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The Effect of the Transmission Grid on Market Power

Steven Stoft
SEStoft@lbl.gov

Environmental Energy Technologies Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

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Executive Summary

If competition could extend without hindrance through the entire extent of an electrically connected power grid, the U.S. would have just two electricity markets, each with a uniform price. These markets would be competitive indeed. Unfortunately, losses and congestion present barriers to competition and thereby provide the likelihood of significantly increased market power. This paper begins the analysis of congestion as it affects the physical extent of markets and thereby affects the degree of market power. This is new territory; very little has previously been written in this area.

Although the theoretical developments reported here rely on complex economic analysis, and although the market behaviors described are extremely subtle, several broad generalizations relevant to policy analysis can be made. From these generalizations one major policy conclusion can be drawn: *In an unregulated market it will be socially beneficial to build a grid that is more robust than what is optimal in a regulated environment.*

- **Unused capacity may be needed.** For a line to support full competition it may need to have a capacity that is much greater than the flow that will take place on it under full competition.
- **Markets do not have sharp boundaries.** Even with only one line the two busses may be in different regions, the same region, or partially in each other's region.
- **Increasing capacity is more effective on a small line.** If connecting two busses with a very strong line will reduce market power, then the first MW of connecting capacity will have the most impact and each additional MW will have less.
- **A congested line will cut a market into two non-competing regions.** In each region the generators will markup according to the elasticity of the demand in only their region.
- **A generator may reduce output in order to congest a line and thereby increase its market power.** This occurs when the line is large enough to support the duopoly line flow but not large enough to stabilize the duopoly equilibrium.

1 Introduction

Although many claim to have the key to predicting market power in the coming deregulated electricity market, their predictions span the spectrum from almost no market power to intense market power. The truth of the matter is that the task is actually quite difficult, and two or three crucial elements of market-power theory have yet to be developed. This paper explains some cutting-edge work on market boundaries conducted by the author and two colleagues at the University of California Energy Institute (UCEI) over the last year and a half.

As in any industry, market concentration and demand elasticity play key roles as determinants of market power. Unfortunately market concentration depends crucially on market boundaries (however fuzzy) and these are difficult to define in a power grid. If physical connection were all that mattered, then the U.S. would contain only two markets: the West; and the East plus Texas. In this case market concentration in each area would be quite low. But we know that line losses and congestion can limit trade within these broad areas. This paper concerns the role of congestion in limiting competition.

Unfortunately, the problem of computing market power in a congested network turns out to be even more difficult that one would have any reason to guess. We will soon see that solving for the markup in the simplest possible model describing two markets connected by a weak line can require a sophisticated computer algorithm and tens of millions of arithmetic operations. (For small enough lines, no algebraic solution is currently known, and we do not know where the cut-off is for “small enough.) In spite of this complexity, a rough qualitative understanding of most results in this area is possible. This paper provides such an explanation and relies heavily on references to the UCEI work for support.

Because of the difficulty of this theory, only simple models have been solved. Nonetheless several interesting general conclusions can be drawn, and some of these point in interesting policy directions. First we will show that a line can be quite useful for mitigating market power even though no power flows on that line. Thus the old requirement of “used and useful” must be rejected in some circumstances. In general, this result says that for the purpose of market-power mitigation, line size may need to be much larger than the observed flow on the line.

A second result is that a line can have a surprisingly large impact for its size. For instance building a 10-MW line could induce two firms that previously had local monopolies to increase their total output by 60 MW. This is not a general result, but there may be many cases where over-building would pay off.

The first two results are based on a symmetric example, which is simple to analyze but which may exhibit special properties because of its symmetry. The final sections of the paper extend the work to asymmetric competition, where we find some new phenomena. In particular, it appears that Path 15, between Northern and Southern California, might actually be subject to the following asymmetric gaming behavior. If PG&E were to assume this path were too large to congest, it would produce in such a way that the actual line would not be congested. But because the line is small enough, PG&E could find it profitable to restrict output in the north thereby congesting the line.

1.1 Assumptions About the Way the Energy Market Works

There is still much controversy over the exact form that the restructured energy market should take. Possibilities include nodal pricing, zonal pricing, bilateral trading with no public market price, or any of these methods coupled with various types of transmission rights. What is a researcher to do? The situation would be hopeless except for one often overlooked fact: Any of these systems, if they work as promised (i.e., efficiently) will produce exactly the same set of nodal prices. This is because they are all free market systems in which firms are not forced to sell unless they find it profitable, and there is only one set of nodal spot prices that induces the efficient output. Now these prices may not be printed in the newspaper, but for the market to function efficiently, market participants must know them with considerable accuracy and must act accordingly.

Of course if one of these systems is adopted and fails to work well, then there will be different prices and different and inefficient patterns of supply and demand. Since we cannot possibly predict the pattern of such mispricing in advance, we will adopt the more optimistic assumption that some form of efficient market will eventually emerge or that this is at least the best approximation currently possible to what will emerge. Because all efficient market mechanisms produce the same prices and quantities, it does not matter which one we select to use in our analysis. So we will select the easiest one to use. This is the standard nodal pricing system developed by Schweppe (1988).

One further assumption is that competition between firms happens according to the Cournot model. That is, firms compete in quantities. Although this model does not capture reality perfectly, it is the most widely accepted and tractable model of competition and market power. No other specific model seems superior, and, given the difficulty of the present analysis, no other approach seems tractable.

Finally, I will ignore TCCs and other forms of transmission rights. While the market for transmission rights can interact with the energy market, especially when market power exists, these interactions introduce too much additional complexity. Future work in this area should look into the impact of a transmission-rights market.

2 The Unused, But Useful Line

As a first example, we consider two markets which we will call North and South. These are connected by a single line of maximum capacity k MW, as determined by reliability ratings. Thus the independent system operator (ISO) will not allow more than k MW to flow on the line. At each end of the line there is a generator and a competitive group of demanders. For simplicity we assume that the two firms are identical and that the two groups of demanders are identical.

If we consider this arrangement first with a line capacity of $k = 0$, we see that because the two markets are identical in every respect, they trade electricity at the same price. Now if we increase the capacity of the line to some positive value we should expect to find no flow on the line. This is because, with the price the same at the two ends, there will be no gains from trade between North and South. Thus if we build a large line between the two locations, no power will flow on that line under normal conditions.

Now let us consider in detail the case of a very large line, larger than would ever be needed. But the complete electricity market with the line is really quite different than the market without the line ($k = 0$). Without the line we have two separate monopolies, while with the line we have one duopoly. With the line, each generator can sell into the other's home territory as easily as into its own. (With a large enough line there are essentially no line losses, and we will assume they are actually zero.) Now a duopoly produces a more competitive market than a monopoly and consequently with the large line we will find a lower price and more output. Thus in spite of the line not being used, it provides a pathway for competition and lowers the price paid by all customers.

2.1 The Mystery of the Weak Line

So far we have developed a good understanding of the effect of a large line that connects two identical markets; it produces a larger and more competitive market. If the connected markets are identical, then no power will flow on this line.

What if the connecting line is very small relative to the size of the markets? What if for instance a 1-MW line is built between two markets that each trade at the 1-GW level? In this case we should expect a much smaller effect on competition, but still no power flow on the line. Again the argument against power flow is the same: since the entire market is symmetrical, there is no more reason for the power to flow north than south. But there is something very puzzling about this view which we must now explore.

We must now return to the logic of duopoly which we know leads to more output and lower prices than does monopoly. So exactly what drives this change in behavior when a strong line is added between two monopoly markets? When this happens, each firm realizes that if the other does not change its output, then it can increase its own output and sell half of the increase into each market. How do we know this would be profitable? We must start with the logic of the monopoly equilibrium. A monopolist produces just to the point where profits would be hurt as much by the price effect of additional output as they would be helped by the quantity effect. With one additional unit of supply the price will be reduced on all units sold, thus reducing profits a tiny amount on every unit. But additional profit will be made on that unit. When these two effects are equal the monopolist cannot increase profit either by selling more or by selling less. But when the large line is built and the monopolist suddenly gains access to its competitor's market the price effect is reduced. When two additional units are produced only one goes into each market, thus the price effect of two in the combined market is the same as the price effect of one in a local market. The quantity effect remains unchanged. Because the negative effect is halved, the balance between the two is upset, and with the additional territory it become profitable to increase output.

We now see that the logic of duopoly competition begins right at the monopoly output level and, as long as it is possible to sell additional output into both markets, output is pushed up until it reaches the duopoly level. Thus the forces of duopoly should apply even across a weak line. So when two monopolies find themselves in competition, even when connected by a small line, they should begin increasing output, and they should not stop until they get to the duopoly level. This logic seems quite compelling. If we imagine that the two firms pick any level of output between the low monopoly level and the high duopoly level, the line will be uncongested. In this case the logic of duopoly competition shows that either could increase profit by increasing output a small amount. Of course the amount must be small enough to leave the line uncongested, but this is always possible.

We have now seen that competition even on a weak line would seem to drive the output all the way to its duopoly level. But this seems more than a little strange. How could a 1-MW line provide such a strong check on the behavior of a 1-GW generator? After all if the North generator, G_N, decided to just ignore the line and continue acting as a monopolist, the worst that could happen is that G_S would start exporting 1 MW into North's territory. This could not damage profits nearly as much as full-fledged duopoly competition.

We have just shown two things about competition on a weak line. First, if both firms produce the same level of output, this cannot be less than the duopoly level. Second, either firm can do much better than to produce the duopoly level of output. It is at first difficult to imagine how the market can operate sensibly without violating either one of these facts.

2.2 The Strange Nature of Competition on a Weak Line

One way out of this apparent contradiction would be to find a market equilibrium near the monopoly equilibrium (this seems necessary with a very thin line), but which requires the two firms to produce at different output levels. For instance, one firm might produce a little more than the monopoly output and the other a little less. This asymmetric production could congest the line and thereby interfere with duopolistic competition. Of course, because the model is symmetric, there would have to be two such equilibria, one in which North produces more and one in which South produces more.

Unfortunately, as shown by Borenstein et al. (1996), there is no such asymmetric equilibrium. The firm that is producing less always can always profit by selling a little bit more because half would be sold into the other's market. The proof of this is not quite as easy as it sounds, but it has been carried out rigorously. This leaves us with even more of a puzzle. We seem to have ruled out all possible equilibria. We have shown that a symmetric equilibrium must gravitate to the duopoly output level, but that at this level does not constitute an equilibrium because either firm can profit by returning unilaterally to the monopoly output level. And, we have shown that no asymmetric equilibrium exists. So what is left? A real market must do something.

The solution to the puzzle was promised forty years ago by Nash. The market we have been analyzing can be considered a non-cooperative game between the two firms. For such a game, Nash proved that there is always an equilibrium of the type we are seeking, now called a Nash equilibrium. In order to prove this he had to expand the concept of equilibrium to include a type that we have so far not considered, one called a "mixed-strategy" equilibrium. The strategies we have considered so far are termed "pure" strategy equilibria because they involve selection of one and only one output level. Nash's theorem assures us that, if we expand our search to include equilibria in which the firms use mixed strategies, we can find an equilibrium.

A mixed strategy is one that chooses randomly between two or more output levels (Q). This randomness must be well defined. For instance, a mixed strategy could be: choose $Q = 6$ with a probability of 20% and choose $Q = 10$ with a probability of 80%. Because of the symmetric nature of our game, the two players will choose the same mixed strategies. Let us consider what this would look like if the above-mentioned strategy were employed by both firms and if the line had a capacity of 1. Since the two strategies employ independent random events to determined their "realized outcomes," we can find the probabilities of the joint outcomes by simply multiplying the appropriate probabilities. These outcomes are shown in Table 1.

Table 1. Realized Outcomes From Two Mixed Strategies

Probability	North's *Q*	South's *Q*	Flow from North to South	Congestion?
4%	6	6	0	No
64%	10	10	0	No
16%	6	10	–1	Yes
16%	10	6	1	Yes

As can be seen, the line is congested a total of 32% of the time and equally in each direction. This gives the general character of the equilibrium we are looking for. Of course there is no guarantee that the firms will randomize over only two output levels. In fact extensive calculations show that with a linear demand function, the number of output levels in the mix increases as the line size diminishes, while with a constant-elasticity demand function, the firms always mix over an infinite number of output levels (e.g., all levels between $Q = 6$ and $Q = 7$).

There is one property of a mixed-strategy Nash equilibrium which is crucial to understanding its random nature. No matter which realized output level the firm chooses (6 or 10 in this example) the firm will, on average, make the same profit. We must say "on average" because profit also depends on what the opposing firm does. So no matter what output level North chooses, its profit will depend on South's choice. Thus if North plays $Q = 6$, it will have a 20% chance of having South play $Q = 6$, which would be relatively good and an 80% chance of having South play $Q = 10$, which would be relatively bad. These two outcomes each determine a profit level for North, and these profit levels can be averaged using their probabilities as weights to compute an expected profit for North. If the above pair of strategies were a true Nash equilibrium, North would find the same expected profit level whether it played $Q = 6$ or $Q = 10$. In fact the definition of a mixed-strategy Nash equilibrium is that each player finds all of the moves (quantities) included in its mix equally profitable (on average) and all other moves (not included in its mix) less profitable. It is the relative profitability of moves within the mixed strategy that stabilizes the equilibrium.

2.3 An Example of Competition Over a Potentially Congested Line

Now that we have discussed the theory of competition across a potentially congested line, it will be useful to study a pair of examples. Both examples will use identical firms and identical markets, as discussed above. The main difference between the examples will be that the first has a linear demand curve, while the second has a constant-elasticity demand curve.

The first example is defined by the linear demand function $P = 10 - Q$, and a constant marginal cost of one. We will solve this example for various line sizes between $k = 0$, and $k = 0.52$; at or above a line size of $k = 0.52$, the line is large enough to support maximum competition, and the pure duopoly equilibrium results. The quantity units of the demand function can be thought of as MW or GW or anything else. All that matters is that the units for line capacity, k, are considered to be the same. The units for price should be thought of as ¢/kWh.

The Pure-Strategy Equilibria

Before presenting the interesting but difficult cases of line capacities between 0 and 0.52, we will solve the two extreme cases. First the monopoly case, where $k = 0$ and the two markets are separate. The price and quantity in each market are solved for as follows:

$$\text{profit} = \pi = P \cdot Q - C = (10 - Q - 1) \cdot Q$$

$$\frac{d\pi}{dQ} = 9 - 2Q = 0 \tag{1}$$

$$Q = 4.5, \qquad P = 5.5$$

Remember that cost is zero in this example. Profit in each market is 20.25 and consumer surplus, the area under the demand curve and above price, is 10.125. Next we consider the case of a large line ($k > 0.573$) and a pure duopoly market. In this case we must find a Nash equilibrium, which requires that each firm adopt a profit-maximizing strategy assuming the other firm keeps producing the quantity it is producing. This equilibrium is solved for as follows:

$$\text{profit} = \pi = P \cdot Q_1 - C = (10 - \frac{Q_1 + Q_2}{2} - 1) \cdot Q_1$$

$$\frac{d\pi}{dQ_1} = 9 - Q_1 - 2Q = 0$$

$$Q_1 = 9 - Q_2/2 \tag{2}$$

$$\text{similarly} \quad Q_2 = 9 - Q_1/2$$

$$\text{Solving simultaneously} \quad Q_1 = Q_2 = 6, \qquad P = 4$$

This gives a profit for each firm of 18, which is a little smaller, and a consumer surplus of 18 which is, as expected, much greater than in the monopoly case.

These two cases have pure-strategy equilibria where each firm chooses a single level of output, $Q = 5$ in the monopoly case and $Q = 6$ in the duopoly case. The only thing we have not determined for the pure-strategy cases is how big a line is needed before pure duopoly competition ensues. I have stated that this critical value is $k = 0.573$, but have not discussed how that is determined. We will learn something about this as we study the mixed-strategy equilibria.

2.4 *The Mixed-Strategy Equilibria*

I will now present a sequence of mixed-strategy equilibria starting with the one produced by a line that is just slightly too small to support a pure duopoly equilibrium, that is, starting with a line of size $k = 0.572$. The following equilibria track what happens as the line gets smaller and smaller. As expected, the equilibria approach the pure monopoly equilibrium as the line size approaches zero. For completeness, I will include a graph of the solution of the pure duopoly at the beginning and the pure monopoly at the end. The following graphs were all generated by the numeric simulation of a dynamic competitive process that converges eventually to the Nash equilibrium. Only the equilibrium is displayed.

A number of remarks are in order regarding Figures 1-6. First they do illustrate a type of gradual transition from the duopoly solution ($Q = 6$) shown in Figure 1 to the monopoly solution ($Q = 4.5$) shown in Figure 6. To see this, it is important to note that Figure 5 uses an expanded scale on the horizontal axis in order to show detail, but all of the positive probability occurs within 0.3 of the monopoly solution.

Next it is important to note that all Qs having positive probability of being chosen as the realized output level have the same level of profit. This can be seen by examining the profit functions shown in each figure. Note that in the case of duopoly and monopoly, the profit function is single peaked, indicating a pure-strategy solution just as we assumed when we did the calculations shown above. Figure 2 shows two profit peaks of equal height corresponding to the two pure strategies that are “mixed” by a firm in this Nash equilibrium. Figure 3 also shows two equal profit peaks, and Figure 4 shows three peaks, although they are quite hard to see. In general, any Nash equilibrium will have profit peaks corresponding to each move (Q) that has a positive probability of being chosen.

Figure 1. Line Size, k = 1. Pure Duopoly

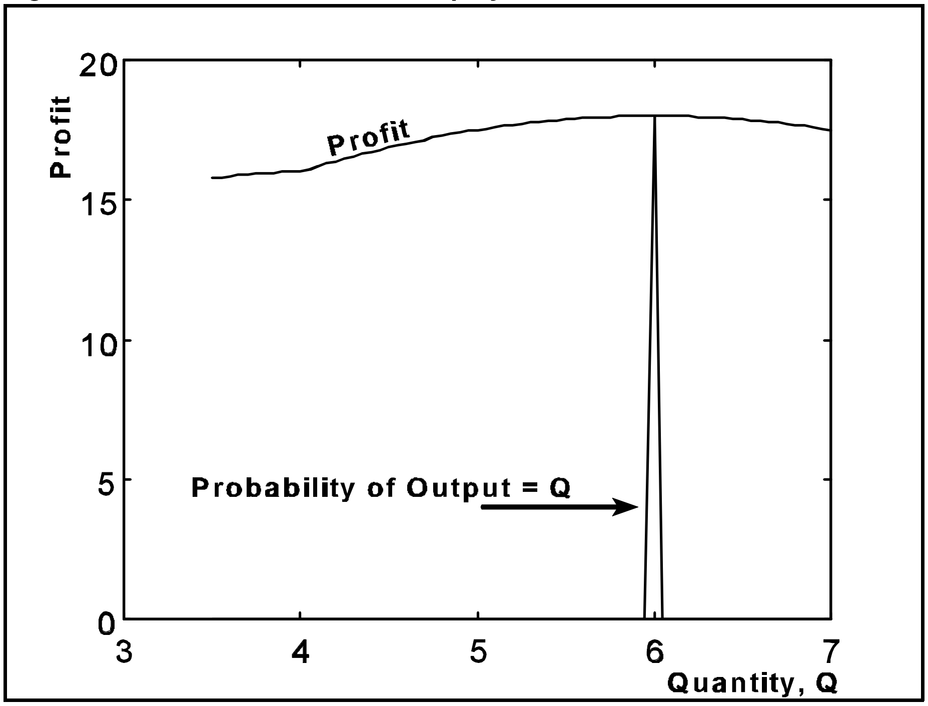


Figure 2. Line Size, k = 0.51.

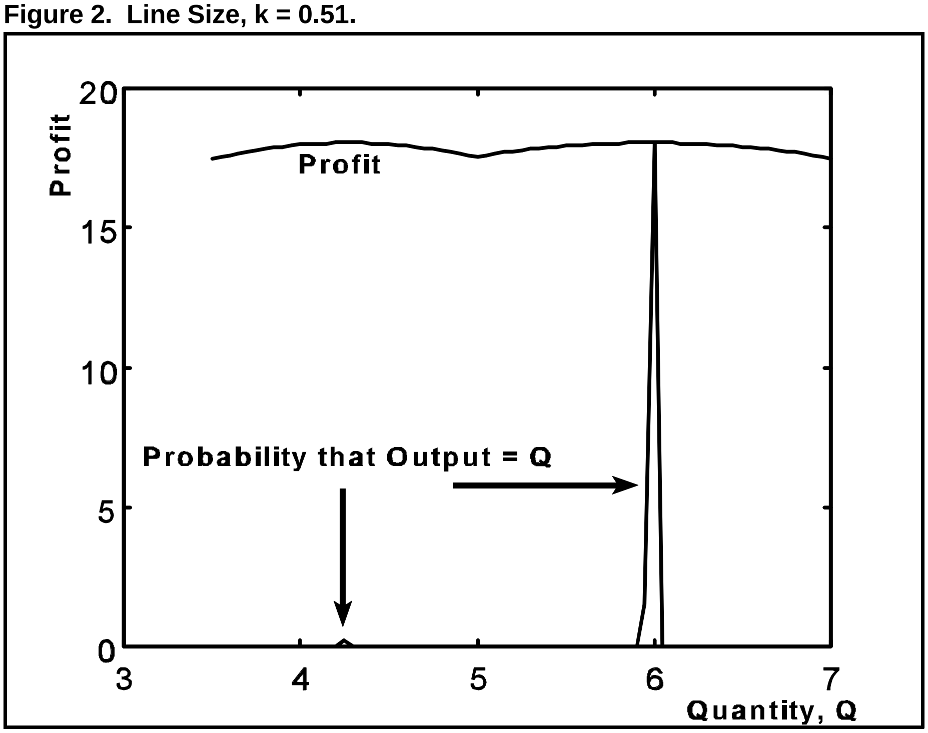


Figure 3. Line Size, k = 0.2.

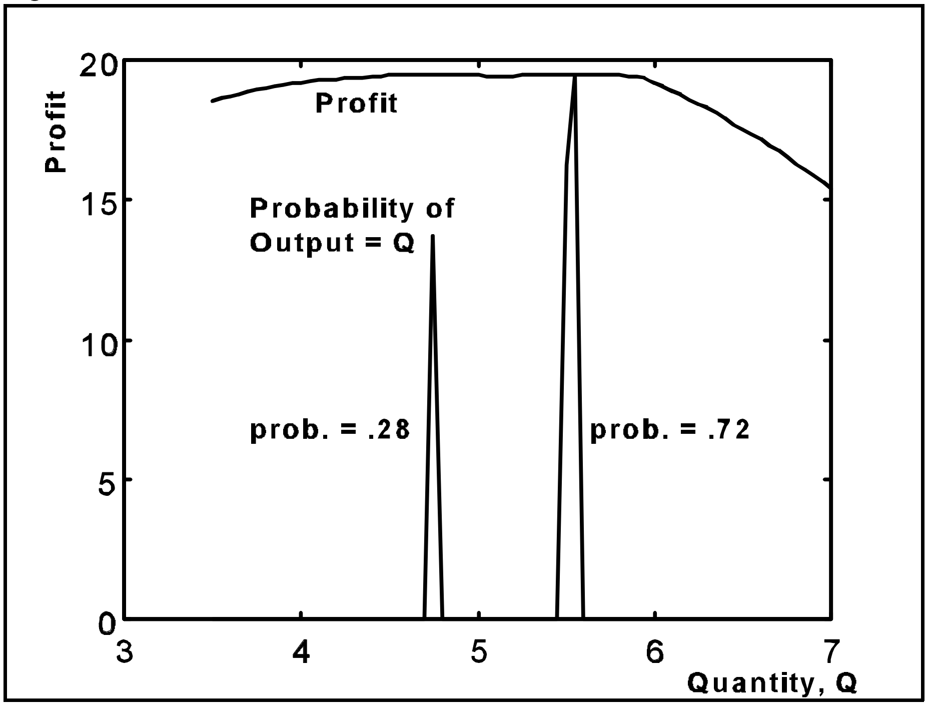


Figure 4. Line Size, k = 0.5.

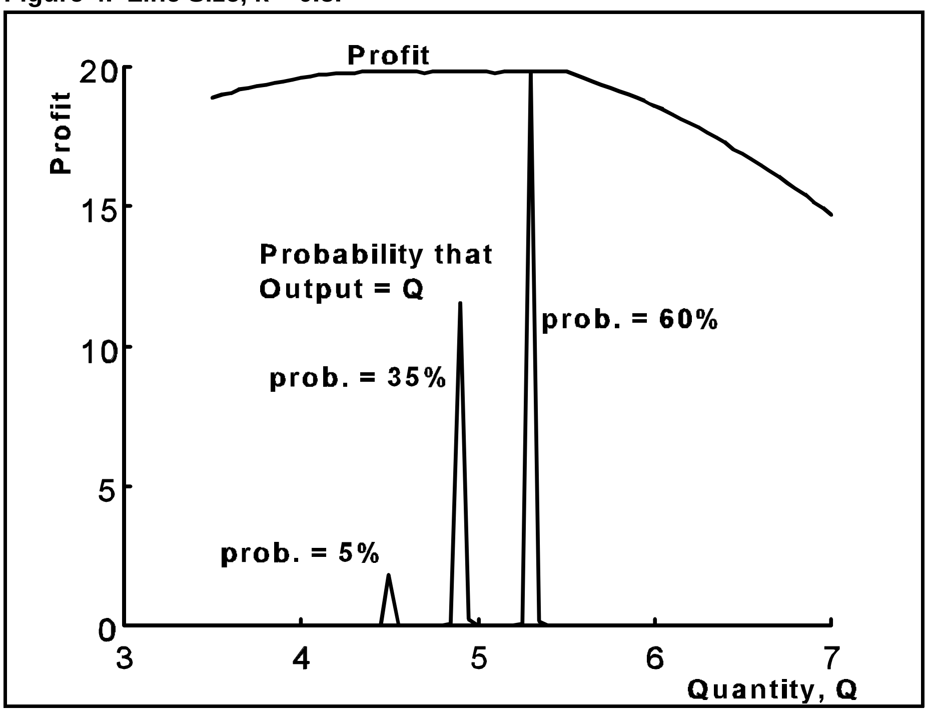


Figure 5. Line Size, k = 0.01.

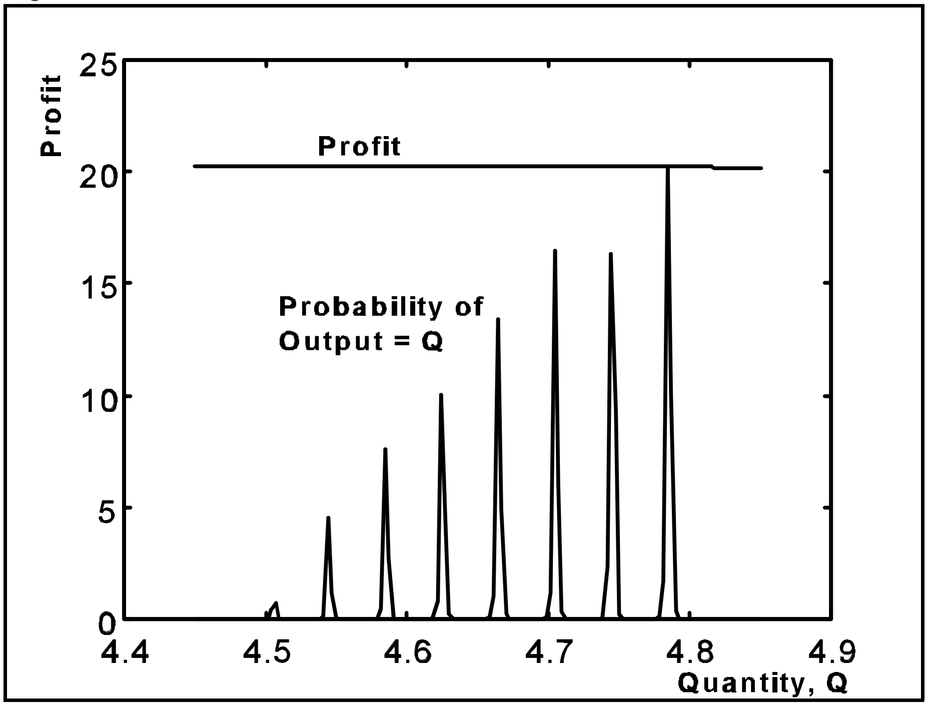
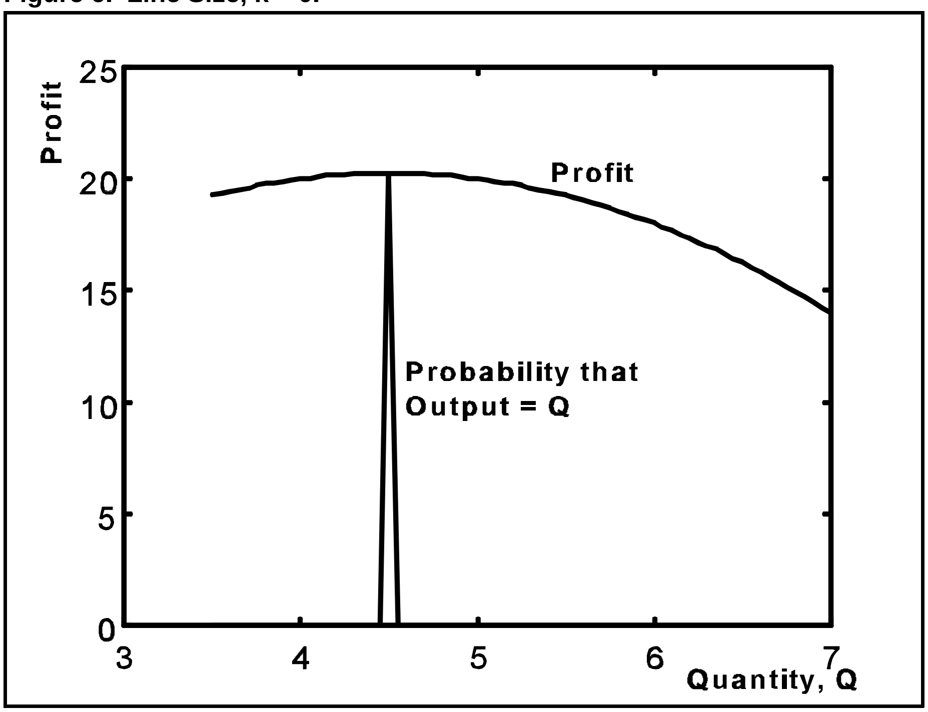
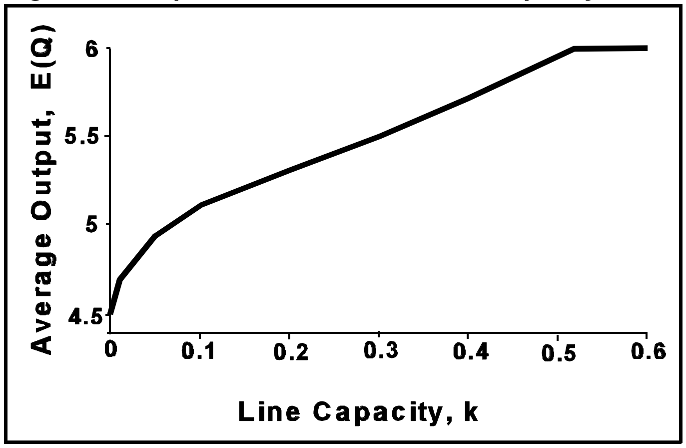


Figure 6. Line Size, k = 0.



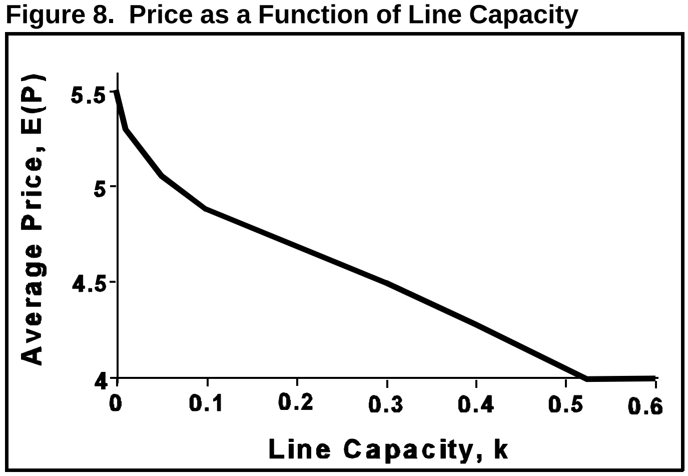
Although both price and output are uncertain in a mixed Nash equilibrium, we can easily find their average or expected value by weighting with the probabilities. Having done this we can graph how each is affected by the size of the line connecting the two markets. Expected output is graphed against line capacity in Figure 7. The most interesting aspect of this graph is the fact that output increases so rapidly with line size for small lines. In fact, when line size increases from 0 to 0.01, the output of both generators increases by 0.2. Probably the slope of this curve is infinite at $k = 0$. Thus a very small line can have a substantial impact on behavior. The two other examples studied in detail do not appear to support the conclusion that this slope approaches infinity as line size approaches zero, but they both exhibit dramatically increased sensitivity of output to line size for small line sizes. (Also note once again that, after the line size reaches 0.52, there is no more effect of increasing k.)

Figure 7. Output as a Function of Line Capacity



For completeness I have included the graph of expected price as a function of k. This exhibits the mirror image behavior because price and quantity are related by a linear demand function. Interestingly, expected profit, a graph of which is not included, is linear as nearly as can be discerned give the accuracy of the plot

Figure 8. Price as a Function of Line Capacity



This completes our analysis of the linear-demand example. A constant-elasticity example was also investigated in detail. The results were qualitatively very similar. One difference that is only of interest mathematically is that all of the mixed strategies involved continuous probability functions. In other words, these mixed strategies drew their randomly chosen output quantities from regions of output, e.g. from the region between $Q = 5$ and $Q = 5.2$, instead of from discrete and separate Q values.

2.5 What Can Be Learned From the Linear Example?

While the details of this example are largely irrelevant to policy considerations, several basic lessons can be drawn from the linear example. The first of these uses a new concept that can be illustrated with our example, that is, the concept of "full competition." By this I mean as much competition as would result if all lines had so much capacity that there was no possibility of congestion. In our example this would mean pure duopoly. In a large network with many competitors, this would mean something very close to perfect competition. Here then are the three lessons of our example.

- **Unused capacity may be needed.** For a line to support full competition, it may need to have a capacity that is much greater than the flow that will take place on it under full competition.

- **Markets do not have sharp boundaries.** Even with only one line, the two busses may be in different regions, the same region, or partially in each other's region.

- **Increasing capacity is more effective on a small line.** If connecting two busses with a very strong line will reduce market power, then the first MW of connecting capacity will have the most impact and each additional MW will have less.

3 Competition in an Asymmetric World

This section generalizes the symmetric two-bus, one-line mode considered in the previous section by allowing asymmetry in both the supply and demand functions. Although this is still far short of a general network, much of this section is suggestive of extensions that would apply to a multi-node network. Some new phenomena will be introduced by this generalization, and we will develop a reasonably simple and systematic approach for solving general asymmetric, one-line networks with one firm at each bus.

3.1 When the No-Line Monopoly Prices Are Equal

First consider two markets that when separate and thus monopolized by their respective firms, reach equilibria with the same price. Still, quantities, elasticities and markups may all differ. Such a pair of markets, if joined by a weak line, will behave similarly to the markets discussed in the last section. With a very weak line each firm will try to sell into the other's market, but by competing this way they simply lower price in each market. This competition could continue until each found it more profitable to return to their monopoly output level and just accept the small amount of imports over the weak line. We have described this process rather loosely. More technically we should say that there is, once again, no pure strategy equilibrium. No matter what pair of outputs we choose, at least one firm will find it advantageous to change its output level. As before, we know that in such a situation Nash has guaranteed that we can find a mixed-strategy equilibrium.

Also as before, when a large enough line connects the two markets, there will be a pure-strategy, completely standard, duopoly equilibrium. Unlike in the symmetric case, this equilibrium may well involve flow on the line, though once again the smallest line that induces this equilibrium generally will be uncongested by the duopoly-equilibrium flow.

3.2 When the Two No-Line Monopoly Prices Differ

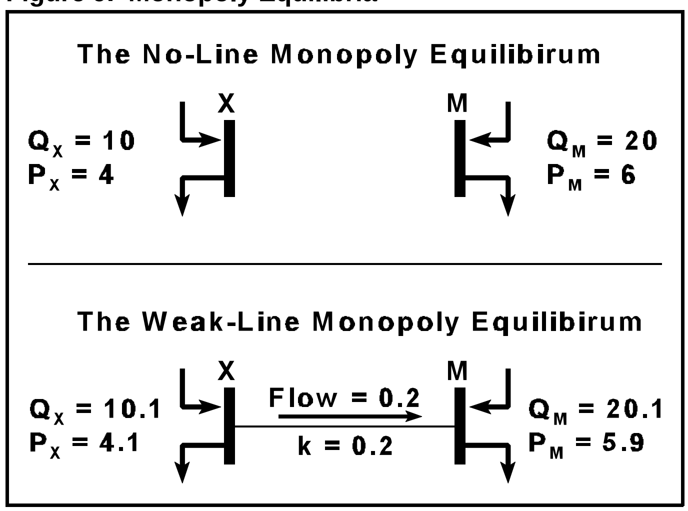
When the no-line monopoly prices differ, one bus will have the lower price. We will call that bus the "export bus" or X for short, and we will call the high-priced bus the "import bus." While this name will prove to be meaningful it should be noted that the export bus may in fact be an importer in the large-line duopoly equilibrium.

The Monopoly Equilibrium

We now introduce a new type of pure-strategy equilibrium: the two-monopoly equilibrium with imports and exports. The no-line two-monopoly equilibrium that we have discussed before is, of course, a special case of this equilibrium (with zero import and exports), and the equal price models discussed in the last subsection are probably capable of such equilibria, but it is in the context of the present models that these equilibrium come to the forefront of our analysis. If the no-line monopoly prices differ, then a small enough connecting line will definitely produce a two-monopoly equilibrium with imports and exports. We will usually just call this a monopoly equilibrium, because in our one-line model, if one firm faces a congested line the other must also, so if one acts as a monopoly so will the other. Thus there can be no one-monopoly equilibrium.

Figure 9 shows both a no-line and the corresponding weak-line monopoly equilibria for a line size of k = 0.2. The numbers do not represent an actual calculation but are plausible. Notice first that the flow is from the cheap bus to the expensive bus. This is always the case for a congested one-line network. Although the flow on the line exactly equals the line's capacity limit, the congestion is substantial in the sense that at these prices much more flow would be desired.

Figure 9. Monopoly Equilibria



At this point the reader may be wondering why we are calling the equilibrium a "monopoly" equilibrium when the market includes two firms both of which could potentially sell into the other's territory. The answer lies in the way they view demand and use it to compute their profit maximizing output. Consider for instance the exporting firm, Generator X, in the weak-line network. This firm will consider producing a little more, say 10.2, or a little less, say 10, and will choose the equilibrium output of 10.1 because that maximizes its profit. But in computing profit it must determine how output affects price and this requires using a demand curve. What demand curve will it use? Will it use the demand curve for loads at Bus X, or will it use the demand curve for the entire market, or something in between? The answer is that it uses only the demand curve for loads at Bus X. This is the same demand curve as is used by Generator X when there was no line and it was in a classical monopoly situation. Of course when the line is present and congested it must shift the Bus-X demand curve left by k units of output, but it does not use any information about demand at Bus M. Why is this? When Generator X considers an output of 10, it knows that this will cause a slight increase in the price at Bus X, but not enough to equal the price at Bus M. Consequently the line will stay congested and the flow on it will stay at exactly 0.2. This means that the 0.1 decrease in output will affect only loads at Bus X, and thus only the demand curve for Bus X will come into play when determining the new price at Bus X. It is also important to remember that all of Generator X's output is priced at P_X, the price at Node X. (Recall our use of the nodal pricing modal, justified in the introduction.)

Line Size and the Three Types of Equilibria

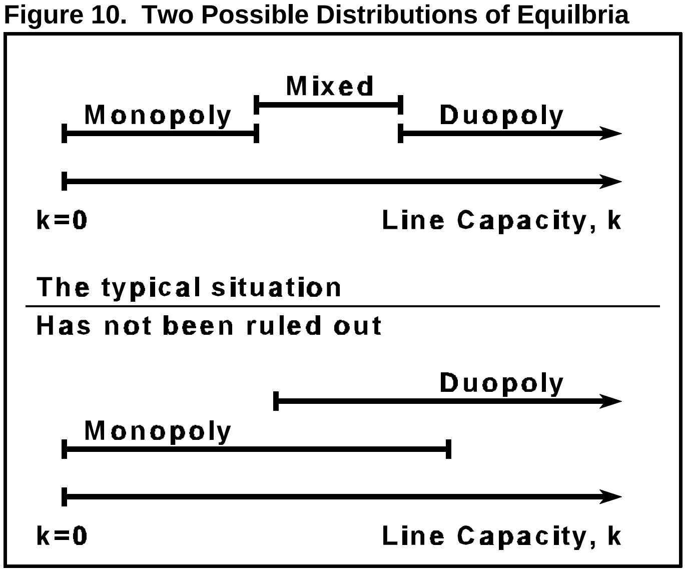
We have now characterized three distinct types of equilibria that can occur in a one-line network: (1) the monopoly equilibrium, (2) the mixed-strategy equilibrium, and (3) the duopoly equilibrium. If the zero-line monopoly equilibrium produces two distinct prices then we will find these three equilibria related to line size as shown in Table 2.

Table 2. How Equilibrium Type Depends on Line Capacity

Line Size	Equilibrium
Small Enough	Monopoly
Medium	Mixed-Strategy or ?
Large Enough	Duopoly

As can be seen there is some ambiguity in this relationship. For a small enough line we know the equilibrium will congest the line and create two monopolies, while for a large enough line we know the two markets are effectively merged into one. But, for intermediate size lines, we are not sure of the outcome. For every example we have examined there is a sizable range of line sizes for which the outcome is a mixed-strategy equilibrium but we have not proved this to be the case in general. The alternative is that there could be both a monopoly and a duopoly equilibrium for some markets and some line sizes. This concept is illustrated again in Figure 10.

Figure 10. Two Possible Distributions of Equilbria



In general, the closer together the prices in the two markets are when there is no line, the more perfunctory the monopoly region is, and the larger the region of mixed strategies is.

Having now classified the various equilibria and learned something about the circumstances under which they are likely to arise, we must now turn to the more difficult question of how we should approach determining the exact qualitative and quantitative nature of an equilibrium in any particular circumstance. We turn now to that problem.

3.3 Calculating Equilibria in a One-Line Network

As we have mentioned the equilibria under consideration are Nash equilibria in a Cournot competition. Also as has been mentioned, a Nash equilibrium is one in which each player finds herself satisfied with her strategy, given her opponent's strategy as fixed. The difficulty with the game under consideration is that each player has an infinite number of strategies corresponding to all the levels of possible output, Q. This complexity is normally dealt with by plotting "best response functions" for each player and finding where they intersect. Although this approach is workable, the present game can be analyzed by considering only three discrete strategies for each player. Of course this approach only allows us to find any pure-strategy equilibria or to show that the equilibrium is of the mixed strategy variety. It does not allow us to compute a mixed-strategy equilibrium. But, the response-function approach does no better.

The three pure strategies we will need are: the duopoly strategy, Q_d; the monopoly strategy, Q_m; and the best response to the other generator's monopoly, Q_b. Let us consider these one at a time for some particular line size, k.

Finding the Duopoly Strategies

Each generator has its own duopoly strategy and these can be found by simply combining the two markets and solving the standard Cournot duopoly problem. We will assume that this produces a unique solution, and thus a unique pair, $(Q_d^X,\ Q_d^M)$. It also produces a market-wide price, P_d.

Finding the Monopoly Strategies

Finding the monopoly strategies is a two-step process: first find the exporting market, then find the two-monopoly equilibria. As mentioned at the beginning of this section, the bus with the lower zero-line-capacity monopoly price is considered the exporting bus and the other bus the importing bus. So the first step requires solving the $k = 0$ model to find the two monopoly prices. The second step is to solve for a new monopoly price at each bus, this time assuming that demand has been increased by k (at every price) at the exporting bus and that demand has been decreased by k at the importing bus. Again, these individual problems are completely standard monopoly problems.

Finding the Best-Response (to Monopoly) Strategies

Finding the "best-response" strategies is only slightly more difficult. First, note that for Generator X we are looking for its best response to Q_m^M, while for Generator M we are looking for its best response to Q_m^X. Second, note that any response will either congest the line or not, so we have two groups of strategies to consider. But we have already searched within the group of strategies that congest the line because this is simply the group of monopoly strategies. Thus if the best response by X to Q_m^M is a strategy that congests the line it must be Q_m^X. But, as will be seen shortly, the best-response strategy is only of interest if it is different than the monopoly strategy, so we can simple ignore responses that congest the line. To be precise, then, we are looking for the best response to the opponent's monopoly strategy that does not cause congestion when paired with the opponent's strategy.

To find the best response by Generator X that does not congest the line, we take Q_m^M as given and then treat Generator X as a monopolist in the combined market. Thus Generator X is viewed as supplying the residual two-market demand, given that Q_m^M is already being supplied. This calculation will yield some output Q. We then check whether or not Q supplied by X and Q_m^M supplied by M will congest the line. If so, then Q is not the output we are looking for. In this case the smallest Q that leaves the line uncongested will generally be the Q we seek, but in that case (as we will soon see) it will turn out to be of no interest. However if we check Q and find that it does not cause congestion, then we have found an interesting "best-response" strategy.

To summarize our procedure: take the opponent's monopoly output as given and treat the responding generator as a monopolist in the combined market. Solve for the optimal Q. Test this Q to see if it leaves the line uncongested given the opponent's output. If so, this is Q_b^X; if not, then Q_b^X will be of no interest.

Setting up the Payoff Table

Having discovered the three crucial strategies of each generator, we can construct the table that defines the Cournot game that describes our market. This is shown in Table 3.

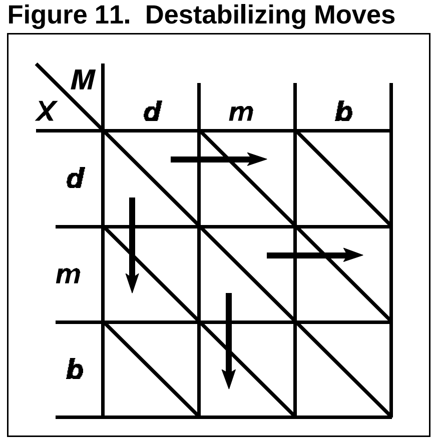
Table 3. Payoff Table for One-Line Market

X \ M	Q_d^M	Q_m^M	Q_b^M
Q_d^X	π_{dd}^X / π_{dd}^M	π_{dm}^M	
Q_m^X	π_{md}^X	π_{mm}^X / π_{mm}^M	π_{mb}^M
Q_b^X		π_{bm}^X	

The profit values used as payoffs in this game are computed using the quantity strategies shown, the demand functions at the two busses, and the line constraint.

Table 3 has many blank cells indicating profit levels that are not needed in the determination of the Nash equilibria. This will become clear as we examine the use of this payoff table. The first thing to note is that only two Nash equilibria are possible: $(Q_d^X,\ Q_d^M)$ and $(Q_m^X,\ Q_m^M)$. This is already a simplification because one might think that there were two possible monopoly (congested-line) equilibria. But in fact it can be shown that if the line is congested, it will be congested from the export bus to the import bus. (Recall that the definition of the export bus was that its price was the lower price when the two $k = 0$ monopoly equilibria were computed.) It is this fact that justifies labeling one bus the export bus without regard to the line size and before computing the actual equilibrium. (As mentioned before, power may not flow from X to M in the duopoly equilibrium.)

Figure 11. Destabilizing Moves



Because there are only these two possible equilibria, we need only check for four possible destabilizing moves and this involves only the four other profit levels shown. The possible destabilizing transitions are shown in Figure 11.

3.4 *An Example*

We will now work an example illustrating the use of the payoff table for determining the equilibrium of an asymmetric market with two busses. The specification of the example is given in Table 4.

Table 4. Specification of Asymmetric Example

	Export Market	**Import Market**
Demand Function	$Q = 10 - P$	$Q = 20 - P$
Constant Marginal Cost	3	3
Line Capacity	$k = 1$	

Our first task is to find the duopoly equilibrium, which is done by summing the two demand functions, maximizing X's profit while holding Q^M fixed and then setting $Q^M = Q^X$. This yields $Q = 8$ and $P = 7$, and a profit for each firm of $8{\cdot}7 - 8{\cdot}3 = 32$. Because the duopoly price must hold throughout the market, demand would be 3 at Bus X and would be 13 at Bus M. But this implies a flow of 5 from Bus X to Bus M, which violates the line's capacity limit. This rules out the duopoly equilibrium as a possibility. Because of this, we omit these profit levels from our payoff table.

Our second task is to find the monopoly equilibrium with the line congested from X to M. This requires adding 1 to the demand quantity at Bus X and subtracting 1 from the demand quantity at Bus M and then working the usual monopoly problem. The result is $Q^X = 4$, $P^X = 7$, $\pi^X = 16$ for the export bus and $Q^M = 8$, $P^M = 11$, $\pi^M = 64$ for the import bus. We can now fill in the first four entries of our payoff table:

M X	$Q_d = 8$	$Q_m = 8$	
$Q_d = 8$	--- ---		
$Q_m = 4$		64 16	

This table tells us what profit each firm would make if there were a duopoly or a monopoly equilibrium, but it tells nothing about whether these potential equilibria are stable. Our next step would be to check the stability of the duopoly strategies by determining what would happen if one generator or the other tried to switch unilaterally to a more profitable strategy. As we have already argued, it would be pointless to look for other strategies that did not congest the line, because by its very definition, the duopoly strategy is better than all of these. So if either generator could find a more profitable strategy, it must be one that congests the line. The most profitable strategy that congests the line is always the monopoly strategy. So

all we would need to do would be to check the profitability of these monopoly strategies when matched with the duopoly strategies, but in this case we need do no checking at all.
Our last step is to compute the "best-response" strategies and use these to check the stability of the monopoly equilibrium. First, consider the best response of Generator M to an output quantity of 4 by Generator X. Since we have already considered all strategies that congest the line when we computed the monopoly strategy, we now need to concern ourselves only with an uncongested network. The combined demand function is given by $Q = 30 - 4 - 2 \cdot P$. We use this to compute profit as a function of Q and then find that the optimal $Q_b = 10$, which implies a P of 8 and a profit of 50. A similar calculation for the best response of Generator X to an output quantity of 8 by Generator M finds that the optimal $Q_b = 8$, which implies a P of 7 and a profit of 32. We now continue the process of filling in the payoff table:

X \ M	$Q_d = 8$	$Q_m = 8$	$Q_b = 10$
$Q_d = 8$	--- ---		
$Q_m = 4$		64 16	[50]
$Q_b = 8$		[32]	

The payoff table now indicates that Generator M would not prefer its best (uncongested) response to its monopoly response, so the monopoly equilibrium is stable with respect to Generator M. However Generator X could double its profits by switching to its best-response strategy of $Q = 8$. But we have overlooked a crucial test and that is the test of feasibility. Could these "best responses" be carried out without violating the line constraint. It is easy to check that they could not, so we did not even need to check their profit levels; they are simply infeasible. (The profit levels were placed in the table only to illustrate the procedure that would be carried out had the best-response strategies been feasible.) The net result is that we have discovered that this market has a single equilibrium and that it is the monopoly equilibrium with the line congested from X to M.

3.5 A California Example

This theory was originally developed to investigate the case of a congested path between Northern and Southern California. In particular, there is a transmission path known as Path 15 that connects the southern part of PG&E's territory and SCE's territory with all of Northern California (PG&E). This path is congested in each direction at various times of the year and day.

Calculations by Borenstein and Bushnell (1997) which are reported in Borenstein et al. (1997) show that a particular type of monopoly equilibrium may occur on this path. Although we have not demonstrated it in this paper, it is possible for a monopoly equilibrium to occur on a line that is large enough to carry the flow that would occur with full duopoly competition. This is not too surprising when we consider that in a symmetric market, no flow occurs in the duopoly equilibrium. In any case, Borenstein and Bushnell have computed that during certain times in September through December, PG&E might profitably cause congestion from south to north on Path 15 by reducing its output.

So far, in this California example, conditions have not been found for a mixed strategy equilibrium, but that may well be because of particular assumptions made by Borenstein and Bushnell concerning the ownership of transmission rights. Nonetheless the monopoly equilibrium that they have found can only be discovered by using the type of game theoretic approach described above. For instance if one computes the equilibrium under the assumption of no transmission constraints and then check to see if any constraints are violated, none will be found to be violated by the computed dispatch. Previous to this work, that would have been accepted as evidence that no constraints would bind and that full competition would take place. Indeed, in a regulated market the grid would be found to be perfectly adequate. However, when the possibility of gaming is considered, it appears that one player could deliberately cause congestion and thereby a separation of the northern and southern markets in order to exercise more market power in its own market. Inadvertently this also allows the exercise of more market power by its competitor.

3.6 Conclusions for the Asymmetric World

The primary extension of our theory necessitated by the introduction of asymmetry is the monopoly equilibrium. In this equilibrium, the line is congested and each generator acts as a standard monopolist but with its demand curve shifted left or right according to whether the flow is in or out of its territory. Thus the congested flow acts as a market barrier and allows the firms in the separated markets to markup according to the elasticities of their own markets. Because these are smaller than the elasticity of the combined market, they are able to exercise more market power, as monopolists do.

Generally, monopoly equilibria occur with smaller line capacities and duopoly equilibria with large line capacities. As with the symmetric case, there are still mixed-strategy equilibria and these occur for line capacities of intermediate size. Our list of lessons can now be expanded by two:

- **A congested line will cut a market into two non-competing regions.** In each region, the generators will markup according to the elasticity of the demand in only their region.

- **A generator may reduce output in order to congest a line and thereby increase its market power.** This occurs when the line is large enough to support the duopoly line flow but not large enough to stabilize the duopoly equilibrium.

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