggests that the North American power system has been operated near criticality. The power tails are of course limited in extent in a practical power system by a finite cutoff near system size corresponding to the largest possible blackout. The distribution of the number of elements lost in North American contingencies from 1965 to 1985 [1] also has a heavy tail distribution [13].

Blackout risk is the product of blackout probability and blackout cost. Here we conservatively assume that blackout cost is roughly proportional to blackout size, although larger blackouts may well have costs (especially indirect costs) that increase faster than linearly [3]. The importance of the power law tail in the distribution of blackout size is that larger blackouts become rarer at a similar rate as costs increase, so that the risk of large blackouts is comparable to, or even exceeding, the risk of small blackouts [5]. For example, if the power law tail for the blackout size has exponent -1, then doubling blackout size halves the probability and doubles the cost and the risk is constant with respect to blackout size. A little less approximately, consider in Figure 3 the variation of blackout risk with blackout size computed from the branching process model at criticality. The pdf power law exponent of -1.5 is combined with the assumed linear increase in costs to give

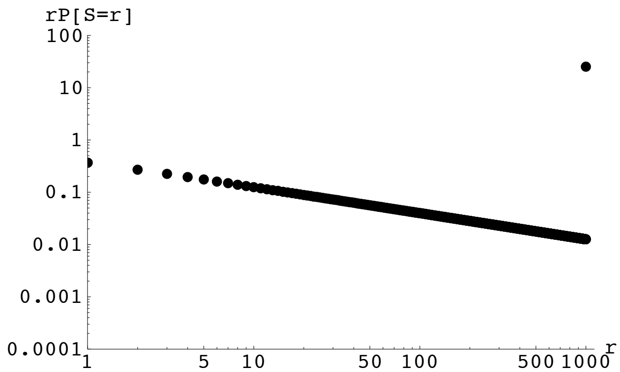

Fig. 3. Blackout risk $rP[S=r]$ as a function of number of failures r. Cost is assumed to be proportional to the number of failures and is measured in arbitrary units.

a modest -0.5 power law decrease in risk before saturation. The risk of the saturated case of all 1000 components failing is substantial. We conclude that the power law tails in both the NERC data and the blackout simulation results imply that large blackouts cannot be dismissed as so unlikely that their risk is negligible. On the contrary, the risk of large blackouts is substantial near criticality. Standard probabilistic techniques that assume independence between events imply exponential tails and are not applicable to blackout risk.

The terminology of "criticality" comes from statistical physics and it is of course extremely useful to use the standard scientific terminology. However, while the power tails at critical loading indicate a substantial risk of large blackouts, it is premature at this stage of knowledge to automatically presume that operation at criticality is bad simply because it entails some substantial risks. There is also economic gain from an increased loading of the power transmission system.

III. Branching process model

One approach models the growth of blackout failures using a branching process and then estimates the branching process parameter λ that measures both the extent to which failures propagate after they are started and the margin to criticality. We first summarize a basic branching process model. Branching process models are an obvious choice of stochastic model to capture the gross features of cascading blackouts because they have been developed and applied to other cascading processes such as genealogy, epidemics and cosmic rays [26]. The first suggestion to apply branching processes to blackouts appears to be in [17].

There are more specific arguments justifying branching processes as useful approximations to some of the gross features of cascading blackouts. Our idealized probabilistic model of cascading failure [22] describes with analytic formulas the statistics of a cascading process in which component failures weaken and further load the system so that subsequent failures are more likely. We have shown that this cascade model and variants of it can be well approximated by a Galton-Watson branching process with each failure giving rise to a Poisson distribution of failures in the next stage [17], [19]. Moreover, some features of this cascade model are consistent with results from cascading failure simulations [6], [20]. All of these models can show criticality and power law regions in the distribution of failure sizes or blackout sizes consistent with NERC data [8]. While our main motivation is large blackouts, these models are sufficiently simple and general that they could be applied to cascading failure of other large, interconnected infrastructures.

The Galton-Watson branching process model [26], [2] gives a way to quantify the propagation of cascading failures with a parameter λ. In the Galton-Watson branching process the failures are produced in stages. The process starts with M_0 failures at stage zero to represent the initial disturbance. The failures in each stage independently produce further failures in the next stage according to an probability distribution with mean λ. The failures "produced" by one of the failures in the previous stage can be thought of that failure's children or offspring and the distribution of failures produced by one of the failures in the previous stage is sometimes called the offspring distribution.

The branching process is a transient discrete time Markov process and its behavior is governed by the parameter λ. In the subcritical case of $\lambda < 1$, the failures will die out (i.e., reach and remain at zero failures at some stage) and the mean number of failures in each stage decreases exponentially. In the supercritical case of $\lambda > 1$, although it possible for the process to die out, often the failures increase exponentially without bound.

There are obviously a finite number of components that can fail in a blackout, so it must be recognized that the cascading process will saturate when most of the components have failed. Moreover, many observed cascading blackouts do not proceed to the entire interconnection blacking out. The reasons for this may well include inhibition effects such as load shedding relieving system stress, or successful islanding, that apply in addition to the stochastic variation that will limit some cascading sequences. Understanding and modeling these inhibition or saturation effects is important. However, in some parts of this paper such as estimating λ, we avoid this issue by analyzing the cascading process before saturation occurs.

Analytic formulas for the total number of components failed can be obtained in some cases. For example, assume that there are M_0 initial failures, the offspring distribution is Poisson with mean λ, and the process saturates when n components fail. Then the total number of failures S is distributed according to a saturating Borel-Tanner distribution:

$$P[S=r] = \begin{cases} M_0\lambda(r\lambda)^{r-M_0-1}\dfrac{e^{-r\lambda}}{(r-M_0)!}; & M_0 \le r < n \\ 1-\displaystyle\sum_{s=M_0}^{n-1} M_0\lambda(s\lambda)^{s-M_0-1}\dfrac{e^{-s\lambda}}{(s-M_0)!}; & r = n \end{cases} \tag{1}$$

Forms of saturation different than that in (1) are described in [17], [20].

Approximation of (1) for large $r < n$ using Stirling's formula and a limiting expression for an exponential yields

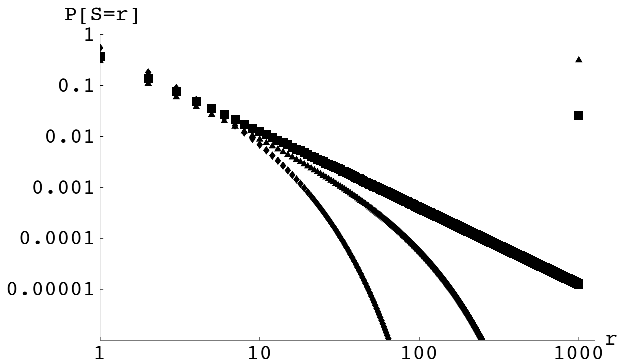

Fig. 4. Log-log plot of PDF of total number of failures in branching process model for three values of λ. $\lambda = 0.6$ is indicated by the diamonds. $\lambda = 1.0$ (criticality) is indicated by the boxes. $\lambda = 1.2$ is indicated by the triangles.

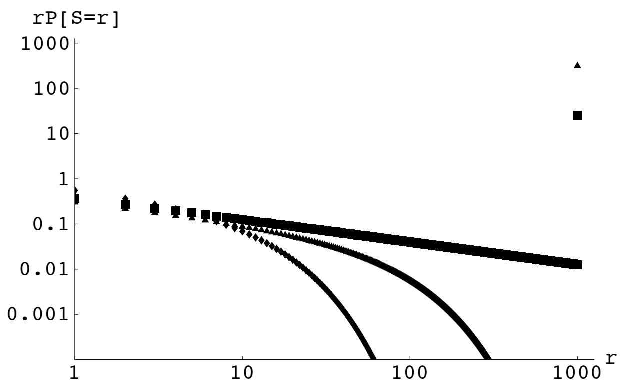

Fig. 5. Blackout risk $rP[S = r]$ as a function of number of failures r for three values of λ. $\lambda = 0.6$ is indicated by the diamonds. $\lambda = 1.0$ (criticality) is indicated by the boxes. $\lambda = 1.2$ is indicated by the triangles. Cost is assumed to be proportional to the number of failures and is measured in arbitrary units.

$$P[S = r] \approx \frac{M_0}{\sqrt{2\pi}} \lambda^{-M_0} r^{-1.5} e^{-r/r_0}; \quad 1 \ll r < n \quad (2)$$
$$\text{where } r_0 = (\lambda - 1 - \ln \lambda)^{-1}$$

In approximation (2), the term $r^{-1.5}$ dominates for $r \lesssim r_0$ and the exponential term e^{-r/r_0} dominates for $r_0 \lesssim r < n$. Thus (2) reveals that the distribution of the number of failures has an approximate power law region of exponent -1.5 for $1 \ll r \lesssim r_0$ and an exponential tail for $r_0 \lesssim r < n$.

The qualitative behavior of the distribution of blackout size as λ is increased can now be described. This behavior is illustrated in Figure 4. For subcritical λ well below 1, r_0 is well below n and the exponential tail for $r_0 \lesssim r < n$ implies that the probability of large blackouts of size near n is exponentially small. The probability of large blackouts of size exactly n is also very small. As λ increases in the subcritical range $\lambda < 1$, the mechanism by which there develops a significant probability of large blackouts of size near n is that r_0 increases with λ so that the power law region extends to the large blackouts. For near critical $\lambda \approx 1$, r_0 becomes large and exceeds n so that power law region extends up to $r = n$. For supercritical λ well above 1, r_0 is again well below n and there is an exponential tail for $r_0 \lesssim r < n$. This again implies that the probability of large blackouts of size near n is exponentially small. However there is a significant probability of large blackouts of size exactly n and this probability of total blackout increases with λ.

Figure 5 shows the distribution of risk with respect to the number of failures for the same values of λ considered in Figure 4. The essential point is that, given an assumption about the blackout cost as a function of blackout size, the branching process model gives a way to compute blackout risk in terms of λ. Both the expected risk of Figure 1 and the distribution of that risk over blackout size of Figure 5 can be computed.

A variant of the branching process produces potential failures at each stage according to the offspring distribution. Then the potential failures fail independently with probability p. For example, if one thinks of each failure as overloading other components according to the offspring distribution, then this corresponds to either the failure overloading and failing only a fraction of the components [19] or only a fraction of the overloaded components failing [20]. This is a simple form of emigration added to the branching process in the sense that the potential failures leave the process [2, page 266]. If the offspring distribution without emigration has generating function $f(s)$ and propagation λ, then the process with emigration is a branching process with generating function $g(s) = f(1 - p + ps)$. It follows that

$$\lambda_{\text{emigration}} = g'(1) = pf'(1) = p\lambda \quad (3)$$

IV. Detecting criticality in blackout models

We suggest and outline methods of detecting subcriticality or supercriticality and the closeness to criticality from a generic blackout simulation model.

A. Blackout model assumptions

For a given initial failure and a given loading or stress level L, the model produces

1) A sequence of failures. The failures correspond to the internal cascading processes such as transmission line outages. Often models will naturally produce failures in stages in an iterative manner. If not, then the failures need to be grouped into stages. In run j, the model produces failures M_{j0}, M_{j1}, M_{j2}, ... where M_{jk} is the number of failures in stage k.
2) A blackout size such as load shed or energy unserved. In run j, the model produces blackout size B_j.

There is a means of randomizing the initial failure and the system initial conditions so that different sequences of failures at the loading level L are generated for each run. There are a number of different blackout models that satisfy these generic assumptions [4], [11], [23], [25], [27].

Although L may often be chosen as an overall system loading such as total system load or total mean of random loads, there are other important ways of parameterizing the overall system stress. L could measure the overall system margin or reserves, as for example in [6], where the system "loading" is measured by the ratio of generator reserve to load variability or the average ratio of transmission line power flow to line maximum power rating. L could also be the amount of a power transfer across a system. In the sequel we will refer to L as "loading" for convenience while retaining its expansive interpretation as a measure of overall system stress.

One important issue is that instead of regarding all the failures as equivalent and counting them equally, one can weight them according to their importance. For example, the relative impact of a transmission line failure on the system is roughly proportional to the power flowing on it, so that an appropriate weight is the maximum power rating. If the maximum power ratings for individual lines are not available, then the nominal voltage squared (proportional to the surge impedance loading) could be used for the weight.

B. Distribution of blackout size

The model is run to accumulate statistics of the pdf of blackout size. Inspection of the probability of a large blackout at saturation and the extent to which there is a power law region reveals whether the pdf is subcritical or supercritical. This method has been applied to several power system blackout models [4], [11] and was also used to process observed blackout data from NERC [8]. The method does not quantify the closeness to criticality and it is very time consuming to approximate the pdf accurately, especially for the rare large blackouts near criticality. For example, in [4] 60 000 runs were used to estimate the pdf of blackout size of a 382 bus network.

C. Mean blackout size

The mean blackout size $\mu(L)$ at the loading level L can be estimated by J runs using

$$\mu(L) = \frac{1}{J}\sum_{j=1}^{J} B_j \tag{4}$$

Then the sharp change in the slope of the expected blackout size at criticality can be exploited to test for subcriticality or supercriticality (this assumes a type 2 phase transition at criticality). Suppose it is known from previous computations that the slope of the mean blackout size with respect to loading L is approximately $\mathrm{slope}_{\mathrm{sub}}$ below the critical value of L and approximately $\mathrm{slope}_{\mathrm{super}}$ above the critical value of L. Define the average slope

$$\mathrm{slope}_{\mathrm{average}} = \frac{1}{2}(\mathrm{slope}_{\mathrm{sub}} + \mathrm{slope}_{\mathrm{super}}) \tag{5}$$

Estimate the local slope by evaluating with the model $\mu(L+\Delta L)$ and $\mu(L)$ for small ΔL and using

$$\mathrm{slope}\mu(L) = \frac{\mu(L+\Delta L)-\mu(L)}{\Delta L} \tag{6}$$

Then

$$\text{stress } L \text{ is} \begin{cases} \text{subcritical if } \mathrm{slope}\mu(L) < \mathrm{slope}_{\mathrm{average}} \\ \text{supercritical if } \mathrm{slope}\mu(L) > \mathrm{slope}_{\mathrm{average}} \end{cases} \tag{7}$$

This approach gives as a useful byproduct the slope of the mean blackout size with respect to loading.

Now the critical loading and hence the margin to critical loading can be found with further computations of $\mu(L)$ at different values of L. Since (7) gives a way to test whether L is less than or above the critical loading, it is straightforward to approximate the critical loading by first finding an interval containing the critical loading and then interval halving. The interval containing the critical loading is found by increasing L until supercriticality if the first tested L is subcritical and decreasing L until subcriticality if the first tested L is supercritical.

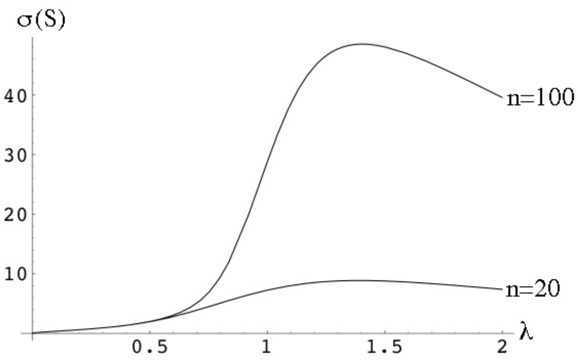

Fig. 6. Standard deviation of the total number of failures S as a function of λ for saturation at $n = 20$ failures and $n = 100$ failures.

We now roughly estimate the number of runs J needed to accurately obtain $\mu(L)$ at a single loading level L. We assume that the runs correspond to independent samples, each starting from one initial failure, and that the failures are generated by a branching process with a Poisson offspring distribution with mean λ and saturation at n failures. Then in run j, the total number of failures S_j is distributed according to the Borel-Tanner distribution (1) with $M_0 = 1$. We also make the simple assumption that the blackout size B_j is proportional to the total number of failures S_j. The standard deviation of $\mu(L)$ is then proportional to $\sigma(S)/\sqrt{J}$, so that the number of runs depends on the standard deviation $\sigma(S)$ of S. If saturation is neglected, $\sigma(S) = \sqrt{\lambda/(1-\lambda)^3}$ becomes infinite as λ increases to criticality at $\lambda = 1$. The saturation makes $\sigma(S)$ larger but finite near criticality as shown in Figure 6. (To obtain Figure 6, the variance of S was obtained via evaluating $D_t^2 E t^S$ at $t = 1$ with computer algebra.) For example, if saturation is at 100 components and $\lambda = 1.3$, then $\sigma(S) = 48$ and a mean blackout size standard deviation corresponding to 0.5 failures requires $(48/0.5)^2 = 9200$ runs. If saturation is instead at 20 components then $\sigma(S) = 9$ and the same accuracy can be achieved with $(9/0.5)^2 = 320$ runs. The number of runs depends greatly on λ, the accuracy required and the saturation.

The mean blackout size $\mu(L)$ was computed for a range of system loadings for several different power system cascading failure models in [4], [11], [27].

D. Propagation λ

We would like to estimate the average propagation λ over a stages. The a stages are limited to the period before saturation effects apply, because the branching process model assumed for the estimation is a branching process model without saturation that only applies to the propagation of failures before saturation. Define the total number of failures in each stage by summing over the J runs

$$M_k = M_{1k} + M_{2k} + ... + M_{Jk}, \qquad k = 1, 2, ..., a \tag{8}$$

Define the cumulative number of failures up to and including stage k to be

$$S_k = M_0 + M_1 + M_2 + ... + M_k \tag{9}$$

Then an estimator for λ is [24], [15]

$$\hat{\lambda} = \frac{M_1 + M_2 + ... + M_a}{M_0 + M_1 + ... + M_{a-1}} = \frac{S_a - M_0}{S_{a-1}} = \frac{S_a - M_0}{S_a - M_a} \tag{10}$$

$\hat{\lambda}$ is a maximum likelihood estimator when observing numbers of failures in each stage for a wide class of offspring distributions, including the exponential family. $\hat{\lambda}$ is biased and its mean underestimates λ, but the bias is inversely proportional to the number of runs J [24, pp. 37-39]. In the special case of $a = 1$, $\hat{\lambda} = M_1/M_0$.

The first stage is usually comprised of the initiating failures. The number of stages a could be limited by one of several methods. For example, to avoid the saturation effects the number of stages could be limited so that the fraction of components failed was below a threshold.

If grouping failures into stages is needed, then, since (10) only requires S_a, M_0, and M_a, it is only necessary to group failures into the first stage to obtain M_1 and into the last stage to obtain M_a. To group failures into stages, the failure data will be assumed to include the time of each failure and perhaps some additional data explaining the causes of the failure and specifying the type and location of the failure. Factors that would tend to group several failures into the same stage could be their closeness in time or location, or being caused by failures in the previous stage.

We now roughly estimate the number of runs J needed to accurately obtain $\hat{\lambda}$. We assume that the runs correspond to independent samples, each starting from one initial failure, and that the failures are generated by a branching process with a Poisson offspring distribution with mean λ. Then as J tends to infinity, the standard deviation of $\hat{\lambda}$ is asymptotically [24, p. 53]

$$\sigma(\hat{\lambda}) \sim \frac{\sigma_{S_{1a}}(\lambda, a)}{\sqrt{J}} = \frac{1}{\sqrt{J}} \sqrt{\frac{\sum_{j=0}^{2a} \lambda^{j+1} - (2a+1)\lambda^{a+1}}{(\lambda - 1)^2}} \tag{11}$$

where $\sigma_{S_{1a}}(\lambda, a)$ is the standard deviation of the total number of failures S_{1a} produced by one initial failure $M_{10} = 1$. That is, $S_{1a} = M_{10} + M_{11} + \ldots + M_{1a}$. Note that $\sigma_{S_{1a}}(1, a) = \sqrt{(a + 3a^2 + 2a^3)/6}$. Figure 7 shows $\sigma_{S_{1a}}(\lambda, a)$. For example, if $\lambda = 1.3$ and the number of stages $a = 5$, then $\sigma_{S_{1a}}(1.3, 5) = 15$ and $\sigma(\hat{\lambda}) = 0.05$ requires $(15/0.05)^2 = 90\,000$ runs. If instead the number of stages $a = 2$ then $\sigma_{S_{1a}}(1.3, 2) = 3$ and the same accuracy can be achieved with $(3/0.05)^2 = 3600$ runs. The number of runs depends greatly on λ, the accuracy required, and the number of stages a.

To illustrate the choice of the number of stages a to avoid saturation, suppose that the failures saturate at $n = 100$ and that we can assume that $\lambda \leq 1.5$. Then in the most rapidly saturating case of $\lambda = 1.5$, the mean number of failures in stage k is 1.5^k. The mean total number of failures in stage 6 is 32 and the standard deviation of the total number of failures is $\sigma_{S_{1a}}(1.5, 6) = 38$. Therefore to avoid saturation we can choose the number of stages a in the computation of $\hat{\lambda}$ in the range $1 \leq a \leq 6$.

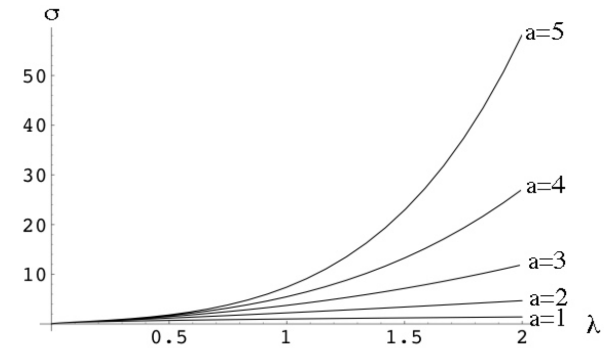

Fig. 7. $\sigma(\hat{\lambda})\sqrt{J} = \sigma_{S_{1a}}(\lambda, a)$ as a function of λ for number of stages $a = 1, 2, 3, 4, 5$.

V. Conclusions and research agenda

This paper discusses branching process models for cascading failure and shows how assuming these models gives a way to roughly estimate expected blackout risk and risk of blackouts of various sizes as a function of the branching process parameter λ. λ describes the average extent to which failures propagate and measures the closeness to criticality. At criticality $\lambda = 1$ and the branching process models show a power tail in the distribution of blackout size and a sharp rise in expected blackout size. The way in which the power law region extends as criticality is approached is described. Then we suggest approaches to determining the closeness to criticality via the expected blackout size or λ from runs of a generic cascading failure blackout model. Some rough estimates of computational effort are made. The approaches in this paper augment previous work relating branching models and other abstract models of cascading failure to power system blackout models and power system data [6], [20], [21]. Further development and testing of measures of closeness to criticality is needed. In particular, estimating λ and assuming a branching process model can yield the distribution of the risk of blackouts of various sizes as well as the average risk.

We now expand our focus and address more generally the research needed to further explore and develop the possibilities of bulk statistical analysis of blackout risk. We consider key research issues for two aspects. In the first aspect the power transmission system is assumed to be fixed and the main objective is to determine how close the system is to a critical loading at which the expected blackout size rises sharply and there is a substantial risk of large blackouts. In the second aspect, the power transmission system slowly evolves subject to the forces of rising demand and the upgrade of the transmission system in response to the blackouts. These dynamics of transmission system evolution can be seen as a form of self-organization in a complex system [7], [5].

A. Measuring proximity to criticality in a fixed network

Some research issues are:

Research access to blackout data. To develop models and methods based on reality, it is essential for blackout data to be collected and for researchers to have access to the data. Although the precise data needs have not yet evolved and will require iteration, it is clear that bulk statistical analysis of blackouts will neglect much of the blackout detail, so that concerns about confidentiality and homeland security can be addressed by only releasing a suitably and substantially filtered record of the blackout events. Discussion about which filters succeed in resolving confidentiality and homeland security concerns would be helpful. One specific goal is to gain research access to the data from the August 2003 blackout of Northeastern America that was collected for the blackout report [38].

Blackout costs. To estimate blackout risk, blackout cost needs to be approximated as a function of blackout size and, while there is considerable information available for smaller blackouts, the direct and indirect costs of large blackouts seem to be poorly known.

Confirm criticality phenomenon. While criticality has been observed in several power system blackout models [4], [11], it needs to be confirmed in power system blackout models representing different interactions and with varying levels of detail in order to be able to conclude that it is a universal feature of cascading failure blackouts. If no criticality or a different sort of criticality is observed, this needs to be understood.

Power system blackout models. The main issues are the tradeoffs between what interactions to model and in what detail to model them, test system size and computational speed.

Abstract cascading failure models. These models presently include branching process models in discrete and continuous time and CASCADE models. These models require substantial refinement and further comparison and validation with real and simulated blackout data to ensure that the main features of blackouts are represented. In particular, blackouts being inhibited and saturating at a fraction of the system size needs to be understood and better modeled.

Monitoring closeness to criticality. Suggested initial approaches are described in this paper and [6], [20], [21]. Much more needs to be done to establish practical statistical methods for monitoring closeness to criticality. Processing of failure data into stages and the appropriate scalings need to be investigated.

The critical loading as a power system limit. The critical loading essentially provides an additional system limit that guides power system planning and operation with respect to the risk of cascading failure. In contrast to an indirect way of limiting cascading failure such as the n-1 criterion, the critical loading directly relates to the risk of cascading failure. The appropriate operating margin to this limit should be based on risk computations and is not yet known. Little is known about the properties of the critical loading as power system conditions change. It would be very useful to be able to identify some easily monitored quantities that are strongly correlated to the critical loading [6], because this would open up the possibility of monitoring the closeness to criticality via these quantities. It would also be useful to evaluate the performance of the n-1 criterion when used as a surrogate for the critical loading limit.

Progression from understanding phenomena to offline models to online monitoring. The research questions above focus on understanding phenomena, developing and validating models and measuring closeness to criticality in power system models and in past blackouts. Once these questions start to be resolved, there is a natural progression to consider the feasibility of schemes to practically monitor closeness to criticality of power systems online.

B. Complex systems dynamics of power systems.

The complex systems dynamics of transmission network upgrade can explain the power tails and apparent near-criticality in the NERC data [8]. The complex system studied here includes the engineering and economic forces that drive network upgrade as well as the cascading failure dynamics. As a rough explanation, below criticality increasing load demand and economic pressures tend to increasingly stress the system. But when the system is above the critical loading, blackout risk rises and the response to real or simulated blackouts is to upgrade the system and relieve the system stress. Thus the system will tend to vary near criticality in a complex systems equilibrium. The system can be said to self-organize to near criticality. A power systems model that incorporates slow load growth and a simple form of transmission upgrade at lines involved in cascading blackouts converges to such a complex systems equilibrium [7]. Moreover, as might be expected in a complex system, simple forms of blackout mitigation can have the desired effect of decreasing small blackouts but also the somewhat counterintuitive effect of ultimately increasing large blackouts [5]. Other theories that can generate power laws or similar behavior include the influence model [34], highly optimized tolerance [35], graph-theoretic network analysis [40] and cluster models for line outages [13].

Some research issues are:

Reframing the problem of blackouts. Instead of simply avoiding all blackouts, the problem is to manage blackout risk both by manipulating the probability distribution of blackout size [5] and by finding ways to minimize blackout costs [36]. Blackout mitigation should take into account complex systems dynamics by which the power system and society slowly readjust themselves to any changes made.

Models for complex system dynamics. For theories such as the influence model, highly optimized tolerance, or graph-theoretic network analysis the challenge is to construct models of power systems and their evolution with an explicit correspondence to the abstract model and study their properties. For the self-organizing complex systems theory, such a model already exists and the challenge is to improve its representation of the engineering and economic forces, and particularly the transmission upgrade, economic investment and human factor aspects. Part of the challenge is understanding cascading failure and complex systems dynamics across several interacting or coupled complex systems [31]. It is necessary to balance the requirements for computational speed and accessibility of data against the requirements of a detailed model. It may be necessary to develop a hierarchy of models of varying detail to accommodate varying emphases on speed versus model detail.

Analysis tools. Diagnostics for monitoring and studying complex systems dynamics need to be developed.

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Ian Dobson received the BA degree in Mathematics from Cambridge, England in 1978 and the PhD degree in Electrical Engineering from Cornell University in 1989. He worked from 1978 to 1983 as a systems analyst for the British firm EASAMS Ltd. In 1989 he joined the University of Wisconsin-Madison, where he is now a professor in electrical and

computer engineering. His current interests are applications of complex systems and nonlinear dynamics, electric power system blackouts and instabilities, and risk analysis of cascading failure.

Benjamin A. Carreras received the Licenciado en Ciencias degree in Physics from the University of Barcelona, Spain in 1965 and the PhD degree in Physics from Valencia University, Spain in 1968. He has worked as a researcher or as a professor at the University of Madrid, Spain, Glasgow University, Scotland, Daresbury Nuclear Physics Laboratory, England, Junta de Energia Nuclear, Madrid, Spain, and the Institute for Advanced Study, Princeton, NJ USA. He is now corporate fellow at Oak Ridge National Lab, TN USA. He is a fellow of the American Physical Society.

David E. Newman received the BS degree in Physics and Mathematics from the University of Pittsburgh, PA in 1983 and the Ph.D. degree in Physics from the University of Wisconsin, Madison, WI in 1993. He was a Wigner fellow and research scientist at Oak Ridge National Laboratory from 1993 to 1998. In 1998 he joined the Physics department at the University of Alaska, Fairbanks, where he is now associate professor.

6.7 The Impact of Various Upgrade Strategies on the Long-Term Dynamics and Robustness of the Transmission Grid

D. E. Newman, B. A. Carreras, V. E. Lynch, I. Dobson

Electricity Transmission in Deregulated Markets
Conference at Carnegie Mellon University
Pittsburgh PA USA December 2004

The Impact of Various Upgrade Strategies on the Long-Term Dynamics and Robustness of the Transmission Grid

David E. Newman, Benjamin A. Carreras, Vickie E. Lynch and Ian Dobson

Abstract— **We use the OPA global complex systems model of the power transmission system to investigate the effect of a series of different network upgrade scenarios on the long time dynamics and the probability of large cascading failures. The OPA model represents the power grid at the level of DC load flow and LP generation dispatch and represents blackouts caused by randomly triggered cascading line outages and overloads. This model represents the long-term, slow evolution of the transmission grid by incorporating the effects of increasing demand and engineering responses to blackouts such as upgrading transmission lines and generators. We examine the effect of increased component reliability on the long-term risks, the effect of changing operational margins and the effect of redundancy on those same long-term risks. The general result is that while increased reliability of the components decreases the probability of small blackouts, depending on the implementation, it actually can increase the probability of large blackouts. When we instead increase some types of redundancy of the system there is an overall decrease in the large blackouts with a concomitant increase of the smallest blackouts. As some of these results are counter intuitive these studies suggest that care must be taken when making what seem to be logical upgrade decisions.**

Index Terms—**blackouts, power system security, cascading failure, reliability, risk analysis, complex system, phase transition.**

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D. E. Newman is with the Physics Department, University of Alaska, Fairbanks, AK 99775 (phone: 907-474-7858; fax: 907-474-6130; e-mail: ffden@uaf.edu).

B. A. Carreras and V. E. Lynch are with the Fusion Energy Division at Oak Ridge National Laboratory, Oak Ridge, TN 37831

I. Dobson is with the ECE Department, University of Wisconsin, Madison, WI 53706 (email dobson@engr.wisc.edu).

I. Introduction

THE recent large scale disruptions to the power transmission network [1] have once again focused a great deal of attention on improving the reliability of the network. However, because of the many different approaches that can be taken in moving toward the goal of improving the robustness of the Electric Power Transmission systems the understanding of the system wide effect of various improvement measures becomes a high priority task for the community. This is both because the expense of these improvements can be enormous and one would like some estimate as to their effectiveness as well as because it is possible that some of the improvements could have counter intuitive results [8].

In this paper we use a global dynamic model (OPA) [2, 3] for the evolution of a large transmission network with which we can explore the long time effects of various improvement schemes. This model is used because it has been found to exhibit long time dynamics with characteristics found in the real power transmission system [4]. As these characteristics include the long time correlations of the system and the frequency of blackouts of various sizes (the blackout PDF), it is appropriate for investigating the impact of the improvement schemes. Specifically, we can characterize the impact of these improvements on the probability or frequency of blackouts of various sizes. The schemes we investigate here are three. First we investigate the impact of increasing the reliability of individual components of the system. Due to the way the components are represented, it is not easy to discriminate from a second improvement method, namely changing the operating safety margin. Finally, we look at the impact of implementing component redundancy on the system. Because of the general nature of the model and because each of these techniques themselves have many ambiguities in their implementation, this should be thought of as an initial survey which perhaps highlights the complexity of the question and the need for further study rather than giving definitive answers.

In the next section we will briefly describe the model and present the results of the different improvement schemes. Finally there is a section on discussion, conclusions and suggestions for further work.

II. Modeling Reliability and Redundancy

A. OPA

The OPA model [2, 3] has been developed as a realization of the global complex dynamics briefly described in the previous section. The OPA model represents the essentials of slow load growth, cascading line outages, and the increases in system capacity caused by the engineering responses to blackouts. Lines fail probabilistically and the consequent redistribution of power flows is calculated using the DC load flow approximation and a standard LP re-dispatch of generation. Cascading line outages leading to blackouts are modeled and the lines involved in a blackout are predicted. The engineering response to the blackout is crudely modeled as an increase in line margin for the lines that were involved in the blackout. The OPA model clearly represents the processes in greatly simplified forms, although the interactions between these processes still yield complex (and very complicated!) behavior. The simple representation of the processes is desirable both to study only the main interactions governing the complex dynamics and for pragmatic reasons of model tractability and simulation run time. This also allows the study of various network configurations, from simple tree type networks that allow some analytic analysis, to a more realistic IEEE test networks such as those shown in Figures 1 and 2.

Blackouts in OPA are complicated events involving line outages and limitations in generation. We can characterize them by two limiting situations each with different dynamical properties [3, 5]. One type of blackout is associated with multiple line outages. The second type of blackout involves loss of load due to generators reaching their limits but no line outages. In general, both effects appear in most blackouts, but for a given blackout, one of these characteristic properties is dominant. The dominance of one type of blackouts versus the other depends on operational conditions and the proximity of the system to one of its two critical points [6]. The first critical point is characterized by operation with lines close to their limits. The second critical point is characterized by the maximum fluctuations of the load demand being near the generator margin capability. When the generator upgrade is suitably coordinated with the line upgrade, the critical points coincide and the model can show a probability distribution of blackout sizes with power tails similar to that observed in NERC blackout data [7]. Similar results are found in both the idealized tree network and a more realistic network (Figs. 1 and 2). One of the important results from these models is that even though the individual causes of each blackout event might vary, the statistics of these events remain remarkably robust. This is because the system rearranges itself to stay near the operational limit at which these statistics (PDFs etc) are characteristic. This rearrangement is likely the result of a combination of the social and economic pressures on the system interacting with the system design and operation and the engineering responses to the blackouts.

Here we look at some different responses and differently engineered systems in order to investigate whether these different systems have similar dynamics and statistics. Note that in this paper we are not studying the short-term effect of the different engineering measures on a fixed network. Instead we are investigating the effect of the different engineering measures on the complex systems equilibrium that is achieved after the system has rearranged itself on the time scale of the dynamics of load growth and network upgrade.

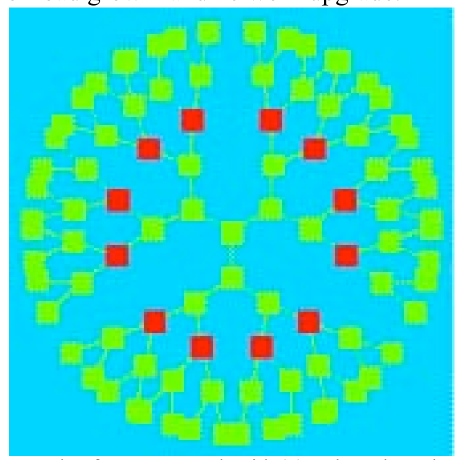

Fig. 1. Example of a tree network with 94 nodes. The red squares are generator nodes.

For the results presented here we work mainly with the IEEE 118 bus network, however, this network is modified for the redundancy studies.

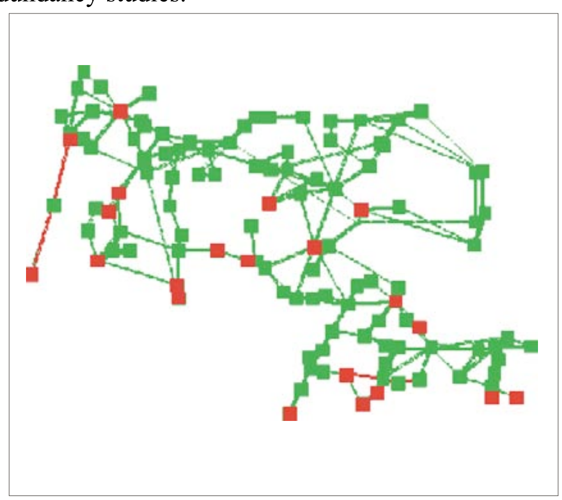

Fig. 2. The IEEE 118 bus network. The red squares are generator nodes.

B. Reliability/Margin Improvements

At the initial level of inquiry, the investigations of the improvements in component reliability are, in this model, an investigation of both component reliability and operating margin. This is because of the way we implement reliability improvement in the model. Due to the general nature of the model we do not model the individual components in any detail. For example, transmission lines and transformers are both considered as part of the lines joining nodes and in this paper when we refer to lines, we mean the lines and the components that make them up. The lines, and their constituent components, have failure probabilities for different situations. For example, each line has a certain probability of random failure (P_0). These can be thought of as failures caused by either uncontrolled external influences (a lightning strike, a squirrel in a transformer etc) or by the random failure of the line due to a defect or ageing. Each line also has a load driven or stress failure specified by P_1. We use the fraction of overloading, $M = F/F_{\max}$, as a measure of the stress on the line, where F is the power flow in a given line and $F_{\max}$ is the limiting power flow. When a component is within a given distance (margin) of its operating limit, M_R, it has a probability of failing (P_1) and then being upgraded. Reducing the random failure probability P_0 does little to the dynamics over a range of values. However changing the margin M_R at which P_1 starts to have an influence can have a significant effect on the system. The margin M_R for onset of P_1 can be interpreted in a number of ways. The first and perhaps most straightforward is that this onset margin is simply the operating margin that the operators strive to maintain given the knowledge that there is an increased failure probability above that point. Because the lines at their onset margins are not yet at their hard limits (emergency ratings) there is some additional margin engineered into the system. In this system if there is a line outage (even if there is no power shed) the line (component) is upgraded. This tends to keep the overall system farther from the critical point. The other way of interpreting the margin M_R is in terms of line reliability. If a line is made more reliable then it has a smaller probability of failing before its hard limit is reached. That can be thought of as a decrease in the margin to the hard limit. That is, a more reliable line can carry higher loadings that have no chance of loading induced failure. There could be a concomitant decrease in P_0 but, as stated before, that has a small effect.

The effect of changing the probability P_1 was studied in detail in [8]. The expectation from this form of increase in the reliability of the lines is an overall decrease in the frequency of the blackouts. Furthermore, large blackouts with many failures are also expected to be less likely because of the decreased probability of cascading line failures. As expected, we saw in previous work [8] that reducing P_1 reduces the probability of large blackouts. However, this is not the only change observed in the dynamics. With the decrease of large blackouts, there is a concomitant increase in the number of small blackouts. The overall result is that there is hardly any change on the frequency of blackouts. As discussed in [8], the increase of reliability through P_1 induces only a logarithmic decrease in cost of the blackouts

When the margin $1\text{-}M_R$ is changed a very noticeable change in the distribution of power shed and outages is seen. Figure 3 shows a large reduction in the largest blackouts when $1\text{-}M_R$ is increased from zero (i.e. it is at the hard limit) to 20%. That is, the local load point at which failures start and upgrades can occur is in the best case 0.8 times the hard limit for the individual lines. This decrease in the largest event probability is up to a factor of five for the largest blackouts. Looked at in the other interpretation, this implies that increasing the component reliability can increase the probability of the largest events by a significant amount.

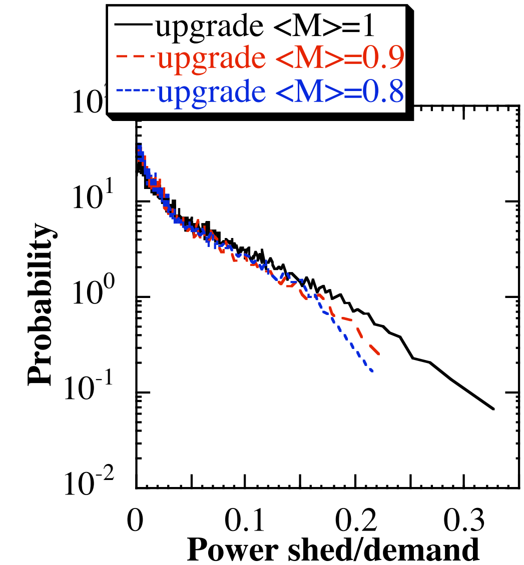

Fig. 3. The probability distribution of blackout size for 3 operating margins, 0, 0.1 and 0.2. The blackout size is measured by the power shed normalized by the total power demand. A marked decrease is seen in the cases with an increases margin, or conversely, a marked increase in the largest events is seen when the system has the most reliable components.

In Fig. 4 the probability distribution of line outages is plotted for the same cases. This shows clearly that as the margin increases the largest outages (those that often cause blackouts) are decreased while there is a concomitant increase in the smaller outages. This is consistent with the power-shed results and again suggests that the increased margin makes the system less prone to large failures, which could be interpreted making the system more robust. Once again, the other way of interpreting this is that as the line reliability increases, the probability of large failures increases which is perhaps a counterintuitive result.

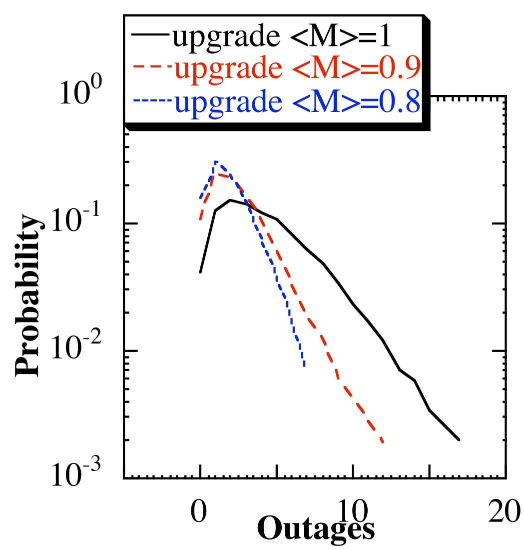

Fig. 4. The probability distribution of number of line outages for 3 operating margins, 0, 0.1 and 0.2. A marked decrease in the largest sizes and an increase in the smallest sizes is seen in the cases with an increased margin, or, conversely, a marked increase in the largest events is seen when the system has the most reliable components.

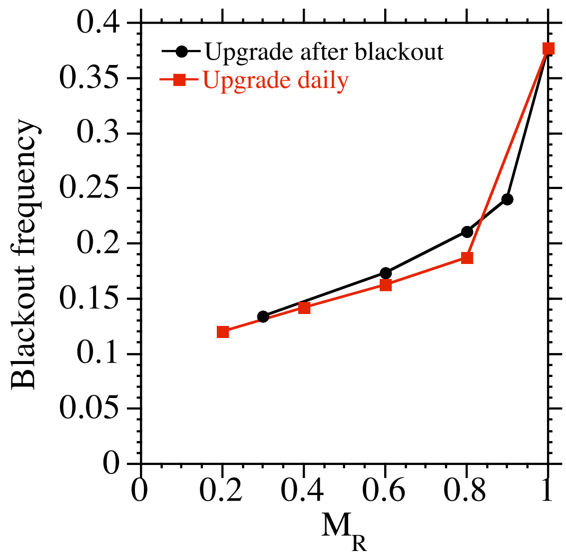

Fig. 5. Frequency of blackouts decreases as the fractional margin point decreases. The two upgrade schemes (failure based or daily prophylactic upgrades) give approximately the same improvement.

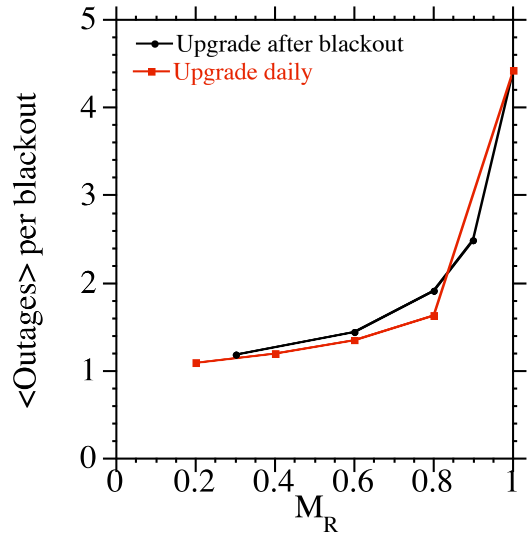

Fig. 6. The mean number of outages per blackout is also seen to decrease as the operating limit decreases. The two upgrade schemes (failure based or daily prophylactic upgrades) again give approximately the same improvement.

The upgrades to this system can be handled in two different ways. The standard method is to wait for a component failure and blackout and then upgrade the components after the failure. This is the standard implementation used for OPA in most cases. However, one can also envisage strengthening the network by increasing the operating margins of stressed lines before they fail. This implementation keeps track of the line loading and those lines that are in their margin region are upgraded preventatively at the end of the day. Surprisingly, both methods had the same effect on the system at least in the parameter range we are using. Figure 5 shows the blackout frequency as a function of the operating limit (M_R) for both upgrade methods. The daily, prophylactic upgrades are a little bit better but are effectively the same as the failure based upgrades in decreasing the blackout frequency.

Figure 6 shows that not only does the frequency of the blackouts decrease, but also the blackout size decreases as the margin is made larger. Once again the two upgrade schemes give approximately the same results.

It should be seen that for both of these measures, the blackout frequency and size, the largest improvement (a factor of more then 2) is found in going from no margin to the 20% margin. After that, the improvement with increasing margin is much slower. Stated using our reliability interpretation of the margin, this means that improving line reliability up to a point does not seriously impact the statistics, but after that point it can have a major effect.

Figure 7 shows the number of blackouts of a given size for the various margins. This shows even more clearly that the largest change in the distribution comes in going from no margin (M_R=1) to a 20% margin (M_R=0.8). After this, the distributions change little except for a modest decrease in the smaller blackouts.

The actual power shed per blackout has a minimum around M_R =0.7-0.8. This is because after the largest events are removed, a further decrease in the smallest blackouts (which are more likely) actually increases the mean size since now the larger blackouts are reduced less. This can be seen in Fig. 7 looking carefully at the smallest sizes or much more easily in Fig. 8.

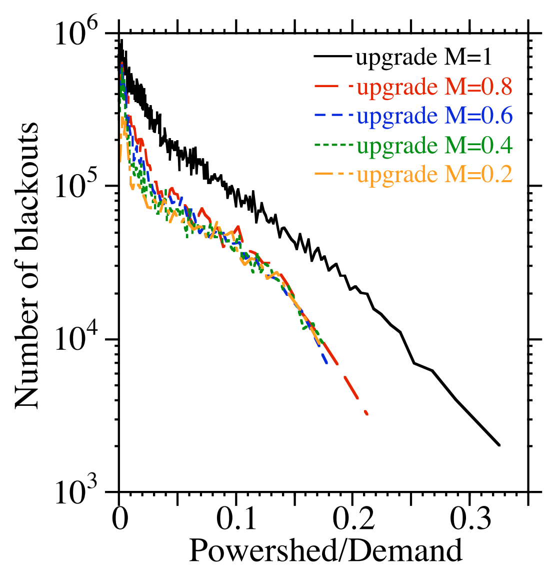

Fig. 7. The number of large blackouts as a function of blackout size for various operating limits M_R The overall decrease in the number of blackouts is much larger for the first 20% increase in margin.

Figure 8 shows the stark difference between the distributions in the first 20% margin increase followed by the overall reduction of the frequency and a slow decrease in the larger events.

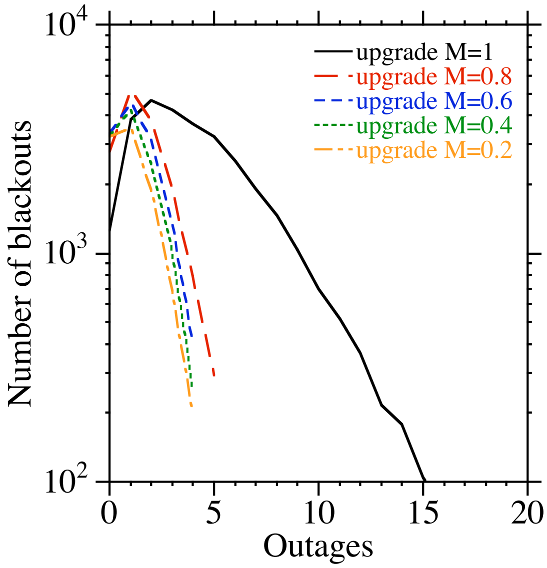

Fig. 8. The maximum number of outages decreases dramatically for the first 20% increase in margin, then the smallest number decreases faster (note the vertical log scale)

This suggests that a working margin of 20-30% is for this model near optimum in terms of both robustness of the overall system and economic efficiency. Likewise, if the component reliability becomes such that the upgrades are not done until just before their hard limit, the system is likely to be more susceptible to large cascading failures.

C. *Redundancy*

Within the OPA model, investigating redundancy has even more ambiguities of definition. For example one can have redundant capacity without having redundant components (lines). This would be accomplished by making the operational margin at least 50%. This would be the same as increasing the margin as in the last section but would do nothing for the random failures. Another possibility is having parallel lines, each of which is able to carry the entire load. In normal operation they will each run at 50% capacity (i.e. with M_R for each line at 0.5). This allows for a failure in one line being fully mitigated by the other line. Finally there is a variant on the last option that involves having a fully redundant second component that is not used unless the main component fails. The first two cases have the difficulty of being susceptible to the strong social and economic pressures to utilize the unused capacity. This would tend over the course of time to remove the redundancy from the system and simply end up with two parallel fully utilized components at which point the system is likely to be in a more vulnerable situation then before [10, 11]. The methods we have investigated are the first 2.

Figure 9 shows the effect of adding redundant lines. Adding the lines around the generators, which tend to be the limiting areas, reduces the frequency of the largest blackouts, with a modest increase in the smallest blackouts. However the largest change in large blackout frequency is seen when all lines are doubled (made redundant). In this case the large blackout frequency is reduced by almost 30% and the overall frequency of blackouts is not much changed.

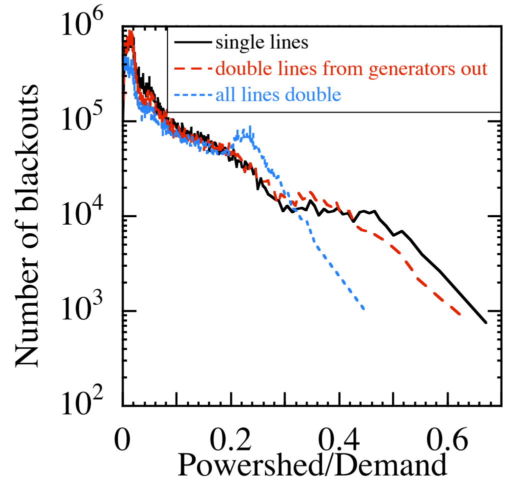

Fig. 9. Doubling lines from the generators decreases the number of large blackouts somewhat. Doubling all lines has the largest effect on reducing the number of large blackouts.

Adding levels of redundancy does little to further protect the system. Figure 10 shows a system in which the lines are doubled and then tripled. The improvement in the doubling of the lines is not enhanced in any significant way by tripling the lines.

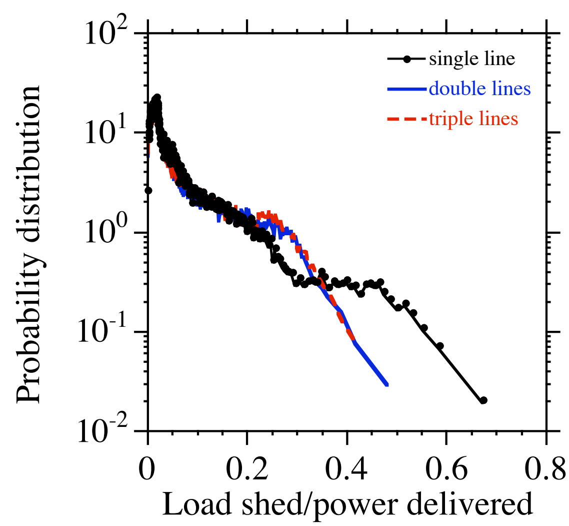

Fig. 10. Reductions in the largest blackouts are seen when adding a set of redundant lines. However adding additional lines beyond that does little additional good.

III. Discussion and Conclusion

In dynamic complex systems models of the power transmission system can reproduce the dynamics, power tails and apparent near criticality observed in the NERC data [4]. The complex system model, studied here includes a representation of the engineering and economic forces that drive network upgrades as well as leading to the cascading failure dynamics. These dynamics come from a competition between two forces. On one side, the increasing load demand and economic pressures that tend to add stress to the system. On the other side, as the system becomes more stressed, the blackout risk rises and the response to blackouts is upgrades to the system which then relieves the system stress. From the competition between the forcing and upgrades, the system tends to organize it self near to the critical point in a complex systems equilibrium. The utility of this type of model is not in the analysis of an individual blackout but rather overall system dynamics as the system responds to slow forcing.

This type of model allows the exploration of various changes in the system engineering and operation in order to investigate the effect of these changes on risk of large failures and system dynamics. In this paper we looked at two of these changes, line component reliability (or margin improvements) and redundancy. The result from these preliminary studies suggests that improving the reliability of lines (or line components) can have a counter intuitive effect. That effect is an increase in large blackouts as the reliability is increased (or the operating margin is decreased). Adding redundant lines on the other hand is found to reduce the probability of large blackouts.

This type of model, with these results, lead naturally to a series of areas for further/future research:

1) System upgrade schemes - Modeling of redundancy and reliability need to be improved and explored in more depth. This should include real reliability characteristics, various redundancy models and a combination of both. Reliability modeling should include at least 4 probabilities associated with component reliability; external random failure, defect failure, aging failure, and stress failure. In addition to these simple system upgrade explorations this type of model allows for the investigation the impact of various islanding schemes on blackout risk.

2) Interacting complex systems – In reality, the complex system model of the power transmission network is one part of the interacting infrastructure system that controls the transmission grid. These interacting infrastructures include economic systems, IT systems and human decision making systems. Incorporating these as separate interacting complex systems or as "agent based models" within the transmission network complex system model needs to be investigated to explore the effect on risk from the system interactions.

3) In order to both compare models to the real system and to develop for real time control and risk assessment techniques, new system state metrics need to be developed. These should be developed for system monitoring and comparison.

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Acknowledgment

The authors thank Raul Sanchez for useful discussions.

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6.8 Risk assessment in complex interacting infrastructure systems

D. E. Newman, B. Nkei, B. A. Carreras, I. Dobson, V. E. Lynch, P. Gradney

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Risk Assessment in Complex Interacting Infrastructure Systems

D. E. Newman
Physics Department University of Alaska, Fairbanks, AK 99775 USA ffden@uaf.edu

Bertrand Nkei
Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA nnb@fed.ornl.gov

B. A. Carreras
Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA carrerasba@ornl.gov

I. Dobson
ECE Department, University of Wisconsin, Madison, WI 53706 USA dobson@engr.wisc.edu

V. E. Lynch
Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA lynchve@ornl.gov

Paul Gradney
Physics Department University of Alaska, Fairbanks, AK 99775 USA

Abstract

Critical infrastructures have some of the characteristic properties of complex systems. They exhibit infrequent large failures events. These events, though infrequent, often obey a power law distribution in their probability versus size. This power law behavior suggests that ordinary risk analysis might not apply to these systems. It is thought that some of this behavior comes from different parts of the systems interacting with each other both in space and time. While these complex infrastructure systems can exhibit these characteristics on their own, in reality these individual infrastructure systems interact with each other in even more complex ways. This interaction can lead to increased or decreased risk of failure in the individual systems. To investigate this and to formulate appropriate risk assessment tools for such systems, a set of models are used to study to impact of coupling complex systems. A probabilistic model and a dynamical model that have been used to study blackout dynamics in the power transmission grid are used as paradigms. In this paper, we investigate changes in the risk models based on the power law event probability distributions, when complex systems are coupled.

1. Introduction

It is fairly clear that many important infrastructure systems exhibit the type of behavior that has come to be associated with "Complex System" dynamics. These systems range from electric power transmission and distribution systems, through communication networks, commodity transportation infrastructure arguably all the way to the economic markets themselves. There has been extensive work in the modeling of some of these different systems. However, because of the intrinsic complexities involved, modeling of the interaction between these systems has been limited [1,2]. While understandable from the standard point of view that espouses understanding the components of a large complex system before one tries to understand the entire system, this approach can unfortunately overlook important consequences of the coupling of these systems that impact their safe operation and overlooks critical vulnerabilities of these systems. At the same time, one cannot simply take the logical view that the larger coupled system is just a new larger complex system because of the heterogeneity introduced through the coupling of the systems. While the individual systems may have a relatively homogeneous structure, the coupling between the systems is often both in terms of spatial uniformity and in terms of coupling strength, fundamentally different (Figure 1). This in the most extreme case leads to uncoupled systems but in the more normal region of parameter space in which the inter-system coupling is weaker or topologically different then the intra-system coupling can lead to important new behavior. Understanding the effect of this coupling on the system dynamics is necessary if we are to accurately develop risk models for the different infrastructure systems individually or collectively.

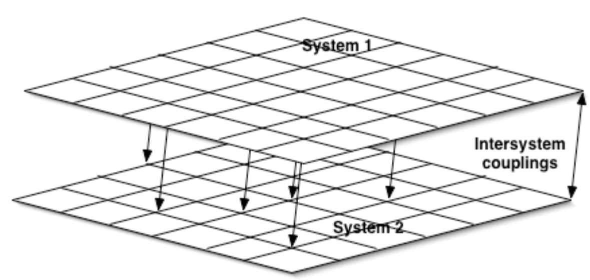

Figure 1: Cartoon of two homogeneous systems with a heterogeneous coupling

Examples of the types of potential coupled infrastructure systems to which this would be relevant include power-communication systems, power-market systems, communication-transportation systems, and even market-market systems. Interesting examples of these interactions are discussed in ref. [3]. The effect of this coupling can be critical and obvious for systems that are strongly coupled such as the power – market coupled system. Perturbations in one can have a rapid and very visible impact on the other. In fact, in many ways such systems are often thought of as one larger system even though the coupling is not homogeneous and each of the component systems (namely the market and the power transmission system) can have their own separate perturbations and dynamics. For other less tightly coupled systems, such as power-communications systems, the effect can be much more subtle but still very important. In such systems small perturbations in one might have very little obvious effect on the other system, yet the effect of the coupling of the two systems can have a profound effect on the risk of large, rare disturbances.

In this paper, we will investigate some of these effects using two different approaches. First we will use a simple probabilistic model for cascading failures (CASCADE) that has been extensively studied for individual systems [4-6]. This model allows us to probe the impact of the coupling on the failure risks and the critical point that has been previously found for the uncoupled systems. This model also has the advantage of allowing some analytic solutions. Next we will present results from a dynamical model of coupled complex systems. This model has dynamic evolution and many of the characteristics found in complex systems.

Throughout this paper for reference purposes we will use the power transmission system as the primary system and the communications systems as the coupled secondary system. In reality, the models discussed have very little specific to these systems. They will be used so the results are more general in nature and we use these reference systems simply to be able to give concrete examples of the actions and effects we discuss.

Many complex systems are seen to exhibit similar characteristics in their failures. While it is useful and important to do a detailed analysis of the specific causes of these failures such as individual blackouts, it is also important to understand the global dynamics of the systems like the power transmission network. This allows some insight into the frequency distribution of these events (e.g. blackouts) that the system dynamics creates. There is evidence that global dynamics of complex systems is largely independent of the details of the individual triggers such as shorts, lightning strikes etc in power systems. In this paper, we focus on the intrinsic dynamics of failures and how this complex system dynamics impacts failure risk assessment in interconnected complex systems. It is found, perhaps counter intuitively, that even weak coupling of complex systems can have adverse effects on both systems and therefore risk analysis of an isolated system must be approached with care.

Several particular issues induced by the interdependence of systems will be addressed in this paper. The first one is how coupling between the systems modifies conditions for safe operation. These systems are characterized by a critical loading [7, 8]. They must operate well-below this critical loading to avoid "normal accidents" [9] and large scale failures. We will explore how the coupling between systems changes the value of this critical loading.

We will also consider the effect of the heterogeneity introduced through in two different ways. Through the different properties of each individual system, like having different critical points, and the coupling of the systems.

Finally we will contrast probabilistic models with dynamical models in order to see the effect of memory in the system impacts the consequences of the couplings.

The rest of the paper will be organized as follows: Section 2 reviews some of the characteristics of complex systems. Section 3 contains a description of the coupled cascade model and results from that model. Section 4 describes the dynamic model with results from that model, followed by section 5 that has a discussion of the implications of these results and conclusions.

2. Coupled CASCADE model

2.1 Individual CASCADE model

The basic CASCADE model [4-6] has n identical components with random initial loads. For each component the minimum initial load is $L_{\min}$ and the maximum initial load is $L_{\max}$. For j=1,2,...,n, component j has an initial load of l_j that is a random variable uniformly distributed in [$L_{\min}$, $L_{\max}$]. $l_1, l_2, \cdots, l_n$ are independent. Components fail when their load exceeds L_{fail}. When a component fails, a fixed amount of load p is transferred to each of the components.

To start the cascade, we assume an initial disturbance that loads each component with an additional amount, d. Components may then fail depending on their initial loads, l_j, and the failure of any of these components will distribute an additional load, $p \geq 0$, that can cause further failures in a cascade. This model describes the cascading failure as an iterative process. In each iteration, loads fail as the transfer load, p, from other failures makes them reach the failure limit. The process stops when none of the remaining loads reaches the failure limit. It is useful to define $\lambda \equiv np$, the total load transferred from a failing component. This system is found to have a transition in the probability of system wide failures (P_∞) at a critical value of λ. As shown in Fig. 2, when $\lambda < \lambda_c$, where λ_c is the critical value of λ, $P_\infty = 0$. However, above the critical value for λ, system wide failures are possible. In the CASCADE model if we assume a uniform random distribution of loads, the critical point is $\lambda_c = 1$.

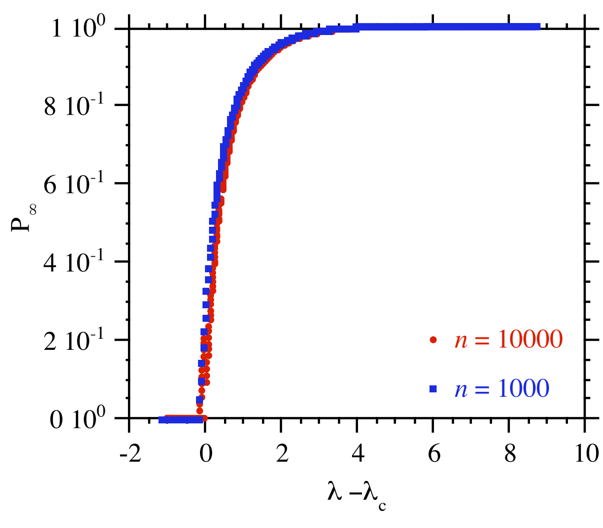

Figure 2: Probability of cascade events of the system size as a function of λ

An important characteristic of the CASCADE model is that around the critical point, the probability distribution function (pdf) of the size of the failures develops a power law tail. In the uniform load case, this power law tail has a characteristic exponent of approximately –1.5. This power law behavior is important because the effect of a failure is proportional to its size so if the probability of failures falls as a power law less steep then –2.0, the large failures dominate the "cost" of failure.

2.2 Coupled CASCADE models

Generalizing the CASCADE model to a pair of coupled CASCADE systems is straightforward. We consider two systems *L* and *M* with random loads (normalized on 0 to 1):

System *L* $\quad l_i \in [0,1]\ i = 1,...n_L$

System *M* $\quad m_j \in [0,1]\ j = 1,...n_M$

At the beginning of each "day" (realization), the random initial loads are generated. We will simplify the situation by considering only initial perturbations in the system *L*. As an initial perturbation, we add an increment *d* to all loads of the components in system *L*. As before, a component fails if its normalized load is greater than 1. For each failed component, we transfer a load p_{LL} to the loads of all other components in the same way that we did in the individual model. Now however, when component i of *L* fails, all loads of the components of system M are increased by an amount p_{ML}. This cross system loading is the inter-system coupling. It should not be thought of as actually distributing the load for L to the other system, rather one can think of it as an increased stress in system M due to failures in system L.

Likewise, when a component in the system *M* fails a load p_{MM} is transferred to all loads of the other components of the system *M* in the same way as was done in system *L*. Finally, we have the back cross loading coming when a component j of *M* fails then all loads of the components of system *L* are increased by an amount p_{LM}.

The basic steps of the algorithm proceed as follows:

At Step t

1) Test stability of all loads in *L* based on their values at step *t*-1.
2) Test possible transfer from *L* to *M* based on the load values at step *t*-1.
3) Test stability of all loads in *M* based on their values at step *t*-1.
4) Test possible transfer from *M* to *L* based on the load values at step *t*-1.

Now update all loads

At the end of each "day" we collect information on how many components failed in *L* and how many in *M*, how long the whole cascade took, and accumulate information for a pdf of failures in both systems. We also accumulate data per iteration from each system, in order to calculate the number of failures per iteration.

The CASCADE model can be re-interpreted as a branching process [10]. This allows the application of the branching process methods [11] to analyze and interpret the results of the cascade model. In trying to understand the consequences of the coupled CASCADES model, we approximate it by a branching process. For simplicity we assume that the two systems have the same size and have symmetric couplings. From the load transfers we can construct the corresponding the transition probability as was done in Ref.[10]. In this case, we define λ_{ij}=n p_{ij}. Then if $F_L(t)$ and $F_M(t)$ are the mean number of failures in systems *L* and *M* respectively, we have

$$\begin{pmatrix} F_L(t) \\ F_M(t) \end{pmatrix} = \begin{pmatrix} \lambda_{LL} & \lambda_{LM} \\ \lambda_{ML} & \lambda_{MM} \end{pmatrix} \begin{pmatrix} F_L(t-1) \\ F_M(t-1) \end{pmatrix} \tag{1}$$

with

$$\begin{pmatrix} F_L(1) \\ F_M(1) \end{pmatrix} = \begin{pmatrix} \theta \\ 0 \end{pmatrix} \tag{2}$$

and

$\theta = nd$

This a 2 type branching process approximation to the evolution of the means in the coupled CASCADE model that generalizes the approximation in [10]. Therefore, iteration of Eq. (1) with the initial condition (2) leads to

$$\begin{pmatrix} F_L(t) \\ F_M(t) \end{pmatrix} = \begin{pmatrix} \lambda_{LL} & \lambda_{LM} \\ \lambda_{ML} & \lambda_{MM} \end{pmatrix}^{t-1} \begin{pmatrix} \theta \\ 0 \end{pmatrix} \tag{3}$$

To solve this system of equations we have to find the eigenvalues of the matrix, they are

$$\lambda_\pm = \frac{1}{2}\left[\lambda_{LL} + \lambda_{MM} \pm \sqrt{(\lambda_{LL} - \lambda_{MM})^2 + 4\lambda_{LM}\lambda_{ML}}\right] \tag{4}$$

Since all λ's are positives the largest eigenvalue is λ_+. Because of the initial conditions,

$$F_L(t) = \theta \frac{(\lambda_+ - \lambda_{MM})\lambda_+^{t-1} + (\lambda_+ - \lambda_{LL})\lambda_-^{t-1}}{\sqrt{(\lambda_{LL} - \lambda_{MM})^2 + 4\lambda_{LM}\lambda_{ML}}} \tag{5}$$

and

$$F_M(t) = \theta \frac{\lambda_+^{t-1} - \lambda_-^{t-1}}{\sqrt{(\lambda_{LL} - \lambda_{MM})^2 + 4\lambda_{LM}\lambda_{ML}}} \lambda_{ML} \tag{6}$$

As an easy test to start comparing the code, we could use $\lambda_{LL} = \lambda_{MM} = \lambda$ and $\lambda_{LM} = \lambda_{ML} = \delta$. In this case, $\lambda_\pm = \lambda \pm \delta$ and

$$F_L(t) = \theta\left[\frac{(\lambda+\delta)^{t-1} + (\lambda-\delta)^{t-1}}{2}\right] \tag{7}$$

and

$$F_M(t) = \left[\frac{(\lambda+\delta)^{t-1} - (\lambda-\delta)^{t-1}}{2}\right] \tag{8}$$

Because of the cascade nature of the process, the average number of failures diverges if the largest eigenvalue is greater than 1 and converges if it is less than 1. Therefore the critical point is now given by

$$\lambda_c = 1 - \delta \qquad (9)$$

This means that the coupling of the systems has shifted the critical point to a lower value of λ. The size of this shift is related to the strength of the coupling. This shift makes the system more susceptible to large failures. It is again important to note that the inter-system load transfer is intrinsically different then the intra-system load transfer. It is this difference that allows the shift in the critical point.

2.3 Numerical results

Numerically one can explore the parameter space to investigate the transition characteristics as a function of these parameters. Initially, we have considered only cases with $\lambda_{LL} = \lambda_{MM} = \lambda$ and $\lambda_{LM} = \lambda_{ML} = \delta$ in order to explore a small space to start with. For this situation we have only to worry about a single new parameter δ. Calculations have been done for two systems of size 400.

For a fixed initial perturbation, θ = 0.2, applied to the system *L*, we can see that the frequencies of the cascades in system *M* increases with λ+δ. This increase is faster when the system is close to the critical point (Fig. 3).

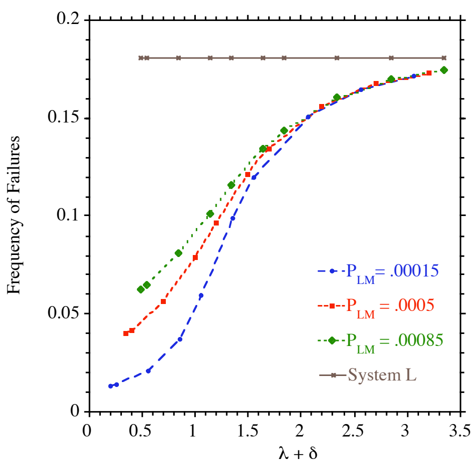

Figure 3: Frequency of failure as a function of λ + δ

Because system M is not perturbed, it is clear that the failures in system *L* drive the failures in system *M*. Below the critical point, the effect is weak. However, at the critical point both systems become strongly coupled. They act more like a single system.

In addition to the drive of system *M* by system *L*, there is clear feedback of system *M* on system *L*, because the critical point is shifted downwards as given by Eq. (9). The numerical results are consistent with the analytical calculation: both systems have the same critical point and the critical point is given by the largest eigenvalue λ+δ. This is shown in Figs. 4 and 5. . In Fig.4, we have plotted the probability of a system-size failure (the system as size 400) for system *L* as a function of λ for the different values of the coupling p_{LM}. Here, p_{LM} is the load transferred to each load of the system *L* by each failure in the system *M* . Then, $\delta = n\, p_{LM}$. We can see that the critical point is shifted to lower λ as p_{LM} increases. Note that with the strongest coupling there is almost a factor of 2 change in the critical point.

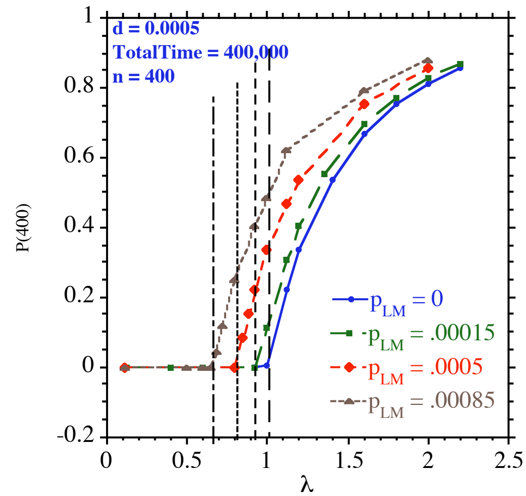

Figure 4: Probability of cascade events of the system size as a function of λ

That the shift in the critical point is given by δ is clearly shown in Fig. 5, where we have replotted the data in Fig. 4 as a function of λ+δ. A universal curve emerges from this plot. Plots of the system-size failure probability for system *M* are identical to the plots for system *L*.

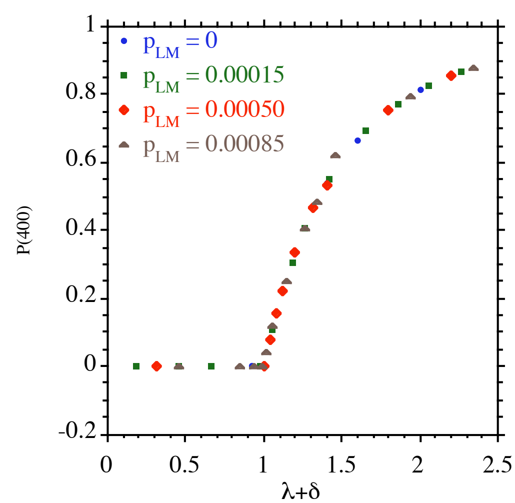

Figure 5: Probability of system size cascade events as a function of λ+δ

In Fig. 6, we have plotted the pdf of the cascade size for λ = 0.95 and δ = 0.06(just 0.01 above the threshold). Keep in mind that for system M there would be no failures at all if the systems were uncoupled while for system *L*, without the coupling the system would still be significantly sub-critical. The pdf of failures for system *L* has the usual slope of –1.5. Remarkably, the slope for system *M* is actually lower than for system L and is close to –1.2. The probability of small cascade in L triggering cascades in M is small. However, large cascades in L often trigger cascades in M. Therefore, the probability of system wide cascades is practically the same in both systems. It is this combination that leads to the shallower slope for system M.

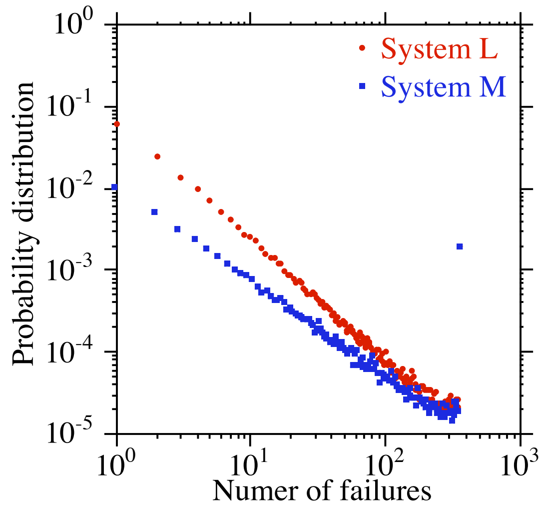

Figure 6: Probability of cascade events of the system size as a function of λ

In Figures 7 and 8 we see the evolution of a cascade for a case in which there would have been no cascade in M and the cascade in L would have stopped after 4 iterations had the systems been uncoupled. Figure 7 shows the number of failures per iteration as the cascade evolves and in this case the two systems are tightly coupled so number of failures per iteration is approximately the same for both systems.

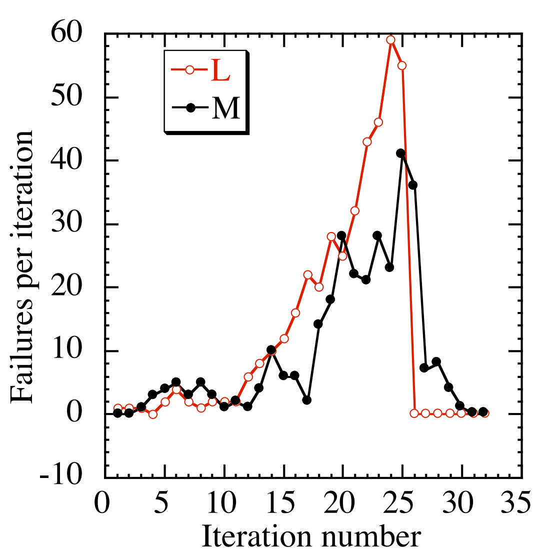

Figure 7: The evolution of failures in a cascade as a function of iteration for both systems.

In figure 8, which shows the cumulative number of failures in each of the two systems, the cascade can be seen to go all the way to the system size (400) in system L at approximately iteration 25. The cascade stops in system M when it reaches the full system size in L because it is no longer being driven by anything. System L is gone!

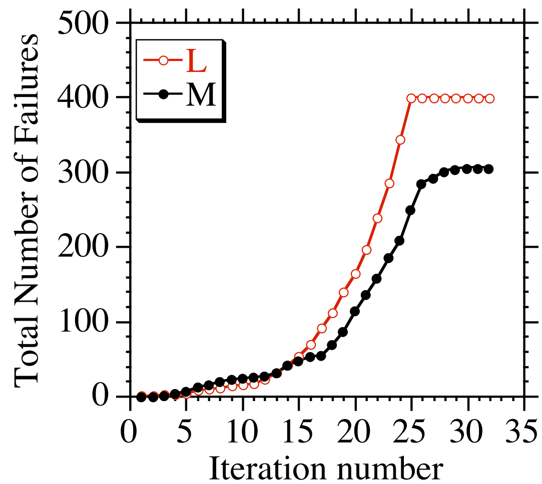

Figure 8: Evolution of the cumulative number of failures in a cascade as a function of iteration for both systems.

If one thinks of system L as a power transmission system and system M as an information communications system the meaning and effect of the coupling is fairly clear. The two systems are coupled in both directions at the simplest level because the communications system uses power to operate and because the communications system carries the information needed to operate the power transmission system. Failure in one increases the probability of failure in the other. For example a power failure increases the probability of a router failing, leading to information packet losses. This failure in the second system then can react back on the first system increasing its probability of further failure. For example, lack of knowledge of the operating state of a line increases the probability of an overload condition. This process facilitates the propagation of the cascade that is the mechanism by which the critical point is lowered.

Both the numeric and analytic approaches to understanding this model can be extended to cases that relax some of the simplifications we have made. Of most interest is relaxing the symmetry assumption in the coupling. This work will be presented in a subsequent paper.

3. A coupled complex system model

3.1 The simple dynamical complex system model

Probabilistic models such as the CASCADE model can shed light on the changes in the critical point and pdf of failures. However, their value is limited by their probabilistic nature. In order to develop sufficient statistics for these measures many realizations with independent initial conditions are performed with no knowledge of earlier cases. We know however that the real systems are deterministic and its state today knows about its state yesterday at least to some degree. Therefore, to investigate the dynamics of these systems we utilize a coupled dynamic complex system model (DCSM).

This DCSM is a cellular automata based model. It is set on a regular grid with fixed interaction rules. The systems we will discuss here are a subset in which the rules are local and the grid is regular. Both of these restrictions are straightforward to generalize (and for some systems other choices make more sense) but we use them as a reasonable starting point.

The rules for the single, uncoupled systems are simple:

1) A node has a certain (usually small) probability of failure (p_f)
2) A node neighboring a failed node has another (higher) probability of failing (p_s)
3) A failed node has a certain (usually high) probability of being repaired (p_r)

The steps taken in the evolution are equally simple:
At step t

1) The nodes are evaluated for random failure based on their state at the end of the t-1 step.
2) The nodes are evaluated for repair based on their state at the end of the t-1 step.
3) The nodes are evaluated for failure due to the state of their neighbors at step t-1.
4) All nodes are advanced to their new state

Outages (failures) in these systems can grow and evolve in non-uniform clusters and display a remarkably rich variety of spatial and temporal complexity. They can grow to all sizes from individual node failures to system size events. The repair rate for nodes is usually slower then the time scale of a cascading failure so repairs to an evolving cascade are unlikely. The main difference between this model and the CASCADE model discussed in Section 2, is the continued evolution of the system after a failure. In this system, the "memory" of previous failures is in the structure of failed and fixed nodes in the system. The characteristic time scales of the system are also captured in the repair time and random failure probability. This type of model gives power law tails in the pdf, as before, in addition to long time correlations and anti-correlations between the failures (Figure 9), something that comes from the dynamical memory of the system.

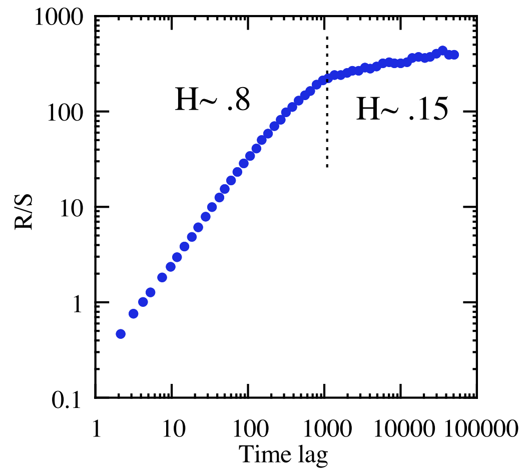

Figure 9: R/S as a function of time lag for a DCSM time series showing a Hurst exponent greater then 0.5 in the mesoscale region, signifying long time correlations.

3.2 The coupled complex system model

The coupling of these systems is achieved along similar line to that done in the CASCADE model. Namely, failures in one system change the probability of failure in the other system. The difference being that since, beyond mean field theory, the details of which will be presented elsewhere, we are unable to make much analytic progress with this model we do not worry about simplifying assumptions. Therefore we couple the two dynamical complex systems models DCSM1 and DCSM2 using two coupling variables. The first of these variables is the spatial structure of the coupling. Since all nodes in one system do not need to be coupled to all nodes in the other systems (in fact usually would not be), we can change the fraction of the nodes coupled (randomly or with a fixed structure). See figure 1 for a cartoon representation of this. The second variable is the strength and direction of the coupling. The strength of the coupling is the cross system probability of failure, similar to the p_{ML} from the coupled CASCADE model. However we do not restrict this coupling to being symmetric. In reality, some systems failures can have a major impact on its counterpart system while a failure in the counterpart system would have little or no effect on the first system. An example of this might be a co-located pipeline/communications system. The communication system is used to monitor the pipeline state. Failure of the communications system can (or often will) cause a failure (or shutdown) of the pipeline system. The converse is usually not true, a failure in the pipeline, unless it is a catastrophic failure, will have no impact on the communications system. Therefore both the strength and direction can be varied.

3.3 Preliminary results from the Coupled DCSM

As described here, the DCSM dynamically arranges itself to sit right at, or near the critical point for a wide range of parameters as long as we are above the percolation limit, which will be discussed below. This is why it is called a self-organized critical system. So unlike the CASCADE model we cannot do a simple λ scan in DCSM to explore the critical point because the system tries to arrange itself arranges to live at that point. However by changing the parameters in both the local coupling and the cross system coupling we can see changes in the failures which can be made explore similar dynamic changes as the lambda scans in CASCADE.

Figure 10 shows the time series of failures for a coupled system and an uncoupled system (with the same parameters other then the coupling), showing a large change in the dynamics of the system.

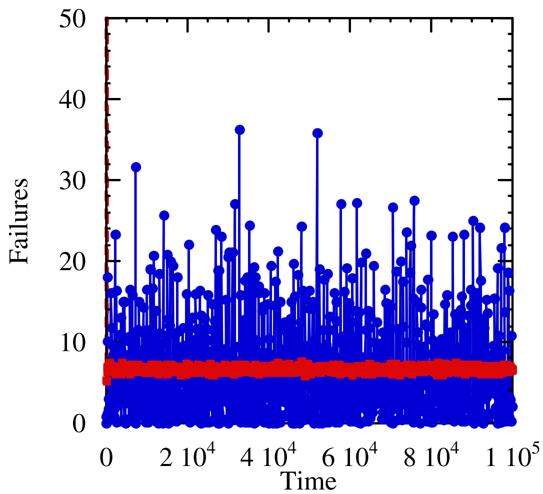

Figure 10: Time series of failure sizes in coupled and uncoupled DCSM

This figure simply illustrates the extreme differences that can be found between coupled and uncoupled systems, in this case when the coupling is strong and 2 way, causing constant small failures in the 2 systems. To begin a systematic understanding of the parameter space we first note a few of the characteristics of the uncoupled system. First is the local coupling parameter p_s, which when below a certain value makes the system sub-critical to the percolation threshold. This means that when the individual elements are coupled to few other elements, or when the coupling is very weak, the cascading failures will be self-limiting. That is, they will have a very low probability of propagating across the entire system and the distribution (PDF) of failure sizes will be exponential (Fig. 11). The threshold is reached when there is at least one failure on average caused by a failed site. This "percolation" threshold can be analytically approximated [12], using mean field theory, as $P_{ncrit} \sim 1/f$, with f being the average number of unfailed sites a site is connected to. This is approximately the number of connections-1 since, during a cascading failure, one of the connections will already be failed. Therefore, for our uncoupled DCSM model with four connections per site, the critical P_n is about 0.333. In reality, mean field theory underestimates the threshold value because long time correlations are not considered but the value is not far from that found as seen in figure 12.

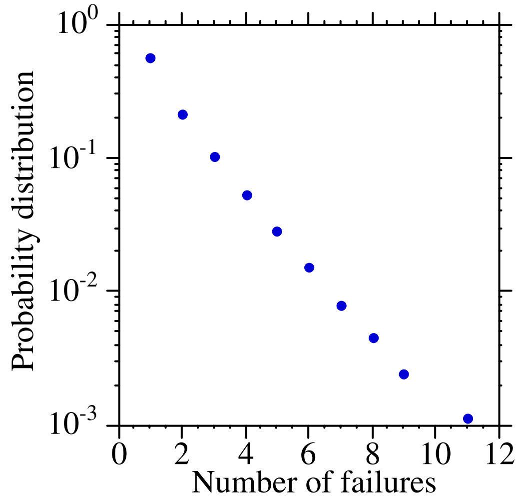

Figure 11: PDF of failure sizes in uncoupled DCSM with coupling parameter P_n=0.1, significantly less then the critical value. The PDF shows an exponential size distribution.

In this figure the critical point can be characterized as the point at which the average number of new failures caused by a failure (λ) equals, or exceeds, one. This is found to be approximately 0.4 for the full DCSM model, just a little above the mean field approximation. Once the system is above the critical point it display all the characteristics of a self-organized complex system.

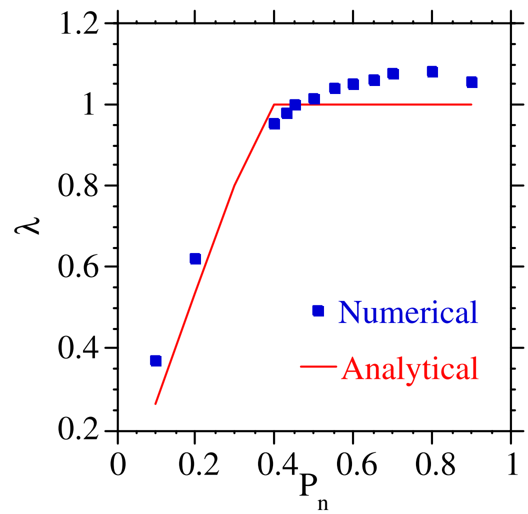

Figure 12: λ vs P_n showing critical point

These include the long time correlations (Fig. 9) and power law PDFs. The appearance of the power law size distribution as we cross the critical point is shown in figure 13 which has PDFs for a just barely critical case and a case with P_n well above the critical point. The power laws found have exponents of approximately –1 and exhibit the standard exponential cutoff at largest sizes due to finite system size effects. It should be noted that the power law of –1 is in contrast to the CASCADE model which, in the uncoupled case, has a power law of approximately –1.5 and is due to the dynamical evolution of the system.

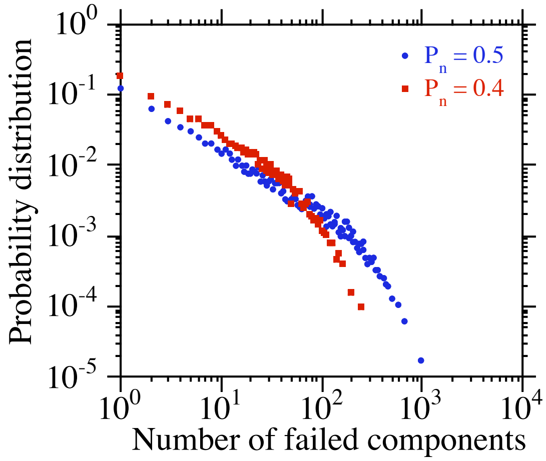

Figure 13: PDFs of failure sizes in 2 uncoupled DCSM calculations with the neighbor coupling parameter P_n=0.4 and 0.5, just at and above the critical value. The PDFs show a power law size distribution.

One of the simplest consequences of coupling the 2 systems is to give another propagation path for failures. If this did in fact occur one would expect that the critical point could be crossed by increasing the cross system coupling as well as by increasing the nearest neighbor coupling in a given system. This consequence can be seen in figure 14 in which the P_n is sub critical but the cross system coupling is able to make the system critical.

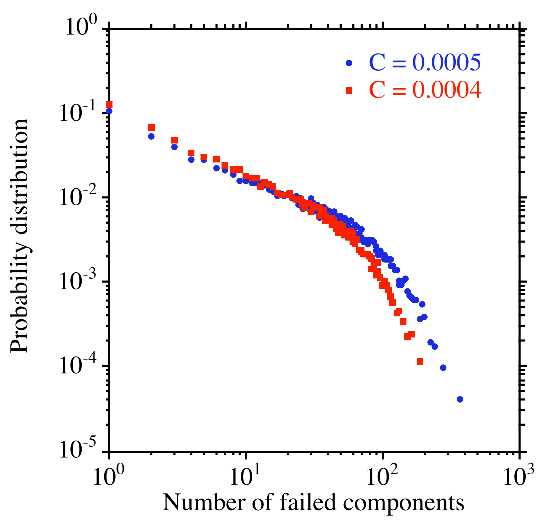

Figure 14: PDFs of failure sizes in 2 coupled DCSM calculations with coupling parameter P_n=0.4 and 0.5, just at and above the critical value.

In the coupled case, the power law found is somewhat weaker then the –1 found for the uncoupled system and is approximately 0.8. The direction of change (ie the weaker power law) is consistent with the effect seen in the coupled CASCADE model discussed in section 2.3, though the coupled DCSM power law is still significantly less steep then the coupled CASCADE result. The actual slope is critical for calculating and understanding the risk of events of various sizes and while changing from an exponential distribution to a power law is much more significant, going from a power law of –1.5 to -.8 will have a large impact on the probability of the largest failures.

Another obvious potential impact of the coupling is the possible synchronization of the failures in the two systems. Using a measure developed by Gann et al in [13] for synchronization, we investigate this effect. Figure 15 shows the synchronization function described in [13] which is basically an average normalized difference between events in the 2 systems. For this measure, a value of 1 means the difference is effectively 100% or no synchronization, while a value of 0 means all events are the same in the 2 systems, or they are synchronized. These values are then plotted as a function of the event sizes. It can be readily seen that small events for all three of the coupling strengths are largely uncorrelated (unsynchronized). The synchronization however increases as the size increases. This makes physical sense since as the even gets larger there are more sites interacting and this increases the probability that a failure in one system will trigger a failure in the other system. It should be noted that this is likely to be sensitive to the spatial homogeneity of the coupling that is being investigated.

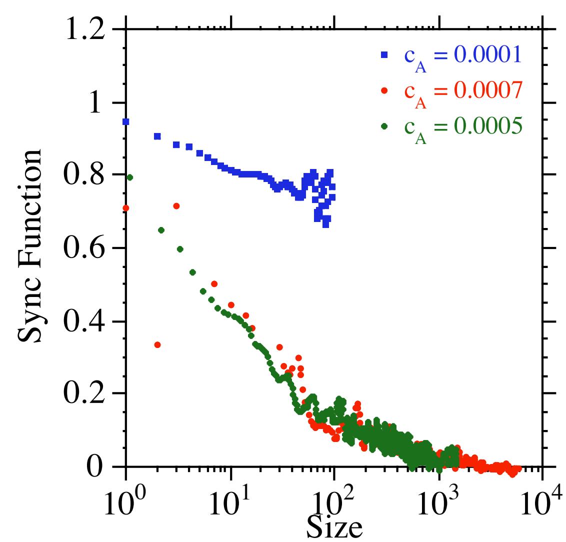

Figure 15: Synchronization functions for coupled DCSM calculations for 3 values of the coupling parameter C_a. A value of 1 is unsynchronized and 0 is synchronized.

This synchronization of large events is important in assessing the impact of the coupling. It may be that small failures in one system are unlikely to trigger a failure in the coupled system, however if a large failure is likely to trigger a coupled failure then the dynamical state of system one (ie it's proximity to a major failure) becomes very critical in assessing the risk of failure of the perhaps more reliable system two.

The results presented here have been for a very small subset of the parameter space. That subset being, symmetric homogeneous coupling with an increased failure probability from an coupled failed or failing site. The rest of the parameter space described earlier is being investigated and will be reported on later.

4. Discussion and Conclusions

Modern societies rely on the smooth operation of many of the infrastructure systems. We normally take them for granted. However, we are typically shocked when one of these systems fails. Therefore, understanding these systems is a high priority for ensuring security and social wellbeing. Because none of these infrastructure systems operate in a vacuum, understanding how these complex systems interact with each other gains importance when we recognize how tightly coupled some of these systems are. Because of the great complexity of even the individual systems it is unrealistic to think that we can presently dynamically model interacting infrastructure systems in full detail.

In this paper, we have investigated some of the general features of interactions between infrastructure system by using very simple models. We look for general dynamical features without trying to capture the details of the individual systems. From this we try to build a hierarchy of models with increasing levels of detail for these systems.

Here, we have shown two such models. One is a probabilistic model, CASCADE. The other model is a dynamic complex system model (DCSM) which can work in a self-organized critical state. Both models are characterize by a percolation threshold above which cascading failures of all sizes are possible, In both models this threshold can be characterized by the branching parameter λ, the average number of new failures caused by a failure. The percolation point is at $\lambda = 1$, where the probability density of failures for CASCADE is a power law with exponent -1.5 while for DCSM it is somewhat closer to –1.0. These exponents are close to the one found in analysis of blackout data.

It has been found that symmetric coupling of these systems actually decreases the threshold. That is, it makes access to the critical point easier, which means that the systems when coupled are more susceptible to large-scale failures and a failure in one system can cause a similar failure in the coupled system. The parameter λ, can be also used to characterize the cascading threshold in the coupled systems. This suggests the existence of a metric that can be generalized for practical application to more realistic systems.

For the DCSM model in addition, it is found that large failures are more likely to be "synchronized" across the two dynamical systems, which is likely to be the reason that the power law found in the probability of failure with size is less steep with the coupling. This means that in the coupled systems there greater probability of large failures and less of smaller failures.

With the DCSM model other important aspects of the infrastructure can be explored, such as non-uniform and non-symmetric couplings. This will be the object of future studies.

With this model there is a large parameter space that must be explored with different regions of parameter space having relevance to different infrastructure systems. There is also a rich variety of dynamics to be characterized. Characterizing the dynamics in the different regimes is more then an academic exercise since as we engineer higher tolerances in individual systems and make the interdependencies between systems stronger we will be exploring these new parameter regimes the hard way, by trial and error. Unfortunately error in this case has the potential to lead to global system failure. By investigating these systems from this high level, regimes to be avoided can be identified and mechanisms for avoiding them can be explored.

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6.9 Understanding the effect of risk aversion on risk

U. Bhatt, D.E. Newman, B.A. Carreras, I. Dobson

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Understanding the Effect of Risk Aversion on Risk

U. S. Bhatt
Atmospheric Science Program, Geophysical Institute, University of Alaska, Fairbanks, AK 99775 USA
bhatt@gi.alaska.edu

D. E. Newman
Physics Department University of Alaska, Fairbanks, AK 99775 USA
ffden@uaf.edu

B. A. Carreras
Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA
carrerasba@ornl.gov

I. Dobson
ECE Department, University of Wisconsin, Madison, WI 53706 USA
dobson@engr.wisc.edu

Abstract

As we progress, society must intelligently address the following question: How much risk is acceptable? How we answer this question could have important consequences for the future state of our nation and the dynamics of its social structure. In this work, we will elucidate and demonstrate using a physically based model that the attempt to eliminate all thinkable risks in our society may be setting us up for even larger risks. The simplest example to illustrate this point is something with which we are all familiar and have known from the time we were very young. When children burn their finger on a hot item they learn the consequences of touching fire. This small risk has taught the child to avoid larger risks. In trying to avoid these small risks as well as larger risks, one runs the dual danger of not learning from the small ones and of having difficulty in differentiating between large and small risks.

We will illustrate this problem with a series of social dynamics examples from the operation of NASA to network operation and then make an analogy to a complex system model for this type of dynamics. From these results, recommendations will be made for the types of risk responses that improve the situation versus those that worsen the situation.

In order to progress, society has to recognize that accidents are unavoidable and therefore an intelligent risk management program must be implemented aimed toward avoiding or reducing major accidents. It is not possible to avoid all risk but it is better to avoid the greater risk situations for society.

1. Introduction

Society must intelligently address the following question: How much risk is acceptable? How we answer this question could have important consequences for the future state of our nation. This world would not have progressed as far as it has if people were not willing to take risks. People took risks in the early days of global exploration, aviation, pioneering, and other facets of life. If pervious generations were not willing to take risks then society would not have progressed to where it is today.

In this work we will elucidate and demonstrate using a physically based model that the desire to eliminate all thinkable risks in our society may be setting us up for even larger risks. Perhaps, the proper strategy in dealing with risk is to develop within society the ability to rationally differentiate acceptable from unacceptable risk [1]. A physical example of the inability to differentiate between risks is when snow piles up on a mountainside. If the stress is relieved through small avalanches (analogous to small accidents) then the probability of a large avalanche (catastrophic accident) is reduced. Conversely, if all the small avalanches (the inconsequential accidents) are suppressed then the probability of a large avalanche (major accident) is increased. Therefore, the ability to differentiate between what is a major and what is a minor accident is of critical importance for society. The reporting style of the news media, which often does not differentiate between major or minor incidents, only foments often irrational hysteria over all issues and in the end leads to excessive aversion to all risk.

All engineered systems have a variety of modes of failure. There are normal accidents such as random failure of a piece of equipment or human error, however, these accidents can be compounded by operator/societal reactions to the accident. These responses are encompassed in the decision making process both on the short and long time scales. If the operators of the infrastructure or the society at large -are particularly risk averse then the responses to the small incidence are likely to be overblown. This can then mask larger problems, which can in turn increase the probability of a larger, even system-size, failure. In order to conduct proper risk assessment, the human decision making component of infrastructure operation must be included as an intrinsic part of that complex system [3].

Why is this issue important? In order to progress, society has to recognize that accidents are unavoidable therefore an intelligent risk management program must be implemented in which the risk of major accidents is reduced. It is not possible to avoid all risk but it is better to avoid the greater risk situations for society.

The remainder of the paper is organized as follows. In Section 2, we will present three examples of human systems in which risk-averse behavior can have counter productive effects. Section 3 will present a model and preliminary results from this model showing the effects of risk-averse behavior. We will conclude in Section 4 with a discussion of the implications for infrastructures.

2. Examples of Risk Averse Operations

To illustrate the effects of behavior based on risk aversion we choose three very disparate systems. These systems are children's learning, corporate safety culture, and NASA space safety culture. These three form a nice hierarchical set, in that we all go through the childhood learning process and on a large scale most of us exist in a corporate safety culture, and on the largest scale, governmental programs like NASA permeate society.

The example of how children's behavior in regard to safety develops is one that is familiar to all of us. Upon first introduction to a dangerous system such as fire, there are two ways that the child's development can be influenced. First a parent can see a child pick up a match, become frightened, display their fear to the child, and then remove the match from the child. This behavior of the parent inculcates into the child the feeling that the match and by extension the fire is an extremely dangerous thing to be feared. Alternatively, the parent could supervise and observe the child playing with the match and even go so far as to allow the child to feel the heat from the match. That latter method, combined with a discussion of fire and heat allows the child to develop a healthy respect and understanding of the real dangers of fire. This way the child can understand that the fire from a match is less intrinsically risky than a forest fire. In the first case the child who learned that fire from a match is something that is very dangerous and something to fear will not be able to differentiate between the risks of lighting a match and the risk of lighting a forest on fire. This is of course an extreme example but illustrates the point that a child must learn to differentiate between the levels of risky behavior. This is done by allowing them to experience some of the consequences of the risky, yet less dangerous behaviors [4], such as falling off of a jungle gym or a bicycle and getting a scraped knee.

Turning to corporate culture, we have all seen the sign, '100 days of accident free operation'. That sign typifies the lack of differentiation between different levels of accidents. Many corporations in their desire to be able report 'X' days of accident free operation, with X getting larger and larger, do not differentiate between different types of accidents. Clearly, there is a difference between getting a paper cut and loosing a finger in industrial operations. And yet, in many industries if medical personnel are involved it is considered a reportable incident. Therefore, in the admirable but misguided desire to increase safety in the workplace, accident avoidance training is often mandatory. As is natural, this accident avoidance training is usually reactive so that the personnel undergo training in response to most recent incidences that have occurred. Many of us, therefore have undergone training sessions on the importance of holding a banister when walking down stairs, or not taking the steps two at a time, or not walking with a pencil of scissors. One might ask what is wrong with reminding people to hold onto the banister in order to avoid a fall. The problem is not that the training attempts to prevent people from falling but rather the problem is that the training does not differentiate between the real risks of different behaviors and different potential accidents. If the operational staff of an organization is trained with the same frequency and intensity about the dangers of paper cuts, tripping down stairs, or electrocution, then they will come to treat all three of those as having the same level of risk. In reality, we would like to prevent catastrophic accidents much more than we would like to avoid minor incidences. The zero-risk tolerance culture which does not differentiate between the severity of risks can actually increase the risk of the catastrophic incidents by overwhelming the personnel with small risk warnings, which mask the important ones. For example, one can imagine operating a table saw and on the table saw there are warnings which are all written red and are all the same size. "Warning, corner of table can poke you"; "Warning, table surface can be hot"; "Warning, splinters can jab you"; "Warning moving saw blade can cut you"; " Warning, will not operate if not plugged in"; "Warning, saw can eject wood"; and "Warning do not eat or drink on surface". Buried in the middle of these warnings, were one or two real, important messages, however, if all the warnings were present, then the user would get used to them as background noise and pay attention to none of them. This then could actually increase the risk of a serious accident.

At the largest scale, governmental organizations, such as NASA, in their desire to avoid bad publicity and to make the operations as safe as possible try to mitigate all risks by extensive planning and training [16]. It is of course ridiculous to argue against planning and training, however, it is also clearly impossible to plan and train for all eventualities. Therefore, the danger in over planning and overtraining for avoiding specific incidences is that then one is completely unprepared for the unexpected. The NASA space program as contrasted to the former Soviet space program provide two extreme examples. In the case of NASA, extensive contingency planning for all foreseeable incidents is done. The astronauts are trained and equipped to deal with all of these foreseeable incidents. For the Soviet program the astronauts were given basic training, some tools, and "duct tape and bailing wire" and are told that they can deal with any problems that arise. Remarkably enough, they were usually able to do so. In a culture like NASAs, which reflects the broader US societal culture, where when something unexpected occurs there can often be difficulty in dealing with the event and even recognizing it because it falls outside of the planned-for and trained-for sphere. Additionally, in a culture like ours, the planning of responses to all foreseen or foreseeable events has the same effect that we were discussing in regard to corporate culture and childhood learning, the lack of differentiation between different risk levels. This is not to say that the planners are not aware of the different risk levels, rather when the culture overwhelms the personnel with the planned responses, it becomes difficult for individuals and by extension the whole organization to rationally differentiate between risks. It is an excellent experience for the NASA astronauts to live on MIR and see how people use innovative ideas when risk has not been minimized. You can only plan for so much before you go into unexplored territory. So it is essential to train people to be innovative under stressful conditions. Being able to differentiate risk is also important for an organization to survive under it's load of regulations. As Bill Weber said about NASA "Many now worry that risk will become the single management metric du jour. It's the obvious reaction to a series of failures. However, risk reduction costs money. At what point do you draw the line? In the space business, no mission will ever be risk free, regardless of the amount of money spent. Thus, too much risk aversion and NASA is on another path to oblivion" [15]

In large complex infra-structures systems, decision making and operations planning are based on an evolving assessment of risk. This assessment of risk, or aversion to risk, depends upon the size and frequency of failures in the recent past and on an overall cultural or societal acceptance of risk. Therefore, understanding how different levels of risk aversion effect decision making and how failures effect the risk aversion are fundamentally important to properly model complex infrastructure systems.

3. A dynamical model of risk averse systems

In order to quantify this type of behavior we have developed a simple deterministic dynamical model for the response to incidents. The dynamical deterministic nature is important since we know that the real behavior is deterministic and reaction today depends on what happened yesterday at least to some degree. At the same time the forcing for the model is a random forcing which is consistent with the idea that it is events outside the system, which are forcing the system.

This model is a cellular automaton based model set on a regular grid with fixed interaction rules. The system we will discuss here are a subset of that general class of models in which the rules are local and the grid is regular. Both of these restrictions are straightforward to generalize, and for some real decision making processes other choices might make more sense, but we use them as a reasonable starting point.

The rules for this simple dynamical system are:

1) A node has a certain (usually small) probability of failure (p_f)
2) A node neighboring a failed node has another (higher) probability of failing (p_s)
3) A failed node has a certain (usually higher) probability of being repaired (p_r)

The steps taken in the evolution are equally simple:

At step t

1) The nodes are evaluated for random failure based on their state at the end of the t-1 step.
2) The nodes are evaluated for repair based on their state at the end of the t-1 step.
3) The nodes are evaluated for failure due to the state of their neighbors at step t-1.
4) All nodes are advanced to their new state

The nodes can be thought of as elements in an "incident" space. The responses to the incident are in the short term a suppression of a repeat of the same incident, followed, after a recovery period, by a return to the former risk level (as memory fades). Incidents (failures) in these systems can grow and evolve in non-uniform clusters and display a remarkably rich variety of spatial and temporal complexity. They can grow to all sizes from individual node failures to system size events. The recovery rate for nodes is usually slower then the time scale of a cascading failure so recovery during an evolving cascade are unlikely. An important feature of this model is the "memory" of previous incidents in the structure of failed and recovered nodes within the system. The characteristic time scales of the system are also captured in the repair time and random failure probability. This type of model gives long time correlations between the failures, a feature that comes from the dynamical memory of the system. This model is not intended to simulate a particular decision making system. Rather the simple nature of the system allows one to investigate the effect of suppression of small events (as a result of risk aversion) on the incidence of larger events.

Figure 1 shows the time history of incidents for 2 systems. The normal system is shown by the solid line and has a variety of incident sizes. The dashed line is for a system with the same parameters but with incidents under a size of ten suppressed. In this case one can see more large incidents.

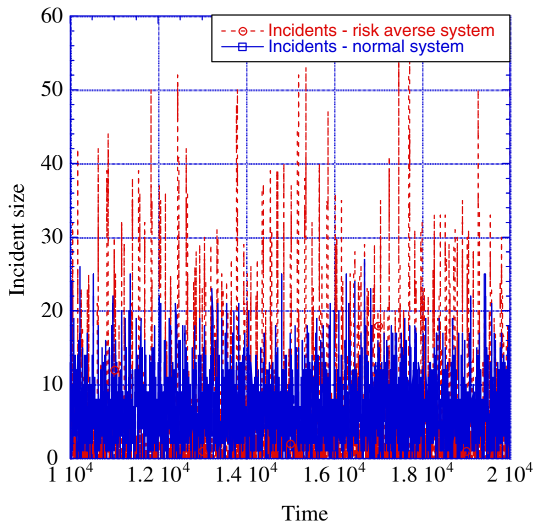

Figure 1: Time evolution of incidents for a risk averse system (dashed) and a normal system (solid), showing increased large events when small events are suppressed.

This is quantified in figure 2 in which the PDF of the incident sizes is shown for the two systems. At smaller scales there is a reduction in incidents (though it should be noted not at the smallest scale). At the larger scales, there are more and larger incidents. From the arguments given earlier, this type of result makes some sense as by focusing on the smaller risks (incidents) the system modeled no longer pays attention to the larger events, this increases the probability of such an incident. This type of model can be coupled to an infrastructure model and driven by the events in that model so that the repair responses in the infrastructure model are modified by the state of the risk/decision making model.

A much more readily understood physical example of this type of phenomena is the dynamics of a forest or brush fire system. In an area with a very efficient fire fighting system, small fires are effectively suppressed. This reduces the probability of small fires. However, it also increases the density of the foliage. Now, when a few random fires are started (lightning strikes, careless people, etc), the fire can spread and get beyond the controllable stage quickly and can lead to larger fires. If you suppress the small-scale events you are more likely to experience large events. The fire control complex has come to understand this so instead of fighting all small fire, some are allowed to burn, others are intentionally lit and clearing (a fire surrogate) is performed.

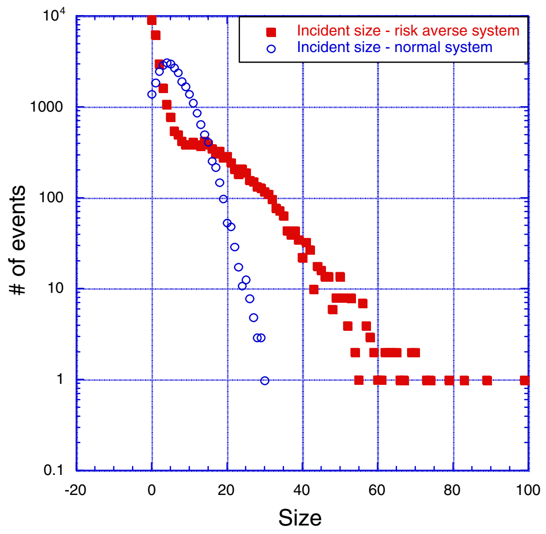

Figure 2: PDF of incident size for a risk averse system (solid squares) and a normal system (open circles), showing more large events when small events are suppressed.

The model can be used to investigate both the effects of risk aversion as well as other types of decision-making paradigms and their effect on overall risk. Which could be used to investigate more intelligent risk management techniques.

4. Implications for coupled infrastructure systems

In the management of complex infrastructure systems the planners and operators of the system both explicitly and implicitly take into account the acceptable level of risk of failures at various levels. While we often make the statement that a system or system component must be made failure free, most planners and operators realize that it is impossible to eliminate all risk and the best we can do is to minimize the most serious risks. However the level of acceptable risk is highly dependent on the time history of incidents in the system (and to some degree outside the system in society as a whole). Therefore in modeling the infrastructure systems the level of risk acceptance or avertion must be taken into account in describing the short/ long term responce of the system (operators/planners) to incidents.

In response to a major incident, risk aversion increases, this leads to operators and planners trying to reduce risk as much as possible. Since this is most easily done in the simplest areas, it can end up reducing the risk of small events but actually increasing the risk of the larger events. To improve the accuracy of the modeling of such complex infrastructure systems, the infrastructure models should be coupled to risk based desision make models in order to capture this important response feedback.

5. Discussion and Conclusions

In a society such as ours, in which major risks have been removed from our lives, it is natural to start focusing on the smaller risks. That combined with the natural human desire to assess blame when something goes wrong, leads to a universal attempt to remove all risks, large and small. Perception of risk is an important component of this desire to remove all risk [14]. Yet, since clearly it is impossible to remove all risks the very act of attempting to do this could have the counter productive effects described earlier. Therefore using models and planning, this instinctive behavior must be kept in check.

These dynamic complex systems range from:

- Congress with legislation
- Employee accidents (OSHA)
- Industrial safety
- Mississippi Floods

In all of these systems what is key is to differentiate between what is important and what is less important. More and more society is moving towards avoiding all risks and this could be a very dangerous thing for our survival.

In order to progress, society has to recognize that accidents are unavoidable therefore an intelligent risk management program must be implemented in which major accidents can be avoided. It is not possible to avoid all risk but it is better to avoid the greater risk situations for society.

The most important remediation to this problem is the ability to differentiate between large and small risks in planning and response:

Without this, "We may wake up one morning and find the human race is in decline, undone by something as simple as being unable to take a risk." [13]

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6.10 Branching process models for the exponentially increasing portions of cascading failure blackouts

I. Dobson, B.A. Carreras, D.E. Newman

Thirty-eighth Hawaii International Conference on System Sciences, Hawaii, January 2005

Thirty-eighth Hawaii International Conference on System Sciences, January 2005, Big Island, Hawaii.

Branching process models for the exponentially increasing portions of cascading failure blackouts

Ian Dobson
ECE Department
University of Wisconsin
Madison WI 53706 USA
dobson@engr.wisc.edu

Benjamin A. Carreras
Oak Ridge National Laboratory
Oak Ridge TN 37831 USA
carrerasba@ornl.gov

David E. Newman
Physics Department
University of Alaska
Fairbanks AK 99775 USA
ffden@uaf.edu

Abstract

We introduce branching process models in discrete and continuous time for the exponentially increasing phase of cascading blackouts. Cumulative line trips from real blackout data have portions consistent with these branching process models. Some initial calculations identifying parameters and using a branching process model to estimate blackout probabilities are illustrated.

1. Introduction

We aim to capture gross features of large, cascading failure blackouts using probabilistic branching process models. Galton-Watson and Markov branching processes are related to the timing of failures and this extends previous work that models the evolution but not the timing of the blackout failures with Galton-Watson branching processes [5]. This overall approach is complementary to the traditional and useful detailed analysis of blackouts and offers a number of possibilities for understanding and monitoring the risk of large blackouts.

Section 2 examines transmission line failure data from three recent North American blackouts for exponentially increasing portions and estimates the exponents of the exponential increases. Section 3 considers Markov branching process models in discrete and continuous time that reflect the exponential increase [1, 8] and suggests methods of identifying branching process parameters. Section 4 shows sample calculations of how a branching process model could be used to explore the likelihood of a particular blackout occurring and the value of including real time data on the cumulative number of line trips in estimates of the blackout propagation.

2. Blackout data

This section examines cumulative high voltage line trips in observed blackout data from the July and August 1996 WSCC blackouts [9, 11] and the August 2003 Eastern interconnect blackout [10].

It is supposed that are three phases to the blackout. The effect of the first phase is summarized as an initial disturbance that causes a certain number of line trips at the beginning of the cascading phase. In the second, cascading phase, the cascading process can cause exponentially increasing cumulative line trips. In the final phase, the cascading process saturates and the blackout starts to slow down and converge to its final extent. The identification of the boundaries between the blackout phases is done by inspection of the data.

For each blackout, we plot the cumulative line trips with respect to time to examine the overall trajectory of the blackout. If there is an exponentially increasing phase, then this should appear as a straight line portion in a plot of the logarithm of the cumulative line trips with respect to time and the slope of the line gives the exponent of the exponential growth.

There is no attempt to filter the data by, for example, combining trips of parallel lines. Generator trips are not included in the data. Trips of lines of different ratings are counted in the same way. These assumptions are made for simplicity in order to make a first analysis of the data from this new perspective.

2.1. July 1996 WSCC blackout

Figure 1 shows cumulative line trips as function of time extracted from the 1996 NERC system disturbance report [9], page 28. The lines tripped include lines of ratings from 120 kV to 500 kV. The initial disturbance is taken as 2 line trips at 14:24 MDT. Examining the logarithm of the cumulative line trips in excess of 2 in Figure 2 suggests an exponential growth between times 14:24 to 14:31 MDT. The exponent of the exponential growth is $\mu \approx 0.47$ min^{-1}. This corresponds to multiplication of the cumulative line trips by a factor of 1.6 every minute.

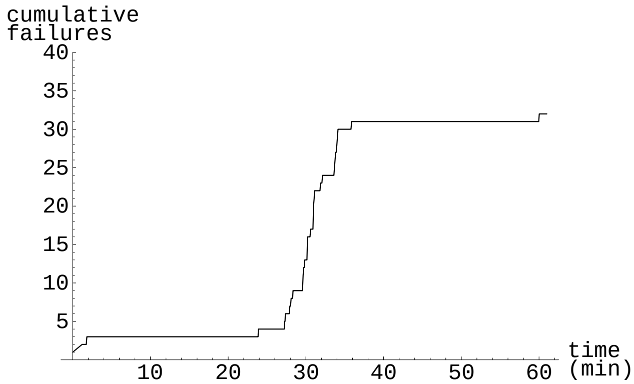

Figure 1. Cumulative line trips in WSCC July 1996 blackout. Time scale is minutes after 14:00 MDT.

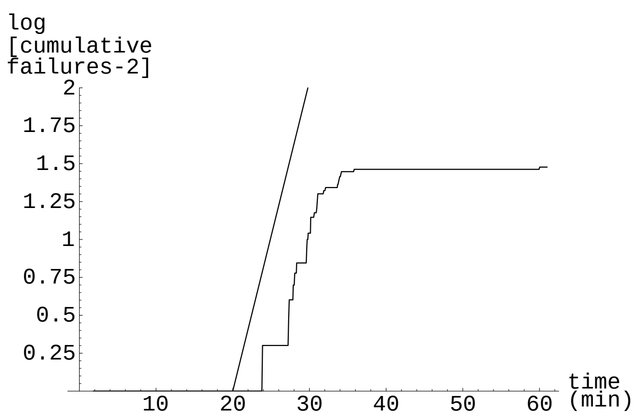

Figure 2. Log[cumulative line trips in excess of 2] in WSCC July 1996 blackout. The straight line growth corresponds to 1.6^{time}. Time scale is minutes after 14:00 MDT.

2.2. August 1996 WSCC blackout

Figure 3 shows cumulative line trips as function of time extracted from the 1996 NERC system disturbance report [9], page 38. The initial disturbance is taken as 2 line trips at 14:46 PDT. Examining the logarithm of the cumulative line trips in excess of 2 in Figure 4 suggests an exponential growth between times 13:46 to 13:49 PDT. The exponential growth is somewhat less clear cut than in the July 1996 blackout because it evolves quickly in only a few jumps. The exponent of the exponential growth is $\mu \approx 1.4$ min^{-1}. This corresponds to multiplication of the cumulative line trips by a factor of 4 every minute.

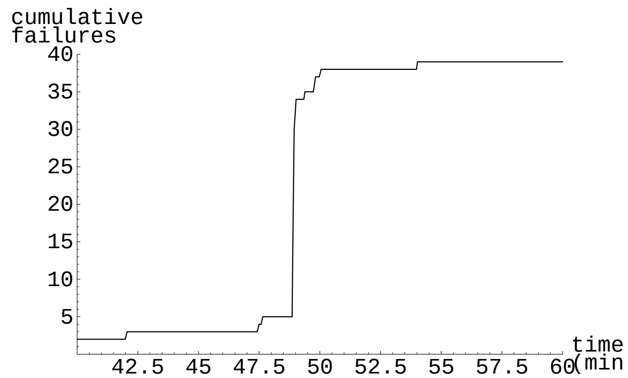

Figure 3. Cumulative line trips in WSCC August 1996 blackout. Time scale is minutes after 15:00 PDT.

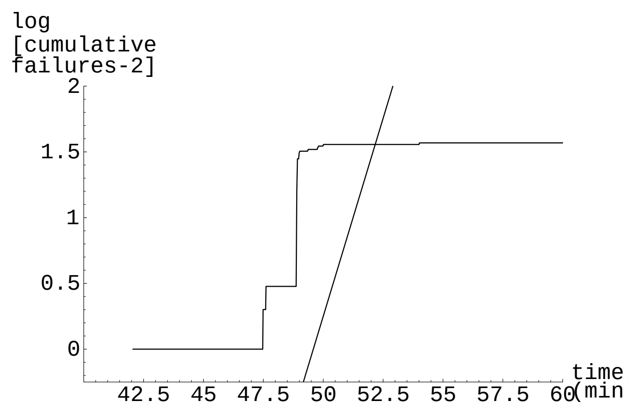

Figure 4. Log[cumulative line trips in excess of 2] in WSCC August 1996 blackout. The straight line growth corresponds to 4^{time}. Time scale is minutes after 15:00 PDT.

2.3. August 2003 Eastern interconnect blackout

Figure 5 shows cumulative line and transformer trips as function of time reprinted from the final blackout report [10]. Since the data underlying Figure 5 is not yet available to us for study, we digitized by hand the cumulative line and transformer trips curve in Figure 5 to obtain approximate data and then replotted the logarithm of the cumulative trips as Figure 6. One way to parse the data in Figure 6 is to consider a slowly cascading phase from time

5.5 to 8.5 and a fast cascading phase from time 8.5 to 9.5, and then saturation of the fast cascading phase. The slow cascading phase fits an exponential more approximately. The slow cascading phase has exponent of the exponential growth $\mu \approx 0.34\ \mathrm{min}^{-1}$. This corresponds to multiplication of the cumulative line trips by a factor of 1.4 every minute. The fast cascading phase has exponent of the exponential growth $\mu \approx 2.9\ \mathrm{min}^{-1}$. This corresponds to multiplication of the cumulative line trips by a factor of 18 every minute.

There are other ways of parsing the data in Figure 6; one could simply fit the data with a single exponential cascading phase from time 5.5 to 9.5. One reason for preferring the fit with two cascading phases considered in the preceding paragraph in an initial exploration of the data is that power system experts identified two cascading phases [10]. However, Figure 6 raises the question of whether the data is best fit by one or two cascading phases.

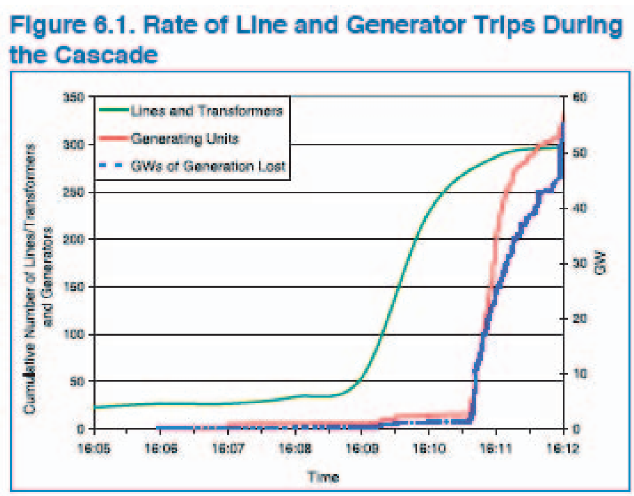

Figure 5. Cumulative line and transformer trips in August 2003 blackout. Reprinted from [10].

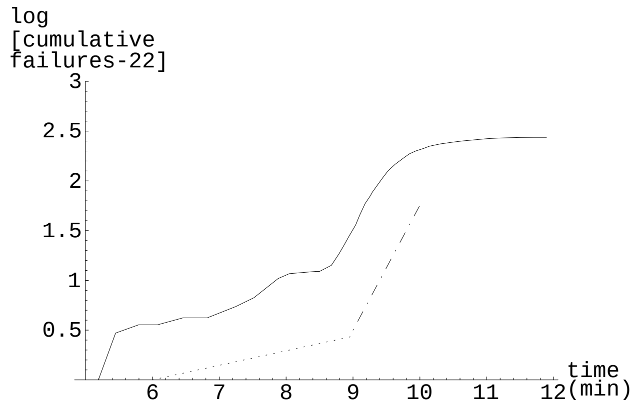

Figure 6. Log[cumulative line and transformer trips in excess of 22] in August 2003 blackout. The straight line growths correspond to 1.4^{time} and 18^{time} respectively. Time scale is minutes after 16:00 EDT.

We conclude that several recent North American blackouts show a region or regions of exponential increase in cumulative line failures.

3. Branching process models

Branching process models are an obvious choice of stochastic model to capture the gross features of cascading blackouts because they have been developed and applied to other cascading processes such as genealogy, epidemics and cosmic rays [8]. The first suggestion to apply branching processes to blackouts appears to be in [5].

There are more specific arguments justifying branching processes as good approximations to some of the gross features of cascading blackouts. An idealized probabilistic model of cascading failure [7, 4] describes with analytic formulas the statistics of a cascading process in which component failures weaken and further load the system so that subsequent failures are more likely. It is known that this cascade model and variants of it can be well approximated by a Galton-Watson branching process with each failure giving rise to a Poisson distribution of failures in the next stage. [5, 6]. Moreover, some features of this cascade model are consistent with results from cascading failure simulations [2, 4]. All of these models can show power law regions in the distribution of failure sizes or blackout sizes consistent with NERC data [3].

All the cascading failure models and branching processes considered above make no reference to the time of failures; the failures are produced in successive stages without reference to the time of each stage. This raises the issues of how to relate the stages to data that arises in real time and whether a branching process model in continuous time can be applied. We consider three possible approaches below. The first two approaches consider a Galton-Watson branching process in which the failures occur in stages and the third approach considers a continuous time branching process. All the standard facts quoted below about branching processes are available in [1, 8].

3.1. Galton-Watson branching process with variable time between stages

The Galton-Watson branching process is assumed to have each failure generate failures in the subsequent stage according to a distribution with mean λ. λ is a measure of the propagation of the failures. There is an initial number of failures θ. The number of failures at stage j is the random variable M_j. The mean number of failures EM_j increases by a factor λ in each stage. More precisely,

$$EM_j = \theta\lambda^j \tag{1}$$

The mean cumulative number of failures at stage j is

$$E\sum_{i=0}^{j} M_i = \theta(1+\lambda+\lambda^2+...+\lambda^j) = \theta\frac{\lambda^{j+1}-1}{\lambda-1} \tag{2}$$

The critical case occurs for $\lambda = 1$ [8, 5]. Moreover, if $\lambda > 1$, as $j \to \infty$,

$$M_j\lambda^{-j} \to \theta W \quad \text{a.s.} \tag{3}$$

where W is a random variable with $EW = 1$ that is constant in time. That is, as $j \to \infty$,

$$\log M_j \sim j\lambda + \log(\theta W) \tag{4}$$

To give some examples of this convergence, we simulate the branching process for various values of λ. This is shown in Figures 7-10. The convergence improves as λ increases away from 1.

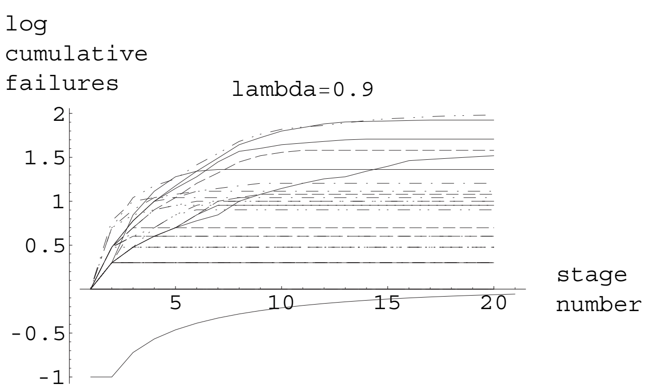

Figure 7. 40 samples of Galton-Watson branching process for $\lambda = 0.9$. The lower curve is $\lambda^i - 1$ where i is stage number to show the form but not vertical placing of (11).

The subcritical case of $\lambda = 0.9$ looks quite different from the other figures as shown in Figure 7. The asymptotic slope is zero as the cascade ends. The supercritical case of $\lambda = 1.1$ contains some samples in which the cascade dies out as shown in Figure 8. This is expected and the probability of this can be computed from λ as explained in section 4.1. The slightly supercritical cases that die out are qualitatively similar to slightly subcritical cases that die out. However, when we identify an exponentially growing

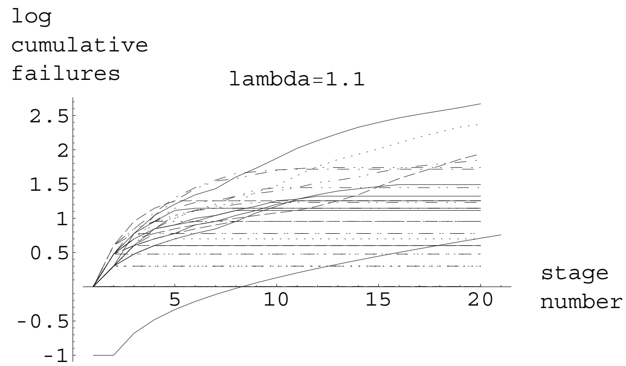

Figure 8. 40 samples of Galton-Watson branching process for $\lambda = 1.1$. The lower curve is $\lambda^i - 1$ where i is stage number to show the form but not vertical placing of (11).

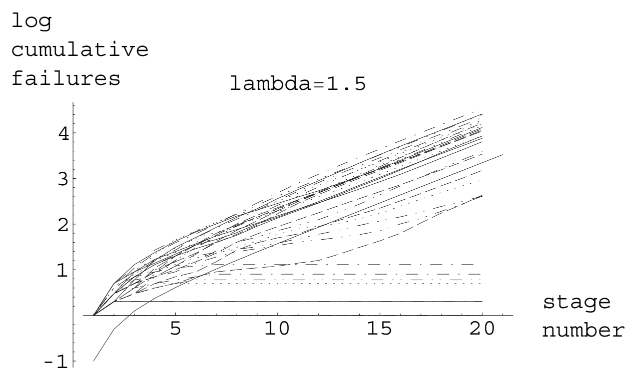

Figure 9. 40 samples of Galton-Watson branching process for $\lambda = 1.5$. The lower curve is $\lambda^i - 1$ where i is stage number to show the form but not vertical placing of (11).

phase in blackout data, we already know that the cascade did not die out and we can expect the measured slope on the log plot to reflect the value of λ.

The discussion so far has not specified the relation of the stages of the Galton-Watson branching process to time and we now outline the first approach to this issue. We suppose that failure data is available that includes the time of each failure and perhaps some additional data explaining the causes of the failure and specifying the type and location of the failure. Then these data used to group the failures into stages. Examples of factors that would tend to group several failures into the same stage could be their closeness in time or location, or being caused by failures in a previous stage. In the initial analysis in this paper we only consider the closeness in time; that is, we group together several failures if they are close in time and neglect the other possible

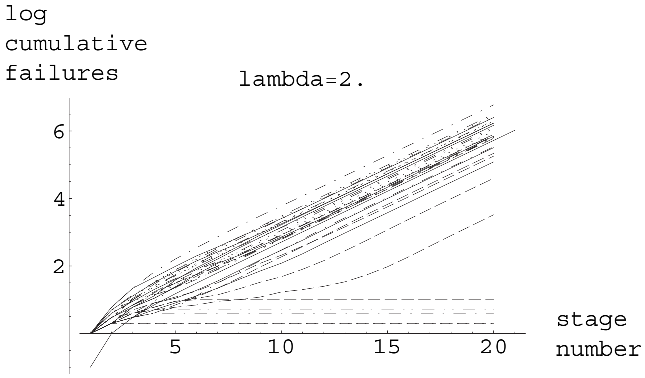

Figure 10. 40 samples of Galton-Watson branching process. The lower curve is $\lambda^i - 1$ where i is stage number to show the form but not vertical placing of (11).

factors. In any case one applies criteria to group the failures into stages and then regards the failures in each stage as arising from a Galton-Watson branching process. In this model, there is no attempt to represent the time at which the stages occur. Indeed the series of times near which failures in each stage occur will generally be non-uniformly spaced. That is, one can regard the stages as occurring with a variable time between stages and this timing is not specified within the branching process model in this approach.

3.2. Galton-Watson branching process with fixed time between stages

We now discuss the second approach to relating the Galton-Watson branching process to time. This approach groups the failures into stages as in the first approach in section 3.1, but then makes the explicit simplification or approximation that the stages occur with fixed time b between the stages. b is chosen to be the average time between stages and is computed by dividing the time interval T over which the branching process model is applied by the number of stages J.

This explicit description of the stage times has several consequences. At each stage of time b minutes, the mean number of failures EM_j increases by a factor λ so that the mean number of failures grows exponentially in time with exponent

$$\mu = \ln(\lambda)/b \tag{5}$$

min^{-1}. More precisely, the mean number of failures is $\theta e^{\mu t_j}$ at the stage times $t_j = jb$.

The mean cumulative number of failures at time jb is given by (2). The mean cumulative number of failures is piecewise constant with jumps at each stage and samples of the cumulative number of failures at each stage are asymptotically exponential with exponent $\mu = \ln(\lambda)/b \text{ min}^{-1}$, the same as the exponent for the mean number of failures.

When fitting this branching process model to failure data, one can fit an exponential $e^{\mu t}$ to a time interval of the data of length T as is done in section 2. This yields an estimate of the number of stages $\hat{J}$ and an estimate of the time between stages $\hat{b} = T/\hat{J}$. Then from (5) we have

$$\hat{\lambda} = e^{\mu \hat{b}} = e^{\mu T/\hat{J}} \tag{6}$$

One consequence of this approach is that in cases where there are several plausible ways to group the failure data into stages, there can be different estimates $\hat{J}$ of the numbers of stages and hence different estimates $\hat{\lambda}$. A larger number of stages yields a $\hat{\lambda}$ closer to 1. The variation of $\hat{\lambda}$ with the estimated number of stages $\hat{J}$ is expected because λ is defined as the expected number of failures per failure in the previous stage and so depends on the stages. In the supercritical case of $\lambda > 1$, increasing the number of stages shortens the time between stages and must decrease the average number of failures that occur over the shorter time between stages. However the supercriticality ($\lambda > 1$) or subcriticality ($\lambda < 1$) is independent of the time between stages.

3.3. Continuous time Markov branching process

The third approach to relating the Galton-Watson branching process to time considers a branching process that produces failures at variable intervals in continuous time. One simple assumption is that each failure causes its subsequent failures at a constant rate $1/a$. That is, when each failure occurs, the next failures "caused" by this particular failure will occur at a random time governed by an exponential random variable with parameter $1/a$. The mean time to this next failures is a. When these next failures occur, their number is governed by a fixed distribution with mean value λ_c. For example, the fixed distribution could be a Poisson distribution. The failures existing at any time propagate to cause more failures independently and at different random times. It follows that if there are $M(t)$ failures at time t, then the next failures occur after a time interval governed by an exponential random variable with parameter $M(t)/a$. This is a standard one dimensional continuous time Markov branching process [1]. Write $Z(t)$ for the number of failures at time t and θ for the initial number of failures at time zero. The mean number of failures is exponential:

$$EZ(t) = \theta e^{\mu t} \tag{7}$$

where

$$\mu = (\lambda_c - 1)/a \tag{8}$$

Moreover, if $\mu > 0$, as $t \to \infty$,

$$Z(t)e^{-\mu t} \to \theta W \quad \text{a.s.} \tag{9}$$

where W is a random variable with $EW = 1$ that is constant in time. That is, as $t \to \infty$,

$$\log Z(t) \sim \mu t + \log(\theta W) \tag{10}$$

(Sampling $Z(t)$ at regular intervals δ of time yields $Z(0)$, $Z(\delta)$, $Z(2\delta)$, $Z(3\delta)$,... and this is a Galton-Watson branching process. However, one does not necessarily recover the original Galton-Watson branching process by this sampling. For example, a Galton-Watson branching process produced with a Poisson distribution is not embeddable in any continuous time Markov branching process [1] and so cannot be the sampled Galton-Watson process.)

It follows from (7) that the mean cumulative number of failures is

$$E\int_0^t Z(\tau)d\tau = \frac{\theta}{\mu}\left(e^{\mu t} - 1\right) \tag{11}$$

If $\mu > 0$, it follows from (9) that

$$\int_0^t Z(\tau)d\tau \sim \frac{\theta}{\mu}\left(e^{\mu t} - 1\right)W \tag{12}$$

and, as $t \to \infty$,

$$\log \int_0^t Z(\tau)d\tau \sim \mu t + \log(\theta W/\mu) \tag{13}$$

so that plotting $\log \int_0^t Z(\tau)d\tau$ against t gives an asymptotic slope of μ. This result supports the procedure in section 2 as long as convergence near to the asymptotic slope is achieved before saturation effects apply.

Examining the cumulative number of failures as a function of time avoids much of the difficulties of grouping blackout data into stages. That is, this approach is largely insensitive to how previous failures were grouped, it only needs to know that they happened in the past.

For the continuous time Markov branching process model, the process evolves in jumps. (See [1], where the jumps are also referred to as splits. Note that the jumps in this process do not correspond to the stages of the fixed stage Galton-Watson branching process model.) At each jump, one of the previous failures is replaced by an average of λ_c failures so that the average number of failures increases by $\lambda_c - 1$. Write S_1, S_2, S_3, ... for the number of failures at jumps $1, 2, 3$, ... Then the increments in the S_r are independent, identically distributed random variables with mean $\lambda_c - 1$. Under suitable conditions assuring that the considered cascades do not die out as detailed in [1],

$$\frac{S_r}{r} \to \lambda_c - 1 \qquad \text{as} \quad r \to \infty \tag{14}$$

This motivates us to group the more nearly simultaneous failures in the exponential increasing phase into jumps to obtain S_1, S_2, S_3, ..., and to examine $S_1/1$, $S_2/2$, $S_3/3$, ... for any indication of convergence to $\lambda_c - 1$.

3.4. Fitting branching models to the blackout data

One can readily conclude that both a supercritical fixed stage Galton-Watson branching process and a supercritical continuous time Markov branching process model are consistent with the exponentially increasing phases of the blackout data in section 2. This conclusion is insensitive to the generating function of the branching process. The criticality for both processes occurs at $\lambda = 1$ or $\lambda_c = 1$ (in the case of the continuous time Markov branching process model $\lambda_c = 1$ corresponds to $\mu = 0$ according to (8)). (Otter's theorem [8] shows that the power tail in the distribution of total number of failures occurs at $\lambda = 1$ for generic assumptions on the generating function.)

To progress beyond this qualitative modeling of the exponential blackout phases as a supercritical branching process, we need to estimate model parameters. Since the WSCC August 1996 blackout has very sparse data and the raw data for the August 2003 blackout is not yet available for study, we illustrate estimating model parameters for the WSCC July 1996 blackout using the discrete and continuous time branching process models. The time period of the exponential growth is chosen to be the 7 minutes from 14:24 to 14:31 MDT. Section 2 fit the exponent of the exponential growth in this time period as $\mu = 0.47$ min^{-1}. The failure times are shown in Figure 11 and Table 1.

For the Galton-Watson branching process models, we group the failures into stages according to their closeness in time. Successive failures are grouped into the same stage if the time between them is less than a fraction δ of the average time between failures. For illustration we choose $\delta = 0.5$. The average time between failures for the failure times in Figure 11 is 0.42 min so that (average time between failures)$\delta = 0.21$ and the corresponding grouping into 8 stages is indicated in the third column of Figure 11.

In the first modeling approach of section 3.1, we plot the logarithm of the cumulative staged failures as a function of stage number and fit these data with an exponential as shown in Figure 12 to obtain $\hat{\lambda} = 1.4$.

In the second modeling approach of section 3.2, the estimated number of stages is $\hat{J} = 8$. Then the estimated stage time $\hat{b} = T/\hat{J} = 7/\hat{J} = 0.875$ and (6) gives $\hat{\lambda} = 1.5$. As discussed in section 3.2, a different assumption about the grouping into stages would give different estimates. For example, choosing $\delta = 1$ would result in fewer stages so that $\hat{J} = 5$, $\hat{b} = 1.2$ and $\hat{\lambda} = 1.9$.

In the third, continuous time modeling approach of section 3.3, we need to group the failure data into jumps.

Although the jumps in the continuous time process do not directly correspond to the stages of the Galton-Watson branching process, we can for illustrative purposes use the same grouping of failure data into jumps as used for stages in Table 1. The consequent evolution of the number of failures and the series (14) is shown in Table 2. The series in Table 2 is too short and noisy for convergence to be verified, but the average of the last four elements estimates the limit of series (14) as ≈ 0.5 and this yields $\hat{\lambda}_c = 1.5$. This implies using $\mu = 0.47$ and (8) that the average time for one failure to split is $\hat{a} = 1.1$.

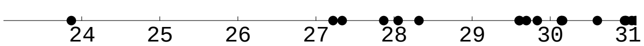

Figure 11. Times of line trips in WSCC July 1996 blackout in minutes after 14:00 MDT.

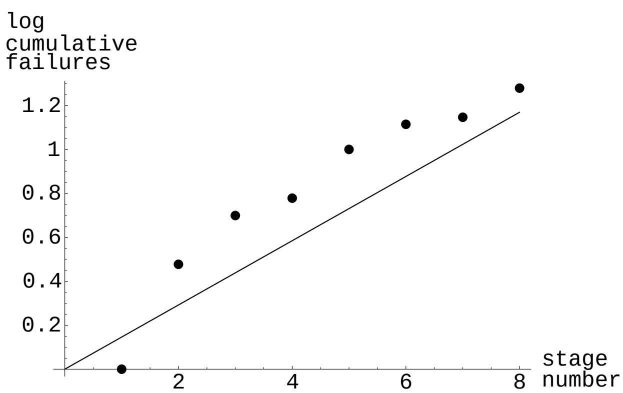

Figure 12. Log cumulative failures in exponential phase of WSCC July 1996 blackout as a function of stage number. The straight line is an exponential with exponent 1.4.

We do not yet have evidence available that the continuous time Markov branching process approximates the time sequence of actual failures; the present argument in favor of this modeling is that the assumptions are simple. Given a longer time series of failure data, such as the failure times in the August 2003 blackout, we could try to discriminate between the models and approaches suggested here and statistically or qualitatively test the fit of the models to the data. The data currently available to us is too limited to attempt this.

Table 1. Line trip times and stage numbers for exponential phase of July 1996 WSCC blackout. Time units are minutes after 14:00 MDT.

trip time	increment in trip time	stage number
23.867		1
27.219	3.352	2
27.336	0.117	2
27.868	0.532	3
28.052	0.184	3
28.319	0.267	4
29.602	1.283	5
29.608	0.006	5
29.695	0.087	5
29.835	0.140	5
30.144	0.309	6
30.145	0.001	6
30.159	0.014	6
30.604	0.445	7
30.953	0.349	8
30.965	0.012	8
30.971	0.006	8
31.045	0.074	8
31.094	0.049	8
31.815	0.721	

Table 2. Evolution of number of failures in jumps

r	1	2	3	4	5	6	7	8
S_r	1	2	2	1	4	3	1	5
S_r/r	1	1	0.67	0.25	0.8	0.5	0.14	0.63

4. Implications of branching model

Now we suppose that blackouts can be approximated by a discrete time Galton-Watson branching process model and explore some illustrative calculations using the model.

4.1. Probability of a given large blackout not happening

One interesting exercise is to use values of λ estimated from the large real blackouts that occurred with a region of exponential increase to compute the probability q of those blackouts *not* having the region of exponential increase. Of course there may be a blackout without the region of exponential increase, but such a blackout will have much more limited size. Thus we consider q to be the probability that the cascade dies out for that given value of λ. (This was alluded to in the discussion of Figure 8 above.) The value of q is the same for the Galton-Watson branching process and for the continuous time Markov branching process, but it does depend on the generating function $f(s)$ that is used to construct these branching processes. Here we will assume that the generating function corresponds to a Poisson distribution so that $f(s) = e^{\lambda(s-1)}$, as suggested by the branching process approximation to the abstract cascading failure model in [7, 5]. The probability q is easily computed as a root of the equation $f(s) = s$ and the results are shown in Table 3.

Suppose that a blackout has an exponential phase with $\lambda \approx 2$. This would imply that the probability that the exponential phase of the blackout did not occur is about 0.2. This calculation is made in hindsight after the blackout, but it does highlight the difficulties of making optimal decisions during the evolution of the blackout, even given good information. Blackouts with lower values of λ will have higher values of q. Suppose that $\lambda = 1.1$ and an exponentially increasing blackout occurred. The probability that it did not occur is 0.82 and one could argue that, in the absence of real time information about risk, a competent and well-informed system operator might well have acted properly by assuming the most likely outcome of no large blackout.

Table 3. Probability q of large blackout not occurring

λ	q
0.9	1.00
1.0	1.00
1.1	0.82
1.2	0.69
1.3	0.58
1.4	0.49
1.5	0.42
1.6	0.36
1.7	0.31
1.8	0.27
1.9	0.23
2.0	0.20
3.0	0.06
4.0	0.02
5.0	0.01

4.2. An initial approach to real time monitoring of cascading blackouts

The exponential cascading phase starts slowly and accelerates later. As we accumulate more failures, the probability of an exponentially accelerating cascade increases. Is it possible to detect this increased probability during the slow part? This subsection outlines an approach to quantify the statistics of this problem by monitoring cumulative line failures. Monitoring cumulative line failures would be practical in real time in a well-instrumented control center.

We regard the λ parameter of a staged branching process as a random variable Λ. Let the probability density function of Λ for a given system condition (i.e. stress level) be $f_\Lambda(\lambda)$ in $[\lambda_{\min}, \lambda_{\max}]$. We assume that for given system condition we have either historical data or off-line simulations giving $f_\Lambda(\lambda)$.

We have set a threshold to limit cascading failure risk of $\lambda < \lambda_t$. Presumably $\lambda_t < 1$ to exclude the possibility of exponentially increasing phases when the power system is operated with $\lambda < \lambda_t$.

Let the cumulative line failures observed in real time be S. Note that it would be necessary to somehow distinguish line failures involved in the initial disturbance from those involved in the cascading phase.

Suppose that we observe in real time that $S = k$. Then we know that $S \geq k$ for the final cascade. So what, knowing that $S \geq k$, is the probability of cascading failure caused by $\lambda > \lambda_t$? That is, what is $P[\Lambda > \lambda_t | S \geq k]$? We give a sample calculation of $P[\Lambda > \lambda_t | S \geq k]$ below. In particular, we numerically evaluate $P[\Lambda > \lambda_t | S \geq k]$ under some assumptions for increasing values of k. This quantifies how the probability of a large cascade increases as the number of observed line trips increases.

The calculations are done by evaluating the formula (27) derived in the appendix. The assumptions are that the cascade is modeled by a Galton-Watson branching process generated by a Poisson distribution. The distribution of Λ on $[\lambda_{\min}, \lambda_{\max}]$, in the absence of any information about the likely form of this distribution, is assumed to be uniform. Saturation effects are neglected.

The data needed is the initial number of failures θ and the range $[\lambda_{\min}, \lambda_{\max}]$ for the uniform distribution of Λ. We choose values of these parameters for a sample calculation and vary k. The results are shown in Table 4. When $k = 1$ there is no information supplied by the line trip that is additional to the information that a cascade has started (the probability of $\lambda > 0.9$ is clearly 0.5 when λ is uniformly distributed in $[0.7, 1.1]$. The probability of $\lambda > 0.9$ increases with k, but the rate of increase is modest in this example. Thus in this example the real time monitoring of k would add little value to the offline calculation of the distribution of Λ. Note that waiting until large k is observed does not help manage the cascading blackout because then the cascading process is well under way and cannot readily be corrected.

Table 4. Probability of λ exceeding threshold λ_t when k line trips are observed.

k	$P[\Lambda > \lambda_t \mid S \geq k]$	parameters
1	0.50	$\lambda_t = 0.9$
2	0.52	$[\lambda_{\min}, \lambda_{\max}] = [0.7, 1.1]$
3	0.53	$\theta = 1$
4	0.55	
5	0.57	
6	0.58	
7	0.60	
8	0.61	
9	0.62	
10	0.63	
15	0.69	
20	0.73	

5. Conclusion

The main contribution of this paper is to observe in recent North American cascading failure blackouts exponentially increasing phases of cumulative line trips and suggest that these be modeled by supercritical Markov branching processes. Simple discrete time branching process models and a continuous time Markov branching process model are considered. Several initial calculations illustrating how parameters may be estimated and these models might be applied are suggested. One interesting consequence of this statistical blackout modeling is that the probability of a given blackout *not* occurring could be estimated after the blackout. A preliminary approach to real time blackout monitoring is considered.

The blackout data sets currently available to us are not long enough to distinguish between the models or definitively estimate the parameters, but, when they are supercritical, all the branching process models do qualitatively reproduce the exponential growth of failures that seems to be the manner in which the 1996 and 2003 North American blackouts became widespread.

6. Acknowledgements

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A. Probability of $\lambda > \lambda_t$ given k failures

For a Galton-Watson branching process with a finite number of components, the probability distribution of total number of failures S for a given value of λ is $P[S = s|\Lambda = \lambda]$ given by saturating generalized Poisson distribution [5].

$$g(r,\theta,\lambda,n) = \theta(r\lambda+\theta)^{r-1}\frac{e^{-r\lambda-\theta}}{r!} \quad ; \quad 0 \le r \le (n-\theta)/\lambda,\ r < n \tag{15}$$

$$g(r,\theta,\lambda,n) = 0\,; \quad (n-\theta)/\lambda < r < n,\ r \ge 0 \tag{16}$$

$$g(n,\theta,\lambda,n) = 1 - \sum_{s=0}^{n-1} g(s,\theta,\lambda,n) \tag{17}$$

Then joint distribution of (S,Λ) is

$$f_{S,\Lambda}(s,\lambda) = P[S = s|\Lambda = \lambda] f_\Lambda(\lambda) \tag{18}$$

$$P[\Lambda > \lambda_t|S \ge k] = \frac{P[\Lambda > \lambda_t \text{ and } S \ge k]}{P[S \ge k]} \tag{19}$$

$$= \frac{\displaystyle\sum_{s=k}^{n}\int_{\lambda_t}^{\lambda_{\max}} f_{S,\Lambda}(s,\lambda)\,d\lambda}{\displaystyle\sum_{s=k}^{n}\int_{\lambda_{\min}}^{\lambda_{\max}} f_{S,\Lambda}(s,\lambda)\,d\lambda} \tag{20}$$

$$= \frac{\int_{\lambda_t}^{\lambda_{\max}} P[S \ge k|\Lambda = \lambda] f_\Lambda(\lambda)\,d\lambda}{\int_{\lambda_{\min}}^{\lambda_{\max}} P[S \ge k|\Lambda = \lambda] f_\Lambda(\lambda)\,d\lambda} \tag{21}$$

Since Λ is assumed to be uniformly distributed on $[\lambda_{\min}, \lambda_{\max}]$,

$$P[\Lambda > \lambda_t|S \ge k] = \frac{\int_{\lambda_t}^{\lambda_{\max}} P[S \ge k|\Lambda = \lambda]\,d\lambda}{\int_{\lambda_{\min}}^{\lambda_{\max}} P[S \ge k|\Lambda = \lambda]\,d\lambda} \tag{22}$$

$$= \frac{\int_{\lambda_t}^{\lambda_{\max}} 1 - F_{S|\Lambda=\lambda}(k-1)\,d\lambda}{\int_{\lambda_{\min}}^{\lambda_{\max}} 1 - F_{S|\Lambda=\lambda}(k-1)\,d\lambda} \tag{23}$$

$$= \frac{\lambda_{\max} - \lambda_t - \displaystyle\sum_{s=0}^{k-1}\int_{\lambda_t}^{\lambda_{\max}} g(s,\theta,\lambda,n)\,d\lambda}{\lambda_{\max} - \lambda_{\min} - \displaystyle\sum_{s=0}^{k-1}\int_{\lambda_{\min}}^{\lambda_{\max}} g(s,\theta,\lambda,n)\,d\lambda} \tag{24}$$

$$= \frac{(\lambda_{\max} - \lambda_t)(1-e^{-\theta}) - \displaystyle\sum_{s=1}^{k-1}\int_{\lambda_t}^{\lambda_{\max}} g(s,\theta,\lambda,n)\,d\lambda}{(\lambda_{\max} - \lambda_{\min})(1-e^{-\theta}) - \displaystyle\sum_{s=1}^{k-1}\int_{\lambda_{\min}}^{\lambda_{\max}} g(s,\theta,\lambda,n)\,d\lambda} \tag{25}$$

Suppose that $k-1$ is small enough to avoid saturation effects. Then

$$\int_{\lambda_t}^{\lambda_{\max}} g(s,\theta,\lambda,n)\,d\lambda = -\frac{\theta}{ss!}\Gamma[s,\theta+s\lambda]\Big]_{\lambda_t}^{\lambda_{\max}} \tag{26}$$

and

$$P[\Lambda > \lambda_t|S \ge k] = \frac{(\lambda_{\max} - \lambda_t)(1-e^{-\theta}) - \displaystyle\sum_{s=1}^{k-1}\frac{\theta}{ss!}\Gamma[s,\theta+s\lambda]\Big]_{\lambda_{\max}}^{\lambda_t}}{(\lambda_{\max} - \lambda_{\min})(1-e^{-\theta}) - \displaystyle\sum_{s=1}^{k-1}\frac{\theta}{ss!}\Gamma[s,\theta+s\lambda]\Big]_{\lambda_{\max}}^{\lambda_{\min}}} \tag{27}$$

6.11 Evidence for self-organized criticality in a time series of electric power system blackouts

B.A. Carreras, D.E. Newman, I. Dobson, A.B. Poole

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Evidence for Self-Organized Criticality in a Time Series of Electric Power System Blackouts

Benjamin A. Carreras, David E. Newman, Ian Dobson, *Senior Member, IEEE*, and A. Bruce Poole

***Abstract*—We analyze a 15-year time series of North American electric power transmission system blackouts for evidence of self-organized criticality (SOC). The probability distribution functions of various measures of blackout size have a power tail and rescaled range analysis of the time series shows moderate long-time correlations. Moreover, the same analysis applied to a time series from a sandpile model known to be self-organized critical gives results of the same form. Thus, the blackout data seem consistent with SOC. A qualitative explanation of the complex dynamics observed in electric power system blackouts is suggested.**

***Index Terms*—Blackouts, complex systems, power system security, reliability, risk analysis, time series.**

I. Introduction

ELECTRIC power transmission networks are complex systems that are commonly run near their operational limits. Major cascading disturbances or blackouts of these transmission systems have serious consequences. Individually, these blackouts can be attributed to specific causes, such as lightning strikes, ice storms, equipment failure, shorts resulting from untrimmed trees, excessive customer-load demand, or unusual operating conditions. However, an exclusive focus on these individual causes can overlook the global dynamics of a complex system in which repeated major disruptions from a wide variety of sources are a virtual certainty. We analyze a time series of blackouts to probe the nature of these complex system dynamics.

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B. A. Carreras is with Oak Ridge National Laboratory, Oak Ridge, TN 37831 USA (e-mail: carrerasba@ornl.gov).

D. E. Newman is with the Physics Department, University of Alaska, Fairbanks, AK 99775 USA (e-mail: ffden@uaf.edu).

I. Dobson is with the Electrical and Computer Engineering Department, University of Wisconsin, Madison, WI 53706 USA (e-mail: dobson@engr.wisc.edu).

A. B. Poole is with the Federal Energy Regulatory Commission, Washington, DC 20426 USA (e-mail: dobson@engr.wisc.edu).

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The North American Electrical Reliability Council (NERC) has a documented list summarizing major blackouts[1] of the North American power grid [1]. They are of diverse magnitude and of varying causes. It is not clear how complete this data is, but it is the best-documented source that we have found for blackouts in the North American power transmission system. An initial analysis of these data [6] over a period of five years suggested that self-organized criticality (SOC) [2], [3], [23] may govern the complex dynamics of these blackouts. Here, we further examine this hypothesis [7], [13] by extending the analysis to 15 years. These extended data allow us to develop improved statistics and give us longer time scales to explore. We compare the results to the same types of analysis of time sequences generated by a sandpile model known to be SOC. The similarity of the results is quite striking and is suggestive of the possible role that SOC plays in power system blackouts. A plausible qualitative explanation of SOC in power system blackouts is outlined in Section VI.

As an introduction to the concept, an SOC system is one in which the nonlinear dynamics in the presence of perturbations organize the overall average system state near, but not at, the state that is marginal to major disruptions. SOC systems are characterized by a spectrum of spatial and temporal scales of the disruptions that exist in remarkably similar forms in a wide variety of physical systems [2], [3], [23]. In these systems, the probability of occurrence of large disruptive events decreases as a power function of the event size. This is in contrast to many conventional systems in which this probability decays exponentially with event size.

It is apparent that large blackouts are rarer than small blackouts, but how much rarer are they? Fig. 1 shows the probability distribution of blackout size from the North American blackout data that is discussed in detail in Section II. Fig. 2 shows a probability distribution of number of line outages obtained from a blackout model that represents cascading failure and complex dynamics [11]. These data suggest a power law relationship between blackout probability and blackout size. For comparison, Fig. 2 also shows the binomial probability distribution of number of line outages and its exponential tail that would be obtained if the line outages were independent. Blackout risk is the product of blackout probability and blackout cost. Here, we assume that blackout cost is roughly proportional to blackout size, although larger blackouts may well have costs (especially

[1]The NERC data arise from government incident reporting requirements. The thresholds for the report of an incident include uncontrolled loss of 300 MW or more of firm system load for more than 15 min from a single incident, load shedding of 100 MW or more implemented under emergency operational policy, loss of electric service to more than 50 000 customers for 1 h or more, and other criteria detailed in the U.S. Department of Energy form EIA-417.

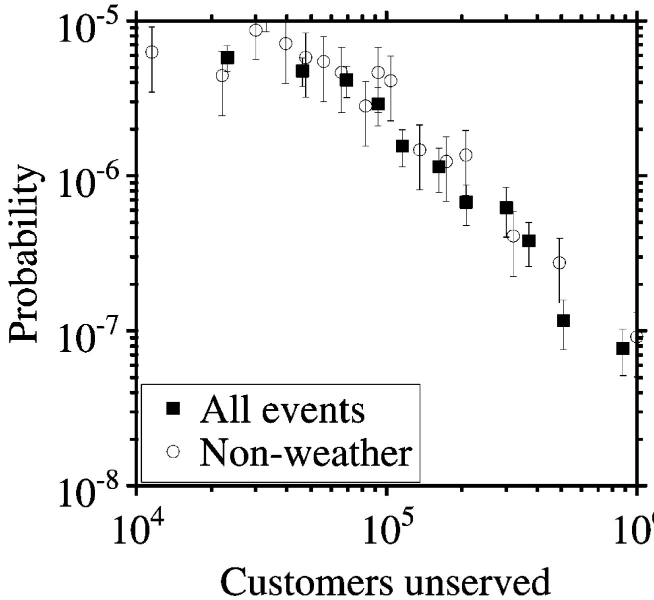

Fig. 1. Log–log plot of PDF of the number of customers unserved comparing the total data set with the data excluding the weather related events.

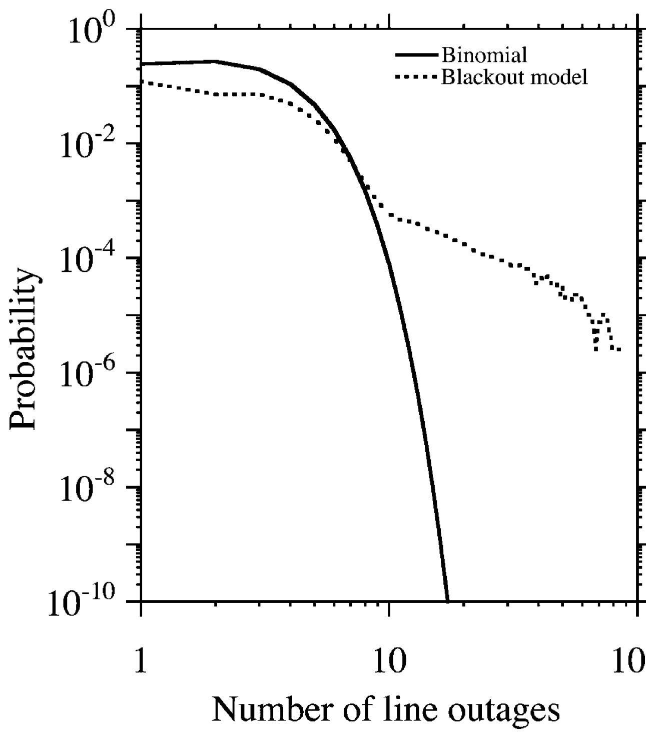

Fig. 2. Log–log plot of PDF of number of line outages from blackout model compared with binomial random variable with exponential tail.

indirect costs) that increase faster than linearly. In the case of the exponential tail, large blackouts become rarer much faster than blackout costs increase, so that the risk of large blackouts is negligible. However, in the case of a power law tail, the larger blackouts can become rarer at a similar rate as costs increase, and then the risk of large blackouts is comparable to, or even exceeding, the risk of small blackouts [11]. Thus power laws in blackout size distributions significantly affect the risk of large blackouts and the evidence for power laws in real blackout data that we address in this paper is pertinent. Standard probabilistic techniques that assume independence between events imply exponential tails and are not applicable to systems that exhibit power tails.

Large blackouts are typically caused by long, intricate cascading sequences of rare events. Dependencies between the first few events can be assessed for a subset of the most likely or anticipated events and this type of analysis is certainly useful in addressing a part of the problem (e.g., [26]). However, this combinatorial analysis gets overwhelmed and becomes infeasible for long sequences of events or for the huge number of all possible rare events and interactions, many of which are unanticipated, that cascade to cause large blackouts. One aim of global complex systems analysis of power system blackouts is to provide new insights and approaches that could address these challenges. As a first step toward this aim, this paper analyzes observed blackout data and suggests one way to understand the origin of the dynamics and distribution of power system blackouts. Indeed, we suggest that the slow, opposing forces of load increase and network upgrade in response to blackouts shape the system operating margins so that cascading blackouts occur with a frequency governed by a power law relationship between blackout probability and blackout size. Moreover, we discuss the dynamical dependencies and correlations between blackouts in the NERC data.

II. Time Series of Blackout Data

We have analyzed 15 years of data for North America from 1984 to 1998 that is publicly available from NERC [1]. There are 427 blackouts in 15 years and 28.5 blackouts per year. The average period of time between blackouts is 12.8 days. The blackouts are distributed over the 15 years in an irregular manner. We have detected no evidence of systematic changes in the number of blackouts or periodic or quasi-periodic behavior. However, it is difficult to determine long term trends or periodic behavior in just 15 years of data. We constructed time series from the NERC data with the resolution of a day for the number of blackouts and for three different measures of the blackout size. The length of the time record is 5479 days. The three measures of blackout size are:

1) energy unserved (MW·h);
2) amount of power lost (MW);
3) number of customers affected.

Energy unserved was estimated from the NERC data by multiplying the power lost by the restoration time.

III. Analysis of Blackout Time Series

In order to gain an understanding of the dynamics of a system from analysis of a time series, one must employ a variety of tools beyond basic statistical analysis. Among other measures which should be employed, the tails of the probability distribution function (PDF) should be investigated for normality and frequency spectra should be viewed in order to begin to look at dependencies in the time domain. The time domain is particularly important as the system dynamics are expressed in time. Periodicities and long-time correlations must both be examined and compared to systems with known dynamics. We will present details of the analysis of the PDFs later; however, the

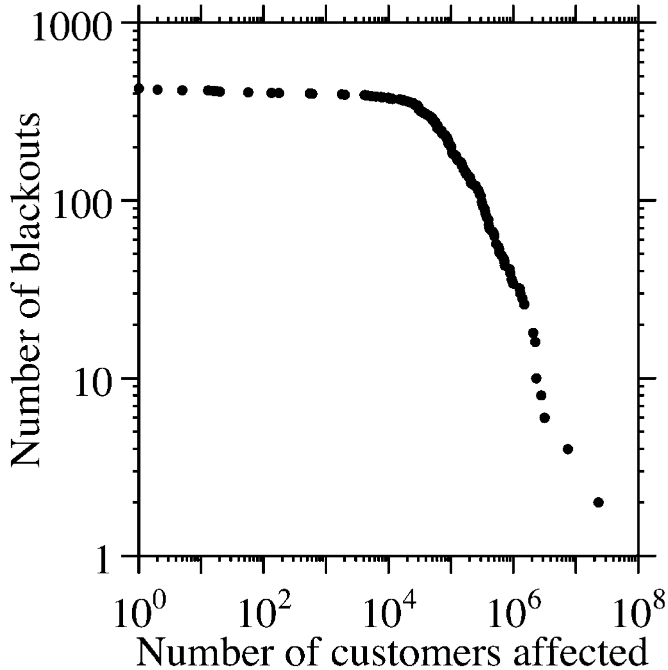

Fig. 3. Complementary cumulative frequency of the number of customers unserved.

first striking characteristic of the data is the power law tail of these PDFs. This power law tail is shown in Fig. 1, where we have plotted the PDF of the number of customers unserved for all events (the squares) on a log–log plot. The PDF falls off with a power of approximately -1.7, which implies a divergent variance. The PDF is clearly not a distribution with exponential tails. In this paper, the PDFs are noncumulative PDFs obtained by binning the data.[2] An alternative way to estimate the distribution is to plot the number of blackouts with more than n customers unserved against n to give the complementary cumulative frequency shown in Fig. 3. The empirical data in Fig. 3 falls off with a power of approximately -0.8 (all tail points considered) or -0.7 (last seven tail points neglected due to sparse data). The relationship for an exact distribution is that a power law exponent α in a PDF yields a power law exponent of $\alpha - 1$ in the corresponding complementary cumulative frequency. Thus the power law exponents obtained from Figs. 1 and 3 are consistent.

Looking in the time domain, a time series is said to have long-range dependence if its autocorrelation function falls off asymptotically as a power law. This type of dependence is difficult to determine because noise tends to dominate the signal for long time lags. One way to address this problem is the rescaled range (R/S) statistics proposed by Mandelbrot and Wallis [24] and based on a previous hydrological analysis by Hurst [21]. The R/S statistics consider blocks of m successive points in the integrated time series and measure how fast the range of the blocks grows as m increases. The calculation of the R/S statistics is further described in the Appendix.

It can be shown that in the case of a time series X with an autocorrelation function that has a power law tail, the R/S

[2]The bins are chosen to require a minimum number of points per bin. The minimum number of points per bin is reduced when the weather-related blackouts are excluded.

TABLE I
HURST PARAMETER H FROM R/S ANALYSIS OF BLACKOUT SIZE TIME SERIES

	H
Events	0.62
Power lost	0.59
Customers	0.57
MWh	0.53

statistic scales proportionally to m^H, where H is the Hurst exponent. Thus, H is the asymptotic slope on a log–log plot of the R/S statistic versus the time lag. If $1 > H > 0.5$, there are long-range time correlations, for $0.5 > H > 0$, the series has long-range anticorrelations, and if $H = 1.0$, the process is deterministic. Uncorrelated noise corresponds to $H = 0.5$. A constant H parameter over a long range of time-lag values is consistent with self-similarity of the signal in this range [32] and with an autocorrelation function that decays as a power of the time lag with exponent $2 - 2H$.

We have determined the long-range correlations in the 15 year blackout time series using the R/S method. The time series has 5479 days and 427 blackouts. The calculated Hurst exponents [21] for the different measures of blackout size are shown in Table I. The H values are obtained by fitting over time lags between 100 and 3000 days. In this range, the behavior of the R/S statistic is power like. The values of H obtained for all the time series are close to 0.6. This seems to indicate that they are all equally correlated over the long range. These values of H are somewhat lower than the previously obtained values [6], but still significantly above 0.5. Note that the "events" in the time series are the events that have produced a blackout and not all the events that occurred. The latter are supposed to be random ($H = 0.5$); however, the events that produce a blackout may indeed have moderate correlations because they depend on the state of the system.

A method of testing the independence of the triggering events has been suggested by Boffetta *et al.* [4]. They evaluated the times between events (waiting times) and argued that the PDF of the waiting times should have an exponential tail. Such is clearly the case for the waiting times of sandpile avalanches (Fig. 4). In the case of waiting times between blackouts, we also have observed the same exponential dependence of the PDF tail (Fig. 5). This observation is confirmed in [13]. This strengthens the contention that the apparent correlations in the events come from SOC-like dynamics within the power system rather than from the events driving the power system dynamics.

Examining the R/S results in more detail, Fig. 6 shows the R/S statistic for the time series of the number of customers affected by blackouts. The average period of time without blackouts is 12.8 days, hence, in looking over time lags of this order we typically find either one blackout or none. For the shorter time lags less than 50 days, we are unable to get information on correlations between blackouts because the time intervals are too short to contain several blackouts. We see a correlation between absence of blackouts, and because these time intervals tend to only contain absences of blackouts, we see H close to 1 (trivially deterministic). For time lags above 50 days, the R/S shows a power behavior and gives a correct determination of blackout correlation. The R/S calculation is sensitive to this change in regime

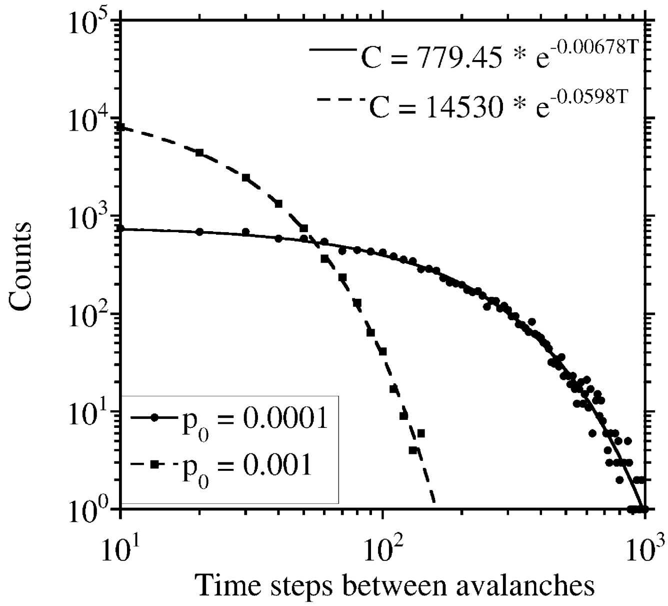

Fig. 4. Distribution of waiting times between avalanches in a sandpile for two values of the probability of adding grains of sand.

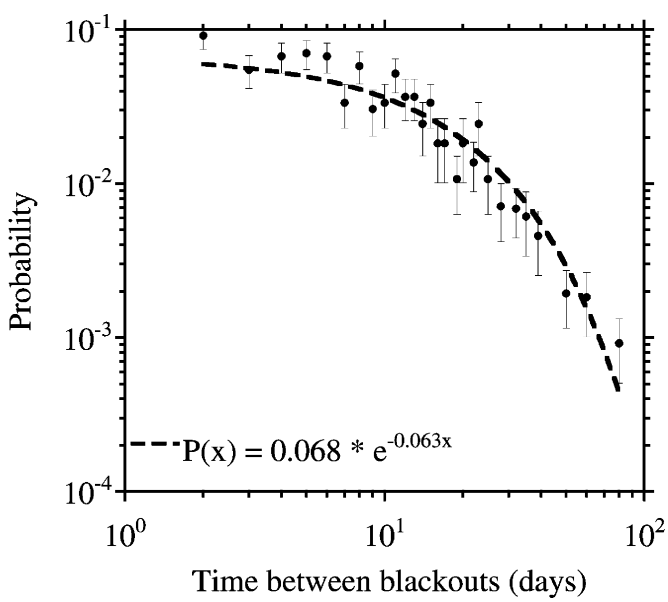

Fig. 5. PDF of the waiting times between blackouts.

and there is an obvious change of behavior for time intervals around 50 days. An alternative method of determining correlations is the scaled window variance method. We do not use the scaled window variance method in this paper because in this method, the correlations between absences of blackouts skew the correlations between blackouts at larger time lags [7].

IV. Effect of Weather

Approximately half of the blackouts (212 blackouts) are characterized as weather related in the NERC data. In attempting to extract a possible periodicity related to seasonal weather, we consider separately the time series of all blackouts and the time series of blackouts that are not weather related. An important

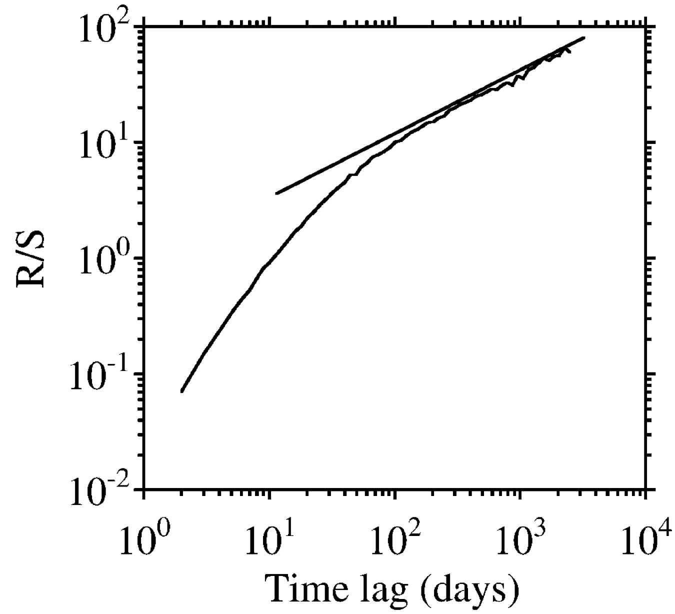

Fig. 6. R/S for the number of customers affected by blackouts.

TABLE II
Hurst Parameter H for Measures of Blackout Size Comparing All Data With Data Excluding Blackouts Triggered by Weather

	H all events	H nonweather events
Events	0.62	0.62
Power lost	0.59	0.64
Customers	0.57	0.58
MWh	0.53	0.57

issue in studying long-range dependencies is the possible presence of periodicities. Both R/S analysis and spectral analysis of this data do not show any clear periodicity. However, since the weather related events may play an important role in the blackouts, one may suspect seasonal periodicities. However, the data combines both summer and winter peaking regions of North America. Because of the limited amount of data, it is not possible to separate the blackouts by geographical location and redo the analysis. What we have done is to reanalyze the data excluding the blackouts triggered by weather related events. The results are summarized in Table II. As can be seen, the exclusion of the blackouts triggered by weather related events does not significantly change the value of H. When looking solely at the blackouts triggered by weather related events, the value of H is closer to 0.5 (random events), although the available data is too sparse to be sure of the significance of this result.

Another question to consider is the effect of excluding the weather related events on the PDF. We have recalculated the PDF for all the measures of blackout size when the weather related events are not included. The PDFs obtained are the same within the numerical accuracy of this calculation. This is illustrated in Fig. 1, where we have plotted the PDFs of the number of customers unserved for all events and for the nonweather related events. Therefore, for both long-range dependencies and structure of the PDF, the blackouts triggered by weather events do not show any particular properties that distinguish them from the other blackouts. Therefore, both the long time correlations and the PDFs of the blackout sizes remain consistent with SOC-like dynamics.

In addition to weather effects, one might expect spatial structure of the grid to have an effect on the dynamics. However, analysis of the NERC data by Chen *et al.* in [13] suggests that similar results are obtained when data for the eastern and western North American power systems is analyzed separately. Since the eastern and western power systems have different characteristics, this interesting result tends to support the notion that there are some underlying common principles for the system dynamics.

V. Comparison to SOC Sandpile Model

The issue of determining whether power system blackouts are governed by SOC is a difficult one. There are no unequivocal determining criteria. One approach is to compare characteristic measures of the power system to those obtained from a known SOC system. The prototypical model of a SOC system is a one-dimensional idealized running sandpile [22]. The mass of the sandpile is increased by adding grains of sand at random locations. However, if the height at a given location exceeds a threshold, then grains of sand topple downhill. The topplings cascade in avalanches that transport sand to the edge of the sandpile, where the sand is removed. In the running sandpile, the addition of sand is on average balanced by the loss of sand at the edges and there is a globally quasi-steady state or dynamic equilibrium close to the critical profile that is given by the angle of repose. There are avalanches of all sizes and the PDF of the avalanche sizes has a power law tail. The particular form of the sandpile model used here is explained in [25] and the sandpile length used in the present calculations is $L = 800$. We are, of course, not claiming that the running sandpile is a model for power system blackouts. We only use the running sandpile as a black box to produce a time series of avalanches characteristic of a SOC system.

It is convenient to assume that every time iteration of the sandpile corresponds to one day. When an avalanche starts, we integrate over the number of sites affected and the number of steps taken and assign them to a single day. Thus we construct a time series of the avalanche sizes. The sandpile model has a free parameter p_0, which is the probability of a grain of sand being added at a location. p_0 is chosen so that the average frequency of avalanches is the same as the average frequency of blackouts.

In evaluating the long-range time dependence of the blackouts, we use the rescaled range or R/S [24] technique described earlier. As stated before, the R/S technique is useful in determining the existence of a power law tail in the autocorrelation function and calculating the exponent of the decay of the tail (see Appendix for details). The same R/S analysis used for the blackout time series is applied to the avalanche time series. Fig. 7 shows the R/S statistic for the time series of avalanche sizes from the sandpile and for the time series of power lost by the blackouts. The similarity between the two curves is remarkable. A similarly good match of the R/S statistics between the blackout and sandpile time series is obtained for the other measures of blackout size.

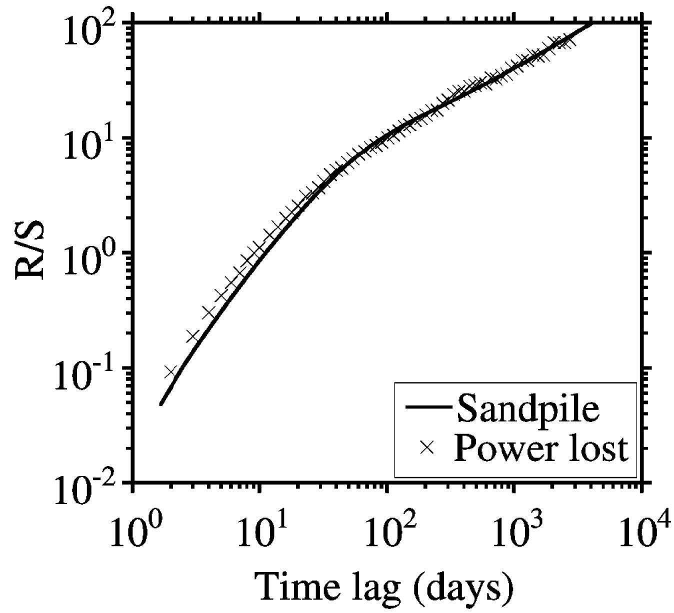

Fig. 7. R/S for avalanche sizes in a running sandpile compared to R/S for power lost in blackouts.

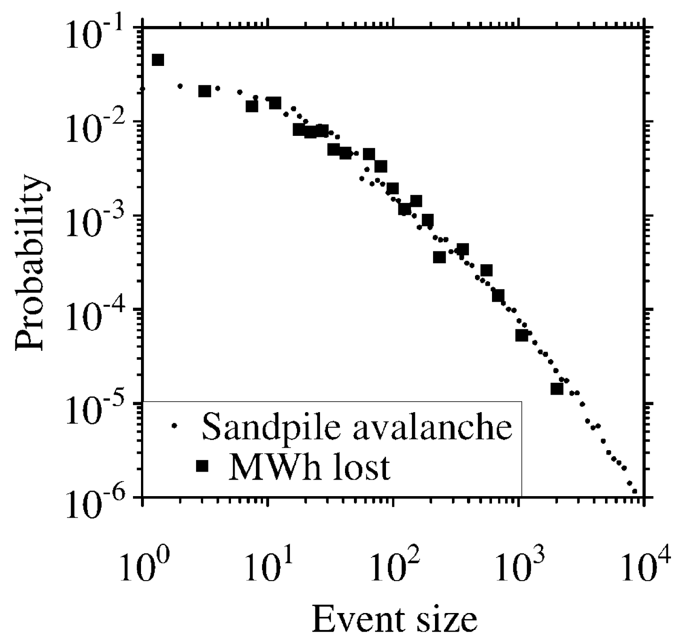

Fig. 8. Rescaled PDF of energy unserved during blackouts superimposed on the PDF of the avalanche size in the running sandpile.

Fig. 8 shows the PDF of the avalanche sizes from the sandpile data together with the rescaled PDF of the energy unserved from the blackout data. The resemblance between the two distributions is again remarkable. The rescaling is necessary because of the different units used to measure avalanche size and blackout size. That is, we assume a transformation of the form

$$P(X) = \lambda F\left(\frac{X}{\lambda}\right). \tag{1}$$

X is the variable that we are considering, $P(X)$ is the corresponding PDF, and λ is the rescaling parameter. If the transformation (1) works, F is the universal function that describes the

PDF for the different parameters. Transformation (1) is used to overlay the sandpile and blackout PDFs.

We can consider PDFs of the other measures of blackout size and use transformation (1) to plot each of these PDFs with the sandpile avalanche size PDF. In all cases, the agreement is very good. Of course, the rescaling parameter differs for each measure of blackout size. The exponents obtained for these PDFs tails are between -1.3 and -2. These exponents imply divergence of the variance, one of the characteristic features of systems with SOC dynamics. In fact, divergence of the variance is a general feature of systems near criticality. This comparison of the PDFs of the measures of blackout and avalanche sizes is useful in evaluating the possible errors in the determination of the power law decay exponent of the PDFs. One can see that for the large size events where the statistics are sparse, there may be deviations from the curve. These deviations can influence the computed value of the exponent, but they are probably of little significance for the present comparisons.

VI. Possible Explanation of Power System SOC

To motivate comparisons between power system blackout data and SOC sandpile data, we suggest a qualitative description of the structure and effects in a large-scale electric power transmission system which could give rise to SOC dynamics. The power system contains many components such as generators, transmission lines, transformers and substations. Each component experiences a certain loading each day and when all the components are considered together, they experience some pattern or vector of loadings. The pattern of component loadings is determined by the power system operating policy and is driven by the aggregated customer loads at substations. The power system operating policy includes short term actions such as generator dispatch as well as longer term actions such as improvements in procedures and planned outages for maintenance. The operating policy seeks to satisfy the customer loads at least cost. The aggregated customer load has daily and seasonal cycles and a slow secular increase of about 2% per year.

Events are either the limiting of a component loading to a maximum or the zeroing of the component loading if that component trips or fails. Events occur with a probability that depends on the component loading. For example, the probability of relay misoperation [13] or transformer failure generally increases with loading. Another example of an event could be an operator redispatching to limit power flow on a transmission line to its thermal rating and this could be modeled as probability zero when below the thermal rating of the line and probability one when above the thermal rating. Each event is a limiting or zeroing of load in a component and causes a redistribution of power flow in the network and hence a discrete increase in the loading of other system components. Thus events can cascade. If a cascade of events includes limiting or zeroing the load at substations, it is a blackout. A stressed power system experiencing an event must either redistribute load satisfactorily or shed some load at substations in a blackout. A cascade of events leading to blackout usually occurs on a time scale of minutes to hours and is completed in less than one day.

It is customary for utility engineers to make prodigious efforts to avoid blackouts and especially to avoid repeated blackouts with similar causes. These engineering responses to a blackout occur on a range of time scales longer than one day. Responses include repair of damaged equipment, more frequent maintenance, changes in operating policy away from the specific conditions causing the blackout, installing new equipment to increase system capacity, and adjusting or adding system alarms or controls. The responses reduce the probability of events in components related to the blackout, either by lowering their probabilities directly or by reducing component loading by increasing component capacity or by transferring some of the loading to other components. The responses are directed toward the components involved in causing the blackout. Thus the probability of a similar blackout occurring is reduced, at least until load growth degrades the improvements made. There are similar, but less intense responses to unrealized threats to system security such as near misses and simulated blackouts.

The pattern or vector of component loadings may be thought of as a system state. Maximum component loadings are driven up by the slow increase in customer loads via the operating policy. High loadings increase the chances of cascading events and blackouts. The loadings of components involved in the blackout are reduced or relaxed by the engineering responses to security threats and blackouts. However, the loadings of some components not involved in the blackout may increase. These opposing forces driving the component loadings up and relaxing the component loadings are a reflection of the standard tradeoff between satisfying customer loads economically and security. The opposing forces apply over a range of time scales. We suggest that the opposing forces, together with the underlying growth in customer load and diversity give rise to a dynamic equilibrium and conjecture that this dynamic equilibrium could be SOC-like. It is important to note that this type of system organizes itself to an operating point near to but not at a critical value. This could make the system intrinsically vulnerable to cascading failures from unexpected causes as the repair and remediation steps taken to prevent a known failure mode are part of the system dynamics.

We briefly indicate the roughly analogous structure and effects in an idealized sand pile model. Events are the toppling of sand and cascading events are avalanches. The system state is a vector of maximum gradients at all the locations in the sand pile. The driving force is the addition of sand, which tends to increase the maximum gradient, and the relaxing force is gravity, which topples the sand and reduces the maximum gradient. SOC is a dynamic equilibrium in which avalanches of all sizes occur and in which there are long time correlations between avalanches. The rough analogy between the sand pile and the power system is shown in Table III. There are also some distinctions between the two systems. In the sand pile, the avalanches are coincident with the relaxation of high gradients. In the power system, each blackout occurs on fast time scale (less than one day), but the knowledge of which components caused the blackout determines which component loadings are relaxed both immediately after the blackout and for some time after the blackout.

TABLE III
ANALOGY BETWEEN POWER SYSTEM AND SAND PILE

	power system	sand pile
system state	loading pattern	gradient profile
driving force	customer load	addition of sand
relaxing force	response to blackout	gravity
event	limit flow or trip	sand topples

VII. CONCLUSION

We have calculated long time correlations and PDFs for several measurements of blackout size in the North American power transmission grid from 1984 to 1998. These long time correlations and PDFs seem consistent with long-range time dependencies and PDFs for avalanche sizes in a running sandpile known to be SOC. That is, for these statistics, the blackout size time series seem indistinguishable from the sandpile avalanche size time series. This similarity suggests that SOC-like dynamics may play an important role in the global complex dynamics of power systems.

We have outlined a possible qualitative explanation of the complex dynamics in a power system which proposes some of the opposing forces that could give rise to a dynamic equilibrium with some properties of SOC. The opposing forces are, roughly speaking, a slow increase in loading (and system aging) weakening the system and the engineering responses to blackouts strengthening parts of the system. Here we are suggesting that the engineering and operating policies of the system are important and integral parts of the system long-term complex dynamics. Carlson and Doyle have introduced a theory of highly optimized tolerance (HOT) that describes power law behavior in a number of engineered or otherwise optimized applications [5]. After this paper was first submitted, Stubna and Fowler [33] published an alternative view based on HOT of the origin of the power law in the NERC data.[3]

The PDFs of the measures of blackout size have power tails with exponents ranging from -1.3 to -2 and therefore have divergent variances. Thus large blackouts are much more frequent than might be expected. In particular, the application of traditional risk evaluation methods can underestimate the risk of large blackouts. R/S analysis of the blackout time series shows moderate ($H \approx 0.6$) long time correlations for several measures of blackout size. Excluding the weather related blackouts from the time series has little effect on the results. The exponential tail of the PDF of the times between blackouts supports the contention that the correlations between blackouts are due to the power system global dynamics rather than correlations in the events that trigger blackouts.

The strength of our conclusions is naturally somewhat limited by the short time period (15 years) of the available blackout data and the consequent limited resolution of the statistics. To further understand the mechanisms governing the complex dynamics of power system blackouts, modeling of the power system is indicated. There is substantial progress in modeling and analyzing the approach inspired by SOC outlined in Section VI [8]–[12], [17] and in modeling blackouts and cascading failure from other perspectives [14]–[16], [18]–[20], [27], [29]–[31], [34].

If the dynamics of blackouts are confirmed to have some characteristics of SOC, this would open up possibilities for monitoring statistical precursors of large blackouts or controlling the power system to modify the expected distribution of blackout sizes [11]. Moreover, it would suggest the need to revisit the traditional risk analysis based on random variables with exponential tails since these complex systems have statistics with power tails.

[3]To apply HOT to the power system, it is assumed that blackouts propagate one dimensionally [33] and that this propagation is limited by finite resources that are engineered to be optimally distributed to act as barriers to the propagation [5]. The one-dimensional assumption implies that the blackout size in a local region is inversely proportional to the local resources. Minimizing a blackout cost proportional to blackout size subject to a fixed sum of resources leads to a probability distribution of blackout sizes with an asymptotic power tail and two free parameters. The asymptotic power tail exponent is exactly -1 and this value follows from the one dimensional assumption. The free parameters can be varied to fit the NERC data for both MW lost and customers disconnected. Moreover, [33] shows that a better fit to both these data sets can be achieved by modifying HOT to allow some misallocation of resources.

APPENDIX

Consider the time series $X = \{X_t; t = 1, 2, \ldots, n\}$. We construct the series $Y = \{Y_t; t = 1, 2, \ldots, n\}$ that is the original series integrated in time: $Y_t = \sum_{s=0}^{t} X_s$. For the series Y and for each $m = 1, 2, \ldots, n$ a new series $Y^{(m)} = \{Y_u^{(m)}; u = 1, 2, \ldots, n/m\}$ is generated. The elements of the series $Y^{(m)}$ are blocks of m elements of Y so that $Y_u = \{Y_{um-m+1}^{(m)}, \ldots, Y_{um}\}$. We then calculate the range R_m^i and standard deviation S_m^i within each of the n/m blocks of m elements of $Y^{(m)}$, and compute for each block R_m^i/S_m^i. The R/S statistic as a function of the time lag m is then the average $(m/n) \sum_{i=1}^{n/m} R_m^i/S_m^i$.

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Benjamin A. Carreras received the Licenciado en Ciencias degree in physics from the University of Barcelona, Barcelona, Spain, in 1965, and the Ph.D. degree in physics from Valencia University, Valencia, Spain, in 1968.

He has been a Researcher and a Professor at the University of Madrid, Madrid, Spain, Glasgow University, Glasgow, U.K., Daresbury Nuclear Physics Laboratory, Warrington, U.K., Junta de Energia Nuclear, Madrid, Spain, and the Institute for Advanced Study, Princeton, NJ. He is now a Corporate Fellow at Oak Ridge National Laboratory, Oak Ridge, TN.

Dr. Carreras is a Fellow of the American Physical Society.

David E. Newman received the B.S. degree in physics and mathematics from the University of Pittsburgh, Pittsburgh, PA in 1983 and the Ph.D. degree in physics from the University of Wisconsin, Madison, in 1993.

He was a Wigner Fellow and Research Scientist at Oak Ridge National Laboratory, Oak Ridge, TN, from 1993 to 1998. In 1998, he joined the Physics Department at the University of Alaska, Fairbanks, where he is now an Associate Professor.

Ian Dobson (M'89–SM'98) received the B.A. degree in mathematics from Cambridge University, Cambridge, U.K., in 1978, and the Ph.D. degree in electrical engineering from Cornell University, Ithaca, NY, 1989.

He was a Systems Analyst for EASAMS Ltd, Frimley, U.K., from 1978 to 1983. In 1989, he joined the University of Wisconsin, Madison, where he is now a Professor in Electrical and Computer Engineering. His current interests are applications of complex systems and nonlinear dynamics, electric power system blackouts and instabilities, and risk analysis of cascading failure.

A. Bruce Poole received the B.S. degree in aerospace/ mechanical engineering from the University of Texas in 1968, and the M. A. degree in management from Tusculum College, Greeneville, TN, in 1992.

He was an Engineer and Manager for a variety of manufacturing and utility companies, including Westinghouse Electric Corporation, Houston Lighting and Power Company, and the Tennessee Valley Authority. He worked at Oak Ridge National Laboratory, Oak Ridge, TN, from 1989 to 2000, where he was a Senior Research Staff Member and Program Manager. He presently works for the Federal Energy Regulatory Commission, Washington, DC.

6.12 Complex dynamics of blackouts in power transmission systems

B.A. Carreras, V.E. Lynch, I. Dobson, D.E. Newman

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Complex dynamics of blackouts in power transmission systems

B. A. Carreras and V. E. Lynch
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

I. Dobson
Electrical and Computer Engineering Department, University of Wisconsin, Madison, Wisconsin 53706

D. E. Newman
Physics Department, University of Alaska, Fairbanks, Alaska 99775

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In order to study the complex global dynamics of a series of blackouts in power transmission systems a dynamical model of such a system has been developed. This model includes a simple representation of the dynamical evolution by incorporating the growth of power demand, the engineering response to system failures, and the upgrade of generator capacity. Two types of blackouts have been identified, each having different dynamical properties. One type of blackout involves the loss of load due to transmission lines reaching their load limits but no line outages. The second type of blackout is associated with multiple line outages. The dominance of one type of blackout over the other depends on operational conditions and the proximity of the system to one of its two critical points. The model displays characteristics such as a probability distribution of blackout sizes with power tails similar to that observed in real blackout data from North America.
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Power transmission systems are complex systems that evolve over years in response to the economic growth of the country and to continuously increasing power demand. In spite of the reliability of these systems, there are widespread disturbances that have significant cost to society. The average frequency of blackouts in the United States is about one every 13 days. This frequency has not changed over the last 30 years. Also the probability distribution of blackout sizes has a power tail; this dependence indicates that the probability of large blackouts is relatively high. Indeed, although large blackouts are rarer than small blackouts, it can be argued that their higher societal cost makes the risk of large blackouts comparable to or exceed the risk of small blackouts. The operation of power transmission systems is studied from the perspective of complex dynamics in which a diversity of opposing forces regulate both the maximum capabilities of the system components and the loadings at which they operate. These forces enter in a nonlinear manner and may cause an evolution process to be ultimately responsible for the regulation of the system. This view of a power transmission system considers not only the engineering and physical aspects of the power system, but also the economic, regulatory, and political responses to blackouts and increases in load power demand. From this perspective, the search for the cause of the blackouts must not be limited to the trigger of the blackout, which is normally a random event, but it must also consider the dynamical state of the power transmission system. A detailed incorporation of all these aspects of the dynamics into a single model is extremely complicated. Here, a simplified model is discussed with some approximate overall representation of the opposing forces controlling the system dynamics. This model reproduces some of the main features of North American blackout data.

I. INTRODUCTION

Power transmission systems are complex systems that evolve over years in response to the economic growth of the country and to continuously increasing power demand. The evolution and reliability of these systems are leading engineering accomplishments of the last century that underpin developed societies. Nevertheless, widespread disturbances of power transmission systems that have significant cost to society are consistently present. An analysis of blackouts[1] done in the 1970s indicated that the average frequency of blackouts in the United States was one every 14 days. More recent analyses[2,3] of 15 years of North American Electrical Reliability Council (NERC) data on blackouts of the North American power grid[4] gave an average frequency of blackouts of one every 13 days. Furthermore, these analyses show that the distribution of blackout sizes has a power tail with an exponent of about -1.3 ± 0.2. These results indicate that the probability of large blackouts is relatively high. Indeed, although large blackouts are rarer than small blackouts, it can be argued that combining their higher societal costs with their relatively high probability makes the risk of large blackouts comparable to or greater than the risk of small blackouts.[5]

It is clear that individual blackouts are triggered by random events ranging from equipment failures and bad weather to vandalism.[4] The blackouts then typically become widespread through a series of cascading events. However, it must be remembered that these individual blackouts occur in a power transmission system that is itself slowly and dynamically evolving in its design, configuration, and operation. For example, the loading of system components relative to their maximum loading is a key factor governing the propagation of component failures and this loading evolves as the system components or operational policies are upgraded. The existence of a power tail in the distribution of blackouts and the long time correlations seen in the system suggests that underlying the large-scale blackouts may be a dynamically caused proximity to a critical point. It should be noted that the size of a given blackout is unrelated to the particular triggering event that initiated that blackout.

To investigate such a possibility, we propose a model for power transmission systems[6,7] that involves not only the dynamics of the generator dispatch but also the evolution of the system under a continuous increase in demand. This model shows how the slow opposing forces of load growth and network upgrades in response to blackouts could self-organize the power system to a dynamic equilibrium. Blackouts are modeled by overloads and outages of transmission lines determined in the context of linear programming (LP) dispatch of a dc load flow model. This model shows complex dynamical behaviors and has a variety of transition points as a function of increasing power demand.[8] Some of these transition points have the characteristic properties of a critical transition. That is, when the power demand is close to a critical value, the probability distribution function (PDF) of the blackout size has an algebraic tail, and the system changes sharply across the critical point. Because of this, the risk of a global blackout increases sharply at the critical transition.

The fact that, on one hand, there are critical points with maximum power flow through the network and, on the other hand, there is a self-organization process that tries to maximize efficiency and minimize risk, may lead to a power transmission model governed by self-organized criticality (SOC).[9] Such a possibility was first explored with a simple cellular automaton model[10] that incorporates neither the circuit equations nor the type of long-term dynamics discussed above. In this paper, we study the dynamical properties of a power transmission model[6,7] that does incorporate these two components.

There have been some other complex system approaches to modeling aspects of power system blackouts. In the most closely related work, Chen and Thorp[11,12] modeled power system blackouts using dc load flow and LP dispatch and represented in detail hidden failures of the protection system. They obtained the distribution of power system blackout size by rare event sampling, and studied blackout risk assessment and mitigation methods. Stubna and Fowler[13] applied a modified ''Highly Optimized Tolerance'' (HOT) model to fit North American blackout data for blackout sizes measured by both power shed and customers disconnected. Using a different approach, Roy, Asavathiratham, Lesieutre, and Verghese constructed randomly generated tree networks that abstractly represent influences between idealized components.[14] In that work, components can be failed or operational according to a Markov model that represents both internal component failure and repair processes and influences between components that cause failure propagation. The effects of the network degree of connectivity and intercomponent influences on the failure size and duration were studied. Similarly, Pepyne *et al.*[15] used a Markov model for discrete-state power system nodal components but had failures propagate along the transmission lines of a power system network with a fixed probability. DeMarco[16] and Parrilo *et al.*[17] addressed the challenging problem of determining cascading failure due to dynamic transients by using hybrid nonlinear differential equation models. DeMarco used Lyapunov methods applied to a smoothed model; Parrilo *et al.* used Karhunen–Loeve and Galerkin model reduction to address the problem.

The rest of this paper is organized as follows: In Sec. II, we introduce a dynamical model of power transmission system evolution over long time scales. Details of the power flow model and of the fast time scale dynamics are provided in the Appendix. In Sec. III, numerical results of the model are reported with an analysis of the time and space correlations introduced by the dynamics. In Sec. IV, we analyze the effect of changing the ratio of generator capacity margin to maximum load fluctuation. This ratio allows the separation of the dynamics into two different regimes. The conclusions are given in Sec. V.

II. DYNAMICAL MODEL FOR POWER TRANSMISSION

In modeling the dynamics of power transmission systems, one must consider two intrinsic time scales. There is a slow time scale, of the order of days to years, over which load power demand slowly increases. Over this time scale, the network is upgraded in engineering responses to blackouts and in providing more generator power in response to demand. As we shall see, these slow opposing forces of load-increase and network-upgrade self-organize the system to a dynamic equilibrium. The dynamical properties of this model are the main topic of this paper. In power transmission systems, there is also a fast time scale, of the order of minutes to hours, over which power is dispatched through the network within which (depending on the conditions of the network) cascading overloads or outages may lead to a blackout.

Over the fast time scale, we solve the standard dc power flow equation for a given distribution of load demand. We use the standard LP method[18–20] with the usual constraints on generating power capability and transmission line limits to solve the generator power dispatch. An example of a power transmission network used in these studies is the IEEE 118 bus network[21] shown in Fig. 1. Details of the fast dynamics can be found in Refs. 6 and 7 and a summary description is given in the Appendix.

In any network, the network nodes (buses) are either loads (L) (black squares in Fig. 1), or generators (G), (gray squares in Fig. 1). The power P_i injected at each node is positive for generators and negative for loads, and the maximum power injected is $P_i^{\max}$. The transmission line connecting nodes i and j has power flow F_{ij}, maximum power flow $F_{ij}^{\max}$, and impedance z_{ij}. There are N_l lines and $N_N = N_G + N_L$ total nodes, where N_G is the number of generators and N_L is the number of loads.

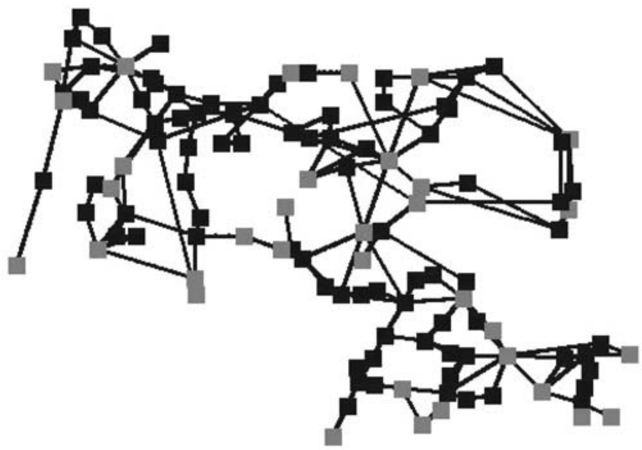

FIG. 1. Diagram of the IEEE 118 bus network. Generators are gray squares; loads are the black squares.

The slow dynamics proposed in Refs. 6 and 7 has three components: (1) the growth of the demand, (2) response to blackouts by upgrades in the grid transmission capability, and (3) response to increased demand by increasing maximum generator power. These components of the model are translated into a set of simple rules. We simplify the time scale by regarding one blackout to be possible each day at the peak loading of that day. At the beginning of the day t, we apply the following rules:

(1) The demand for power grows. All loads are multiplied by a fixed parameter λ that represents the daily rate of increase in electricity demand. On the basis of past electricity consumption in the United States, we estimate that $\lambda = 1.00005$. This value corresponds to a yearly rate of 1.8%,

$$P_i(t)=\lambda P_i(t-1) \quad \text{for} \quad i \in L. \tag{1}$$

To represent the daily local fluctuations in power demand, all power loads are multiplied by a random number r, such that $2-\gamma \leqslant r \leqslant \gamma$, with $1 \leqslant \gamma \leqslant 2$. The power transmission grid is improved. We assume a gradual improvement in the transmission capacity of the grid in response to outages and blackouts. This improvement is implemented through an increase of $F_{ij}^{\max}$ for the lines that have overloaded during a blackout. That is,

$$F_{ij}^{\max}(t)=\mu F_{ij}^{\max}(t-1), \tag{2}$$

if the line ij overloads during a blackout. We take μ to be a constant greater than 1 and in the present studies we have varied μ in the range $1.01 \leqslant \mu \leqslant 1.1$.

It is customary for utility engineers to make prodigious efforts to avoid blackouts, and especially to avoid repeated blackouts with similar causes, which we have simplified into this one parameter μ. In general, these responses to blackouts occur on a range of time scales longer than one day. Responses include repair of damaged equipment, more frequent maintenance, changes in operating policy away from the specific conditions causing the blackout, installing new equipment to increase system capacity, and adjusting or adding system alarms or controls. The responses reduce the probability of events in components related to the blackout, either by lowering their probabilities directly or by reducing component loading by increasing component capacity or by transferring some of the loading to other components. The responses are directed toward the components involved in causing the blackout. Thus the probability of a similar blackout occurring is reduced; at least until load growth degrades the improvements that were made. There are similar but less intense responses to unrealized threats to system security, such as near misses and simulated blackouts.

By simplifying all engineering responses into a single parameter μ we crudely represent all these responses to a blackout. The response is modeled as happening on the next day, but the effect is eventually cancelled by the slow load increase. Because of the disparity between these two time scales, at this level of modeling it does not seem crucial to have an accurate estimate of the response time, and the one-day time scale may be reasonable.

(2) The maximum generator power is increased in response to the load demand as follows:

(a) The increase in power is quantized. This can reflect either the upgrade of a power plant or the addition of generators. The increase is taken to be a fixed ratio to the total power. Therefore, we introduce the quantity

$$\Delta P_a \equiv \kappa (P_T/N_G), \tag{3}$$

where P_T is the total power demand, N_G the number of generators, and κ is a parameter that we have taken to be a few percent.

(b) To be able to increase the maximum power in node j, the sum of the power flow limits of the lines connected to j should be larger than the existing generating power plus the addition at node j. This requirement maintains the coordination of the maximum generator power ratings with the line ratings.

(c) A second condition to be verified before any maximum generator power increase is that the mean generator power margin has reached a threshold value. That is, we define the mean generator power margin at a time t as

$$\frac{\Delta P}{P}=\frac{\sum_{j \in G} P_j - P_0 e^{(\lambda-1)t}}{P_0 e^{(\lambda-1)t}}, \tag{4}$$

where P_0 is the initial power load demand.

(d) Once condition (c) is verified, we choose a node at random to test condition (b). If the chosen node verifies condition (b), we increase its power by the amount given by Eq. (3). If condition (b) is not verified, we choose another node at random and iterate. After power has been added to a node, we use Eq. (4) to recalculate the mean generator power margin and continue the process until $\Delta P/P$ is above the prescribed quantity $(\Delta P/P)_c$.

(3) We also assign a probability p_0 for a random outage of a line. This value represents possible failures caused by phenomena such as accidents and weather related events.

After applying these three rules to the network, we look for a solution of the power flow problem by using linear programming as described in the Appendix.

It is also possible to introduce a time delay between the detection of a limit in the generation margin and the increase in maximum generator power. This delay would represent construction time. However, the result is the same as increasing the value of κ in Eq. (3), which can also give an alternative interpretation for κ.

Five basic parameters control the dynamics of this model. One is the rate of increase in power demand, λ, which we keep fixed at 1.8% per year on the basis of the averaged value for the U.S. grid in the last two decades.[22] A second parameter is the improvement rate of the transmission grid, μ. This is not an easy parameter to estimate. However, once μ is given, there is a self-regulation process by which the system produces the number of blackouts that would stimulate the engineering response needed to meet demand. This is a necessary condition for the dynamical equilibrium of the system. The rate of increase in power demand for the overall transmission system is essentially given by $R_D \approx (\lambda - 1) N_L$. The system response is $R_R \approx (\mu - 1) f_{\text{blackout}} \langle \ell_o \rangle N_L$, where f_{blackout} is the frequency of blackouts and $\langle \ell_o \rangle$ is a weighted average of the number of lines overloaded during a blackout. Dynamical equilibrium implies that $R_D = R_R$. That is, the increase in demand and the corresponding increase in power supply must be matched by improvements in the transmission grid. Because those improvements are in response to real or simulated blackouts, this relation implies that μ must be greater than λ; otherwise, the system would be collapsing with constant blackouts. In the numerical calculations and for the value of the demand increase of 1.8% per year, we found that μ must be >1.01 in order to avoid this collapse regime. In the present calculations, we keep μ in the range 1.01–1.1. In this regime, results depend weakly on μ.

A third parameter Γ is a measure of the generation capacity of the power system in response to fluctuations in the power demand. Γ is the ratio of the reserve generator power to the maximum daily fluctuation of the power demand. The averaged power demand increases exponentially in time as $\overline{P_D}(t) \equiv P_0 e^{(\lambda - 1)t}$. However, the real instantaneous demand is $P_D(t)$, different from the averaged power demand because of daily fluctuations. The generator power installed $P_G(t)$ is also different from the averaged power demand. The difference $\Delta P(t) \equiv P_G(t) - \overline{P_D}(t)$ is the generator capability margin used to cope with fluctuations in power demand. In our calculations, the generator capability margin is varying in time, but we require it to be larger than a minimum prescribed value ΔP_c. Because the power demand is continuously increasing, it is convenient to normalize all these quantities to $\overline{P_D}(t)$. Thus we define Γ as the ratio of the normalized minimal generator capability margin $\Delta P_c / \overline{P_D}(t)$ to the maximum fluctuation of the load demand $g \equiv \max([(P_D(t) - \overline{P_D}(t))/\overline{P_D}(t)]^2)^{1/2}$,

$$\Gamma = [\Delta P_c / \overline{P_D}]/g. \tag{5}$$

There is a simple relation between g and the load fluctuation parameter γ. The parameter Γ is the main parameter varied in the calculations presented here. In the U.S., the generator power capability margin has had a wide variation over the years, but an estimated mean value[22] falls into the range of 15%–30%.

The fourth parameter is the probability of an outage caused by a random event (p_0). This parameter can be used to partially control the frequency of blackouts, although the relation between them is not linear. The fifth parameter is the probability for an overloaded line to undergo an outage (p_1). We keep this parameter in the range $0.1 \leqslant p_1 \leqslant 1.0$.

Since each calculation can be done for different specific network configurations, in this work we will use idealized treelike networks, which were discussed in Ref. 8, as well as more realistic networks, such as the IEEE 118 bus network depicted in Fig. 1.

The time evolution of a power transmission system represented by this model leads, after a transient, to a steady-state regime. Here "steady state" is defined with relation to the slow dynamics of the blackouts because the power demand is constantly increasing, as shown in Fig. 2. The time evolution in the model shows the transient period followed by steady-state evolution. This is illustrated in Fig. 2, where we have plotted the number of blackouts per 300 days as a function of time. We can see a slight increase in the average number of blackouts during the first 20 000 days. This transient period is followed by the steady state where the number of blackouts in an averaged sense is constant. The properties in the slow transient are not very different from those in the steady state. However, for statistical analysis, we use the steady-state information to avoid contamination of the statistics. The length of the transient depends on the rate of growth in power demand. In the following calculations, we evaluate the blackout statistics by ignoring the initial transients and doing the calculations for a time period of 80 000 days in a steady state. Of course, the use of these long time scale steady-state results is driven by the need for large statistical samples and it is arguable whether the real electric power grid ever actually reaches a steady state.

III. DYNAMICAL EVOLUTION OF THE POWER TRANSMISSION MODEL

Looking at the time evolution of the different parameters that characterize the blackouts, one observes a noisy signal that could be mistaken for random. One could assume that this is in fact the situation because many of the blackouts are triggered by random events with probability p_0. However, that is not the case. It is instead found that there are significant space and time correlations resulting from the underlying dynamics of the power transmission model.

To investigate the time correlations in this apparently noisy system we calculate the Hurst exponent[23] of time series of blackout sizes. Here, we consider two measures of the size of a blackout. One is the load shed during a blackout normalized to the total power demand; the other is the number of line outages during a blackout.

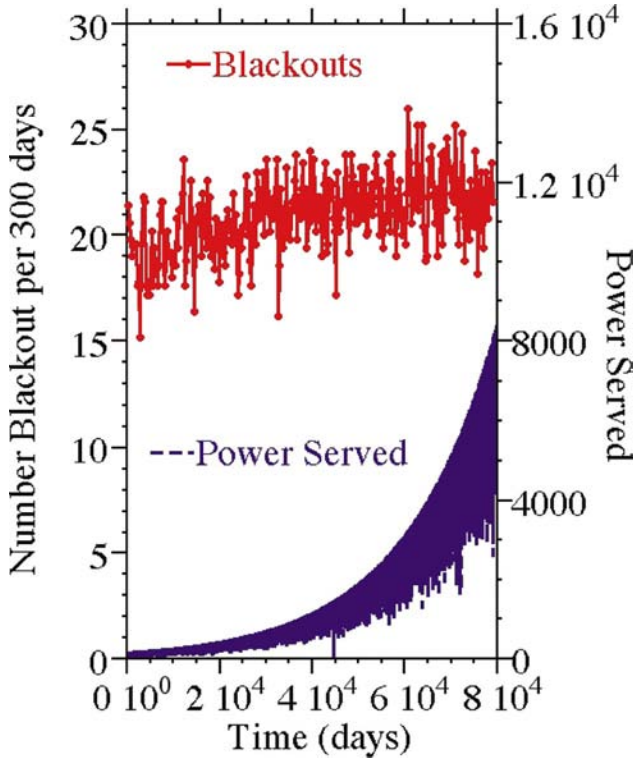

FIG. 2. (Color) Time evolution of the power served and number of blackouts per year from the model.

We use the R/S method[24] to calculate the Hurst exponent. An example of the result of this analysis is shown in Fig. 3. For times of the order of a few days and a few years, both series show weak persistence. They have the same Hurst exponent ($H=0.55\pm0.02$). This result is close to the one obtained in the analysis[2] of NERC data on blackouts of the North American power grid.[4] In this range of time scales, the value of the exponent does not depend on the value of Γ. For longer times, each time series shows a different behavior. The load shed has a nearly random character with $H=0.5$ for $\Gamma<1$. For $\Gamma>1$, the value of H decreases and in many cases is below 0.5. For these longer time scales, the time series of the number of line outages has a clear antipersistent character with H ranging from 0.2 to 0.4, depending on the network structure. In Fig. 4, the value of H resulting from a fit of R/S

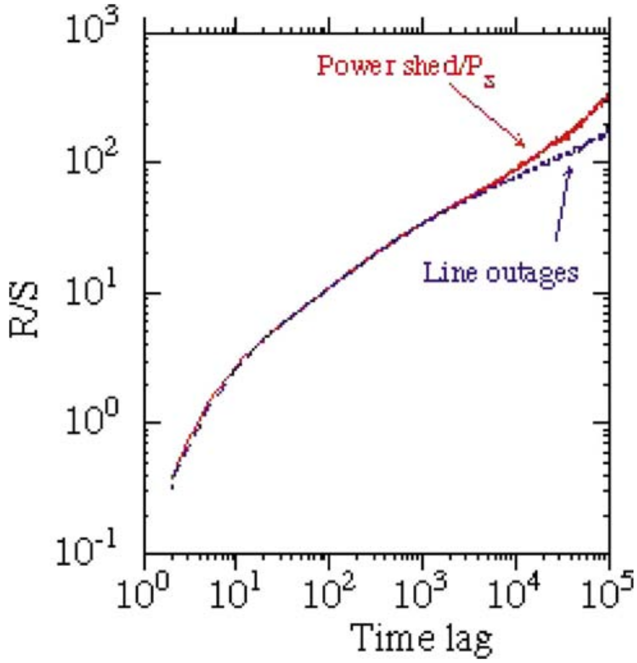

FIG. 3. (Color) R/S for the time series of normalized load shed and line outages for a 46-node tree network.

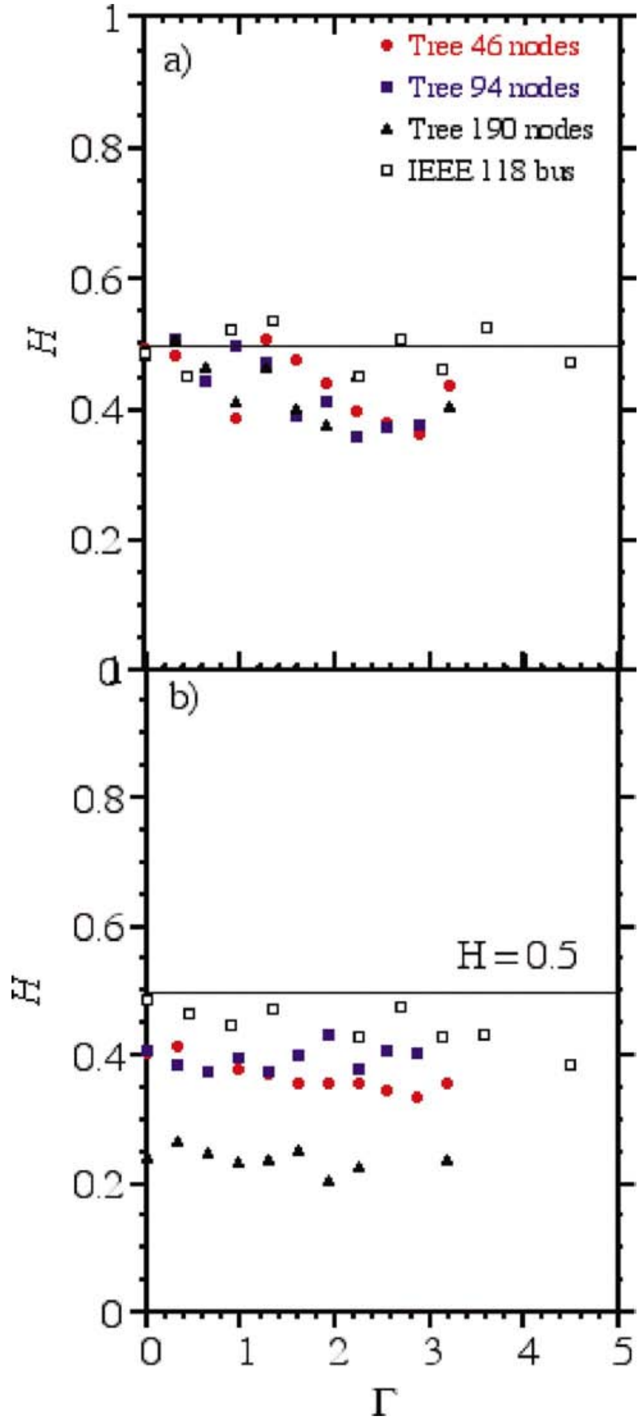

FIG. 4. (Color) The Hurst exponent H as a function of Γ for the time series of normalized load shed (a) and line outages (b). The exponent is calculated from a fit of R/S in the time range $600\leqslant t\leqslant 10^5$ for 46, 94, and 190 nodes tree networks and for the IEEE 118 bus network.

in the time range $600 \leqslant t \leqslant 10^5$ is given as a function of Γ for three of the tree networks and for the IEEE 118 bus network. Antipersistency in the time sequence of number of line outages can be expected from the model. Blackouts with a large number of line outages happen rarely, only once every few years. When they happen, there is a great deal of repair and enhancement of many transmission lines. As a consequence, blackouts with a large number of line outages become less probable after one of those events. Therefore, there is antipersistency at that time scale. In the present model, load shed does not have a direct impact on the repair and upgrade of the system. Therefore, time correlations are weak. As we will discuss in the next section, for $\Gamma > 1$, blackouts with large load sheds are associated with a large number of line outages. Therefore, in this Γ range we see some level of antipersistency due to the coupling of load shed and the number of line outages. The available data from NERC are limited to 15 years, and we therefore do not have any direct way of confirming this long-term behavior of the model in the real power system.

The time lag during which the number of line outages changes from weak persistency to antipersistency is independent of the network size but depends on the repair rate (μ). As μ increases, it takes longer time lags for the change to occur. Increasing μ causes a slight increase in H, but H remains less than 0.5.

Within this model the correlations are not limited to time correlations. The PDFs of the load shed and the number of line outages both have power-scaling regions implying spatial correlations. The correlations responsible for these power tails are the result of the system being near a critical point.

In Ref. 7, we studied the critical points of the power transmission model as the total load demand was varied. The slow dynamics described in Sec. II were not modeled. We found two types of critical points: one type was related to the limiting power flows in the transmission lines; the other type was related to the limit in the power generation. When these types of critical points are close to each other, the probability distribution of the blackout size as measured by the amount of load shed has a power law dependence for a range of values of the load shed. Away from the critical point, this power law dependence no longer exists.

When the dynamical evolution over long time scales is included and the value of Γ is about 1, the system naturally evolves to a situation in which these critical points are close to each other. In this situation, the PDFs of the power shed will have a region of algebraic decay. In Fig. 5, we have plotted the relative cumulative frequency calculated from the time series of the blackout data from the numerical results. The cumulative frequency has been calculated directly from these data using the rank function. In Fig. 5, the load shed is normalized to the total power demand. The calculation was done for three of the tree network configurations. These distributions are compared with those obtained from a load scan without dynamical evolution when the load value was at the critical point. We cannot distinguish between the two calculations; the relative cumulative frequencies are practically the same. The overlap between the two results indicates that the dynamical model described in Sec. II intrinsically leads to operation of the system close to the critical points. A similar result has been obtained for the IEEE 118 bus network. In Fig. 5, we have given an arbitrary shift to the relative cumulative frequencies for a given size network to better observe the three different cases.

The relative cumulative frequency plotted in Fig. 5 has three characteristic regions. They all have an exponential tail reflecting the finite size effect of the network (region III). Region II is characterized by an algebraic decay. This power-law-scaling region increases with the number of nodes in the network, suggesting that it is a robust feature of the system. The power decay index is practically the same for the four networks and is close to -0.55. The particular values of the decay index for each tree network are given in Table I, in which the range of the power tail region is defined as the ratio of the maximum load shed to the minimum load shed described by the power law. From the values obtained for the four networks listed in Table I, we can see that this range scales with the network size.

The functional form of the relative cumulative frequency, or at least their power-scaling region, seems to have a universal character. Therefore, we can compare the relative cumulative frequency of the normalized load shed obtained for the largest network with the relative cumulative frequency of the blackouts obtained in the analysis of the 15 years of NERC data.[4] In Fig. 6, we have plotted the relative cumulative frequency of the NERC data together with the relative cumulative frequencies for the 382-node tree and IEEE 118 bus networks. We have normalized the blackout size to the largest blackout over the period of time considered. We can see that the present model, regardless of the network configuration, reproduces quite well the power-scaling region from the NERC data. The size of this region is shorter for the calculations. This is because the calculations are done for relatively small networks. The level of agreement between the algebraic scaling regions of the relative cumulative frequencies is remarkable and indicates that the dynamical model for series of blackouts has captured some of the main features of the NERC data.

IV. DYNAMICAL REGIMES

Calculations carried out with this model show the existence of two different dynamical regimes. The first regime is characterized by the low value of Γ (that is, a low generator capability margin and/or large fluctuations in the power demand). In this regime, the available power is limited and has difficulties in meeting demand. Blackouts are frequent, but they affect only a limited number of loads. In this regime, there are very few line outages. In the opposite limit, Γ is large and the blackouts are less frequent, but they tend to involve multiple line outages when they happen. This latter regime is interesting because there are many cascading events that can cause blackouts in a large part of the network. This suggests a possible separation between regimes of few failures and regimes with cascading failures both of which are physically interesting.

Let us investigate in a quantitative way the separation between these two regimes by varying the parameter Γ.

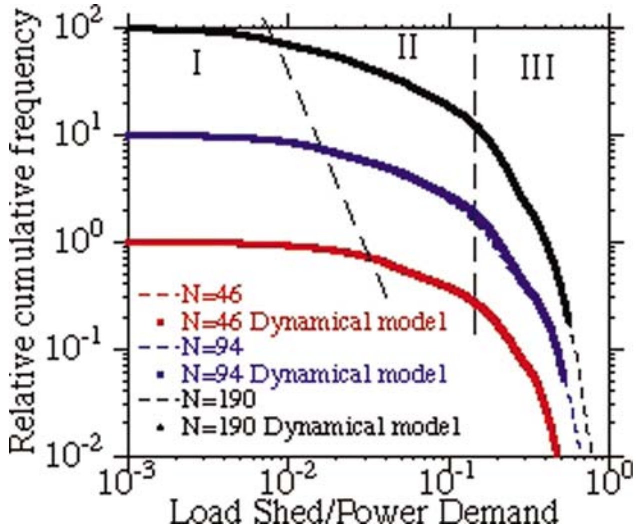

FIG. 5. (Color) Relative cumulative frequency of the load shed normalized to the total power demand for three different tree networks. The relative cumulative frequencies obtained from a load scan near the critical point are compared with the relative cumulative frequencies obtained from the dynamical model discussed in this paper.

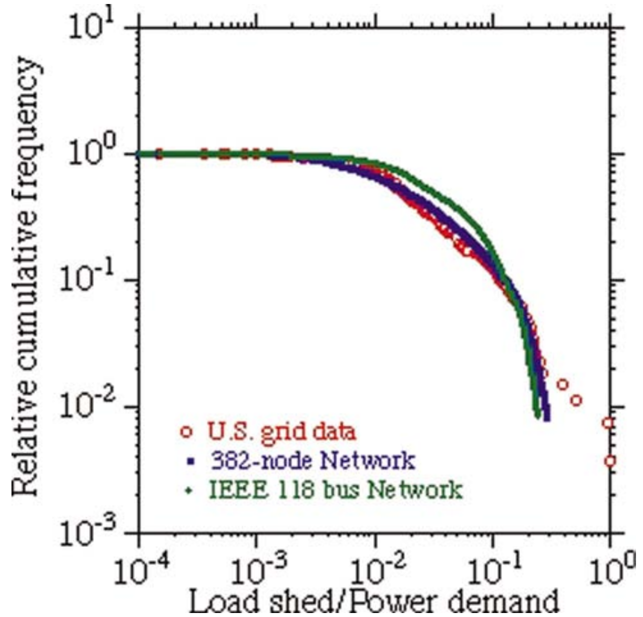

FIG. 6. (Color) Relative cumulative frequencies of the normalized load shed for the 382-node tree, the IEEE 118 bus networks, and the North American blackouts in 15 years of NERC data normalized to the largest blackout.

Varying Γ is not necessarily a realistic way of modeling the transmission system but it allows us to understand some features of the dynamics of the model. For several tree networks, we have done a sequence of calculations for different values of the minimal generator power margin $(\Delta P/P)_c$ at a constant g. We have changed this margin from 0 to 100%. For each value of this parameter, we have carried out the calculations for more than 100 000 days in a steady state regime. One way of looking at the change of characteristic properties of the blackouts with Γ is by plotting the power delivered and the averaged number of line outages per blackout. For a 94-node tree network, these plots are shown in Fig. 7. We can see that at low and high values of Γ, the power served is low. In the first case, because of the limited generator power, the system cannot deliver enough power when there is a relatively large fluctuation in load demand. At high Γ, the power served is low because the number of line outages per blackout is large.

Looking at averaged quantities is not a good way of identifying the demarcation between single failures and cascading events. To have a better sense of this demarcation, we have calculated the PDF of the number of line outages per blackout. In Fig. 8, we have plotted these PDFs for different values of Γ. The calculation was done for a 94-node tree network. We can see that at very low Γ there is a clear peak at 4 outages per blackout with very low probability for blackouts with more than 10 outages per blackout. As Γ increases, a second peak at about 17 outages emerges and the height of the peak increases with Γ. At the highest Γ value, this second peak is comparable to the peak at low number outages per blackout. In Fig. 9, we have plotted the ratio of the frequency of blackouts with more than 15 outages to the mean frequency of blackouts. We can see that for $\Gamma > 1$, this ratio reaches 0.007. This gives a measure of the frequency of what we can consider large-scale blackouts (more than 16%

TABLE I. Power law exponent of the PDF of the normalized power shed.

Number of nodes	PDF decay index	Range of power tail
46	−0.56	4
94	−0.51	8
190	−0.55	13
382	−0.58	31

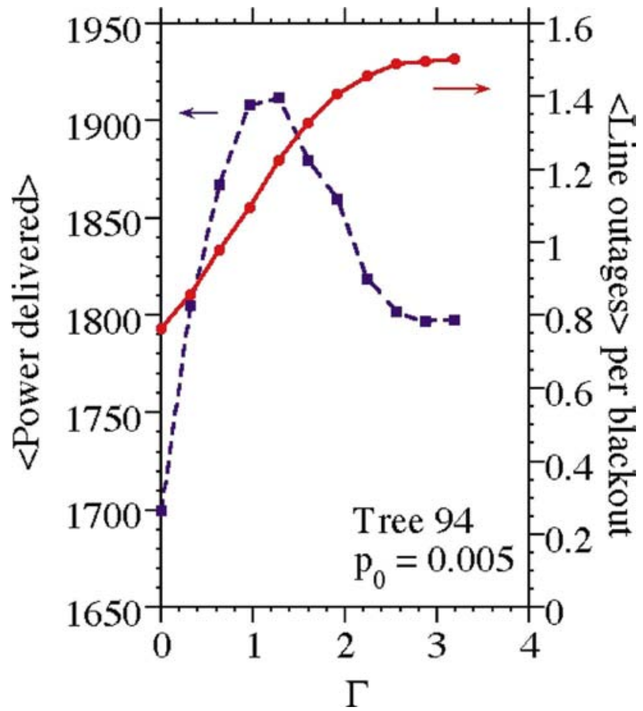

FIG. 7. (Color) Averaged power delivered and number of line outages per blackout for the 94-node tree network as a function of $\tilde{\Gamma}$

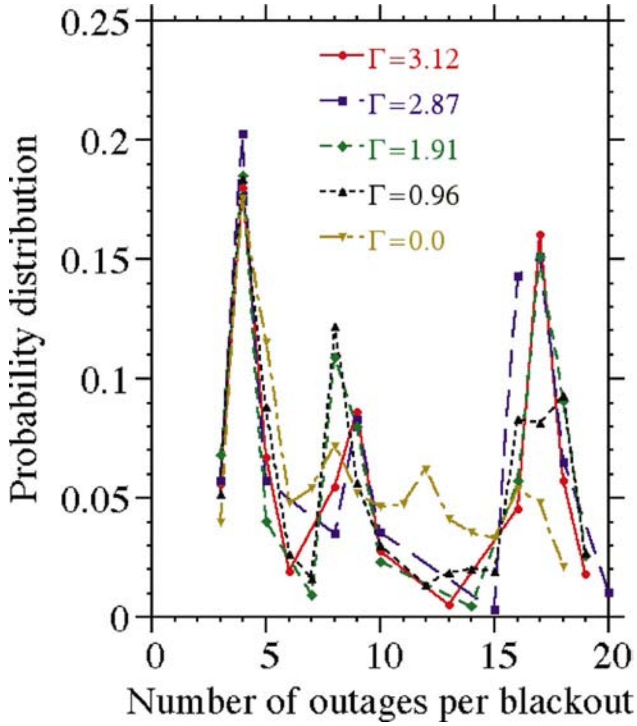

FIG. 8. (Color) PDF of the number of outages per blackout for the 94-node tree network for different values of Γ.

of the whole grid). We can apply this result to the U.S. grid, taking into account that the average frequency of blackouts is one every 13 days. In the low-Γ regime, the ratio is about 0.001; this would imply that a large scale blackout is likely every 35 years. In the high-Γ regime, the ratio goes up to 0.007; this implies a frequency of one large-scale blackout every 5 years.

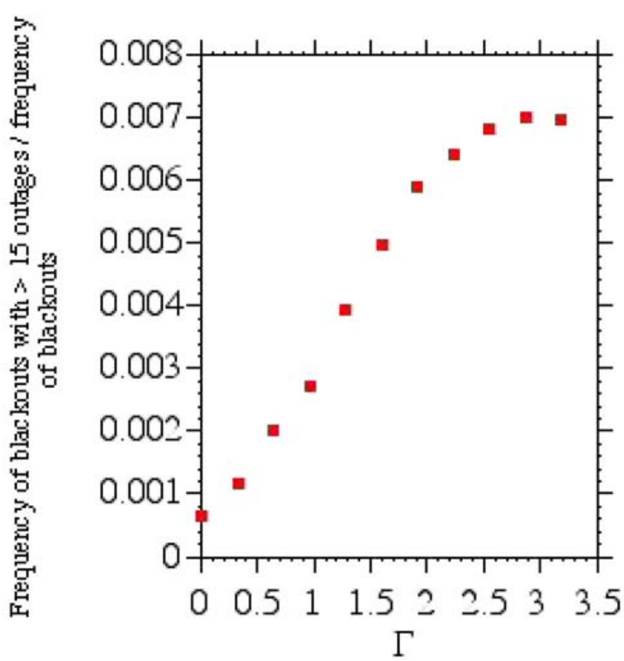

FIG. 9. (Color online) Ratio of the frequency of blackouts with more than 15 outages to the frequency of blackouts for the 94-node tree network as a function of Γ.

V. CONCLUSIONS

The simple mechanisms introduced into the power transmission model and representing the economical and engineering responses to increasing power demands are sufficient to introduce a complex behavior in the power system. The results of the complex dynamics, time correlations, and PDFs of blackout sizes are consistent with the available data on blackouts of the North American electrical grid.

This model suggests that the real cause of the blackouts in the electric power system should not be identified just with the immediate random events that trigger them; instead, the real underlying cause is at a deeper level in the long-term forces that drive the evolution of the power system.

An important parameter in the system, Γ, is the ratio of the generator margin capability to the maximum daily fluctuation of the loads. This is a surrogate for the systems ability to absorb fluctuations. We do not yet have an economic model for the time evolution of Γ which would be the next level of self-consistent evolution for the system. This parameter allows us to classify the dynamics of the model into two regimes. At low Γ, blackouts and brownouts are frequent, and a typical blackout is characterized by very few line outages. For $\Gamma > 1$, blackouts are less frequent, but large cascading events involving many line outages are possible.

The dynamical behavior of this model has important implications for power system planning and operation and for the mitigation of blackout risk. The present model has some of the characteristic properties of a SOC system, although one cannot unequivocally prove that is strictly the case. The success of mitigation efforts in complex systems is strongly influenced by the dynamics of the system. One can understand the complex dynamics as including opposing forces that drive the system to a "dynamic equilibrium" near criticality in which disruptions of all sizes occur. Power tails are a characteristic feature of this dynamic equilibrium. Unless the mitigation efforts alter the self-organizing dynamical forces driving the system, the system may be pushed toward criticality. To alter those forces with mitigation efforts may be quite difficult because the forces are an intrinsic part of our society and therefore the power system. Therefore, we expect that feasible mitigation efforts can move the system to a new dynamic equilibrium which will remain near criticality and preserve the power tails.[5] Thus, while the absolute frequency of disruptions of all sizes may be reduced, the underlying forces can still cause the relative frequency of large blackouts to small blackouts to remain the same.

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APPENDIX

The blackout model is based on the standard dc power flow equation,

$$F = AP, \tag{A1}$$

where F is a vector whose N_L components are the power flows through the lines, F_{ij}, P is a vector whose $N_N - 1$ components are the power of each node, P_i, with the exception of the reference generator, P_0, and A is a constant matrix. The reference generator power is not included in the vector P to avoid singularity of A as a consequence of the overall power balance.

The input power demands are either specified deterministically or as an average value plus some random fluctuation around the average value. The random fluctuation is applied to either each load or to ''regional'' groups of load nodes.

The generator power dispatch is solved using standard LP methods. Using the input power demand, we solve the power flow equations, Eq. (A1), with the condition of minimizing the following cost function:

$$\text{Cost} = \sum_{i \in G} P_i(t) - W \sum_{j \in L} P_j(t). \tag{A2}$$

We assume that all generators run at the same cost and that all loads have the same priority to be served. However, we set up a high price for load shed by setting W at 100. This minimization is done with the following constraints:

(1) Generator power $0 \leq P_i \leq P_i^{\max}$ $i \in G$,
(2) Load power $P_j \leq 0$ $j \in L$,
(3) Power flows $|F_{ij}| \leq F_{ij}^{\max}$,
(4) Power balance $\Sigma_{i \in G \cup L} P_i = 0$.

This linear programming problem is numerically solved by using the simplex method as implemented in Ref. 25. The assumption of uniform cost and load priority can of course be relaxed, but changes to the underlying dynamics are not likely from this.

In solving the power dispatch problem for low-load power demands, the initial conditions are chosen in such a way that a feasible solution of the linear programming problem exists. That is, the initial conditions yield a solution without line overloads and without power shed. Increases in the average load powers and random load fluctuations can cause a solution of the linear programming with line overloads or requiring load power to be shed. At this point, a cascading event may be triggered.

A cascading overload may start if one or more lines are overloaded in the solution of the linear programming problem. We consider a line to be overloaded if the power flow through it is within 1% of $F_{ij}^{\max}$. At this point, we assume that there is a probability p_1 that an overloaded line will cause a line outage. If an overloaded line experiences an outage, we reduce its corresponding $F_{ij}^{\max}$ by a large amount (making it effectively zero) to simulate the outage, and calculate a new solution. This process can require multiple iterations and continues until a solution is found with no more outages.

This fast dynamics model does not attempt to capture the intricate details of particular blackouts, which may have a large variety of complicated interacting processes also involving, for example, protection systems, and dynamics and human factors. However, the fast dynamics model does represent cascading overloads and outages that are consistent with some basic network and operational constraints.

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