Consortium for Electric Reliability Technology Solutions

Scenarios for Distributed Technology Applications with Steady State and Dynamic Models of Loads and Micro-Sources

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Executive Summary

This report defines two distributed energy application scenarios with the necessary models for micro-turbines, fuel cells, inverters and induction machines. The two scenarios described in Section 1 are distribution support and a sensitive load. Each of the scenario descriptions begins with a list of objectives and constraints. There are essentially two components that are required in developing the scenarios of interest; the use of appropriate models and associated parameters for studying phenomenon of interest, and the criteria and methods for evaluation of performance.

Scenarios

The basic assumption of the distribution system support scenario is that DER can be used to help utilities solve performance problems on the distribution system. This scenario requires that the distributed resources operate in parallel with the utility at all times. Issues that this scenario address include; distribution system upgrade deferrals through the use of reduced equipment loading, such as, peak shaving or other load factor improvements; loss reduction; local power quality and reliability improvements.

The sensitive load scenario is a result of modern industrial equipment. These facilities depend on sensitive electronic equipment that can be shut down suddenly by power system disturbances. A large number of these disturbances on the power system are a result of line faults, which can cause momentary voltage sags. This results in equipment malfunctioning and high restart cost. Small distributed resources can increase reliability and power quality by allowing them to be placed near the load. This provides for a stiffer voltage at the load and uninterruptible power supply functions during loss of grid power. The power electronics interfaces found on most small DRs, can also control voltage dips and unbalances. Currently, systems for controlling volt disturbances use a voltage sources inverter which injects reactive power into the system to achieve voltage correction. These systems are effective in protecting against single phase voltage drops (or swells) due to distant faults or unbalanced loads. These systems are costly, complex and are needed only during voltage events. Alternatives to these systems are micro-sources with a more robust inverter to protection against single-phase voltage drops and swells.

Steady State Models

The evaluation of applications of micro-grids requires extensive use of steady state techniques. For example the placement of micro-sources to reduce losses and/or enhance voltage along with determining their ratings can be achieved using load flow methods. Small micro-sources are normally applied at 480 volts or less. At these voltage levels the distribution network is a mix of three phase and single phase loads with different power flows in each phase along with unbalance voltages between the phases. In such cases three phase loads, such as induction machine, need to be modeled in more detail that a constant PQ injection model. The issues of three phase load models during voltage unbalance is important, but is not taken in account during this phase of work due to lack of adequate commercial tools.

Section 2 looks at solutions to the steady state problem when there is balanced phase voltage with the possibility of unbalance load flows in each phase. Micro sources are assumed to be in one of two control modes either regulate real power and local voltage, or regulate real and reactive power. It is import in such studies to include PQ limits based on the rating of the micro-source and the interface inverter. For isolated application the formalism of the load flow is modified to allow for the proportional load sharing among micro-sources.

Dynamic models

Micro-source dynamics are fundamentally different from conventional central station generation technologies. For instance, fuel cells and battery storage devices have no moving parts and are linked to the system through electronic interfaces. Micro-turbines have extremely lightweight moving parts and also use electronic system interfaces. The dynamic performance of such inertia-less devices cannot be modeled simply as if they were scaled down central station units

Virtually all micro-sources require power electronics to interface with the power network and its loads. In all cases there is a D.C. voltage source, which must be converted to an ac voltage source at the required frequency, magnitude and phase angle. In most cases the conversion will be performed using a voltage source inverter. Hence the inverter and its control is a key component of the systems dynamics. However, the properties of the primary power source represent an important limitation on dynamics of micro-sources. For instance, a micro-turbine requires about 10-15 seconds for a 50% change in power output. A fuel cell requires a few seconds for a 20% change in power output, but requires a thermal recovery period of a few minutes to establish equilibrium before it can provide another step change in power output. Hence it is rather important to include such limitations in the dynamics of the power sources. To remove such limitations some form of storage is necessary at the ac or dc bus to cope with instantaneous changes in power demand. This is critical in the case of sensitive loads. In an island mode micro-grids will be incapable of meeting load requirements if storage is not included.

Induction machines models

Induction machines represent large proportion of power consumption in many manufacturing plants. It is not uncommon to have many machines operating in electrical proximity of each other in a local grid. When analyzing a micro-grid with distributed resources, it is very important to be able to model the behavior of the induction machines for both steady state and dynamic studies in order to include their non-linear behavior. Section 4 describes the steady state and slow dynamic models of the induction machine. The steady state model can be either balanced or unbalanced in line-to-line voltage. The dynamic model is suited only for slow dynamics. Both single phase, balanced voltages, and three phase general models are described.

1. Scenario Description

This subtask lays the groundwork to define target scenarios for the DER test beds. The test beds must provide sufficient technical detail to allow meaningful studies of the performance of distribution systems with DER under varying operating conditions and objectives. There are essentially two components that are required in developing the scenarios of interest: one, the use of appropriate models and associated parameters for studying phenomenon of interest, and two, the criteria and methods for evaluation of performance. In the current state-of-the-art, both the models and the performance criteria require further development. Thus, the following lays out the required models and data acquisition as well as possible analysis methodologies for the scenarios. Specific data is not provided here but, where available, references are provided.

1.1 Overview

Two scenarios are described in this section: distribution support and a sensitive load. Each of the scenario descriptions begins with a list of objectives and constraints. These objectives are used to lead to a description of the required scenario data. Areas requiring further development in modeling and data are identified.

Distribution system support scenario

The basic assumption of this scenario is that DER can be used to help utilities solve performance problems on the distribution system. This scenario requires that the distributed resources operate in parallel with the utility at all times. Issues that this scenario address include: distribution system upgrade deferrals through the use of reduced equipment loading, such as, peak shaving or other load factor improvements; loss reduction; local power quality and reliability improvements; generation reserve for conventional distribution; and the providing of ancillary services under deregulation. A number of practical concerns drive the study of this scenario. First, distribution systems constitute nearly 50% of the capital costs of energy service - contrasted with around 30% for transmission and 20% for generation, so that, infrastructure savings here may have great impact [1]. Second, more than 90% of customer outages are associated with the distribution system and thus, improved reliability from a system point-of-view is best achieved at the distribution level [2-4]. Third, many ancillary services, such as regulation and voltage support, are most effective if applied locally. The developed scenario should allow for investigation of these benefits.

The required data for the support scenario is developed in the following way. The usefulness of, or objectives for, applying DER on the system is broken down into two categories: traditional support objectives (listed in Table 1) and ancillary service objectives (listed in Table 2). The breakdown is necessary from an analysis point of view, as the ancillary service objectives require coordination with transmission system studies that are not needed for the more traditional distribution system objectives. Table 1 identifies for the traditional support objectives: the required models for studies, performance criteria, needed tools for analysis and a brief description of the system characteristics associated with such studies. Footnotes are provided, where appropriate, to give some indication of standard utility practice. Table 2 lists, as needed to provide ancillary services: the required models for studies, NERC or similar performance criteria, presently used tools for analysis and a brief description of the system characteristics associated with the service. Here, footnotes are used to clarify the provision of these services as currently defined in deregulated markets. The required models under these different objectives are then described in terms of data needs and availability in Table 3. Table 4 summarizes the data requirements for the scenario.

Sensitive load scenario

The basic assumption of this scenario is that DER can provide a generic solution to the sensitive load problem. This scenario focuses on issues associated with the quality of supply, including: UPS and custom power functions; and both satellite and islanded operation. The motivation for this scenario stems from the well-known fact that as industries incorporate more complex technologies to optimize their production they become sensitive to disturbances on the electrical power system. A good example is the advanced facilities for the production of semiconductor devices, where single events can lead to costs in the millions of dollars. More traditional industries, such as aluminum smelters and plastics production, also experience expensive outage costs, although more typically on the order of a hundred thousand dollars or less per event. Recently, financial service industries have also become more concerned with disturbances that lead to computer system shutdowns [5]. Estimates of outage costs in this industry range up to several million dollars per hour with desired availability rates of 99.9999% (several orders of magnitudes beyond the most reliable distribution systems). The developed scenario should allow for testing a variety of customers needs in power quality and reliability.

The required data for the support scenario is developed in the following way. The objectives for applying DER is broken down into categories based on specific customer needs (Table 5). Table 6 addresses various performance assessment issues.

Comment on Scenarios

The underlying assumption in the previous development for both scenarios, although to a lesser extent for the sensitive load scenario, is that much of the basic distribution infrastructure and characteristics will remain as they are today. Under this assumption, extant distribution circuit and load models will be useful for subsequent analysis. Further, it is assumed that performance criteria currently applied at the systems level will be extended into the distribution system for assessing interconnected operation; however, assessment of the operation of islands will not be so restricted. The possible interconnections and approaches to DER are obviously extremely large. It is not feasible to anticipate all the practical concerns of specific future installations. Instead, the data summaries in Tables 4 and 8 attempt to identify the components a system will need to test likely performance requirements.

1.2. Proposed Studies for Support Scenario

This section identifies studies of static and slow dynamic scenarios that would be constructive for understanding the use of DER under the support scenario. The first subsection details those studies. Section 1.2 selects from the information provided in Tables 1-4 that are needed for studies to investigate the support studies. For static studies see Sections 2.4, 2.6 and 2.8. For dynamic studies see the models in Sections 3.3, 3.6 (Figures 33 and 34) and 3.7 (Figures 45, 47, and 48).

Static and Slow Dynamic Studies

Four general studies are sketched in this section: freed capacity and loss reduction, voltage support enhancements, reliability improvement and the supply of a load following service.

Freed capacity and loss reduction. A comprehensive study of the economic benefits in terms of freed capacity and loss reductions is needed. While feeder loading conditions vary widely, the cost of freeing capacity to deliver real power on heavily loaded feeders using DER should be compared to the typical costs for substation and feeder upgrades. Similarly, the energy costs are needed to assess the value of reduced losses. For assessment, both placement and control of the DER devices must be considered. Specifically:

- Placement of units for optimal loss reduction tends to correlate with optimal placement for freed capacity only if the heavily loading period is of long duration [11]. Assessment needs to consider placement as a trade-off between loss reduction and freed capacity. More realistic investigation requires consideration of placement at less than optimal location as dictated by such practical considerations as easy access and rights-of-way.
- The fundamental control methods for these studies should include four approaches. The simplest scheme is to base load units so that the units are run at full capacity without any dispatch. A slightly more sophisticated scheme is to use an open loop time based control. Here, the units would be run at full (or otherwise specified) capacity for set hours each day. A more involved approach is to assume each unit can be dispatched economically from the substation at set time intervals, say, hourly. The most sophisticated scheme is to assume active demand side management (either through real-time price signals or direct utility control of loads) to achieve capacity savings and loss reductions.

Voltage support. A study of the economic benefits of improved voltage support is needed. The study should include both estimates of the benefits of reduced imbalances and improvements in voltage profiles. Voltage support tends to be mostly a concern on longer feeders with remote loads. In this case, appropriate placement simply requires locating a unit near the load center and so investigating different siting approaches is not particularly meaningful; however, proper control of the units will be critical to prevent large voltage fluctuations during the day. Control should consider base loaded approaches with fixed VAr settings as well as control of voltage within set tolerances.

Reliability. A comprehensive study of the reliability impact is needed. The support scenario assumes that units are run in parallel at all times so reliability assessment must focus on events, such as voltage quality and capacity shortages rather than equipment outages. The value of reliability from different classes of customers can be used to place value on reduced outage rates. Conversely, desired outage rates, or economic costs of outages, from the utility perspective using standard measures can be used. Placement and control of units for reduced outages due to capacity shortage should coincide with freed capacity studies, while for voltage quality with that of the voltage support studies. Greater reliability improvements are expected under the sensitive load scenario where islanding is allowed.

Load following. The viability of providing a load following service needs to be assessed. The market rates for load following services are published regularly for several systems, including California Performance requirements for load following have been proposed by NERC [7]. The primary concern for such services under NERC criteria are existence of communications to receive control signals and sufficiently fast response to satisfy the 10-minute criteria. Control strategies should look at centrally dispatched control signals versus decentralized signals. Decentralized control signals could be based on bilateral contracts, or hierarchical control, say, via the substation, that is still governed by the utility. To avoid criticism of near term impracticality, systems with very restricted communications should be studied as well as systems with more extensive controls and communications.

Data Requirements

This section selects from the information provided in Tables 1-6 that are needed for studies identified above.

Three phase network data. The three phase network branch data includes branch impedance and topology (phase connections). There are several published systems [e.g., 10] that provide realistic parameters. Still, mixed phase (single, two phase and three phase) connections are more realistic for the problems at hand. Mutual impedances between phases is also needed. Few of the published systems include mutual impedances. Data from utility partners would be the preferable approach but most utilities do not have accurate mutual impedance parameters. Assumptions of line geometry (distance between phases) can be used to determine parameters with a fair degree of confidence and allow three phase unbalanced calculations. Neglecting mutual impedances leads to less accurate determination of capacity usage when unbalanced flows are significant.

Static studies. Load models need to be developed. Three phase models of static load characteristics are available but many researchers use constant load (P and Q) models. While generally useful in the static studies of the transmission system, the voltage dependencies are more pronounced in the distribution system. Published data on exists for voltage dependencies [e.g., 12] and these should be incorporated. Conversely, component load models [8] can be constructed. For the purposes of these static studies, it is probably sufficient to use a simple polynomial voltage dependency relationship. In terms of load imbalances, three phase loadings with accurate descriptions of imbalances are not widely available. Reasonable assumptions on load imbalances, such as, less than 20% along the main feeder, can be used for studies with a fair degree of confidence. Extreme unbalances are usually addressed by system design (switching loads between phases), and generally not common in the United States. Load duration curves are needed assessment of placement and controls. Load patterns can vary significantly from feeder to feeder and essentially no utilities accurately track daily load patterns on a given feeder. Still, for the purposes of these static studies, assuming a few load levels with specified yearly durations is sufficient. These should include at a minimum base case, light load and heavy load periods. This is particularly important for economic studies of freed capacity that may only be relevant for the few hours of peak load each day. Further, studies of losses should be integrated across the load duration curves, rather than mere peak load studies, to reflect the actual energy costs. Various details are needed for the DER units. For the limited proposed studies, this still must include capacity and failure rates for microturbines, fuel cells, battery storage and energy storage capacitors. Similar data for renewables (wind and solar) may also be beneficial. Further, models of the voltage dependencies of units of traditional designs (without inverters) and the voltage characteristics of converters are needed.

Slow dynamics (load following) studies. In terms of loads, three phase models with accurate representations of dynamic characteristics are not widely available. Models used in the static studies may be adequate for load following studies but damping characteristics of loads could be important. For the purposes of slow dynamics, simple damping models are probably adequate. Still, induction machines exhibit frequency and voltage unbalance dependencies that might interact unfavorably with load following service. For the DER units, ramp rates (or similar time constants) for microturbines, fuel cells, battery storage and energy storage capacitors are needed

Support objective or constraint	**Required model(s) for study**	**Performance criteria**	**Analysis tools**	**System description**
Reduced distribution system losses	- Three-phase network - Load duration curves	- Losses at different load levels	- Three phase distribution load flow - Optimization routines	Long lines to remote loads
Reduced equipment stress via peak shaving, improved load factors, etc.	- Three-phase steady-state network - Load growth forecast	- Magnitude and duration of equipment overloads	- Three phase distribution load flow - Mid-term distribution load forecast - Assessment of overloading on useful remaining life	Heavily loaded systems
Voltage support and three phase balance [1]	- Three-phase network	- Maximum voltage drop - Three phase balance	- Three phase distribution load flow	Long lines to remote loads and unbalanced loads
Voltage quality [2]	- Transients	- Voltage flicker - Harmonic analysis	- Time domain simulation (EMTP)	Industrial and commercial loads on system
Reliability [3]	- Reliability models	- Reliability indices (e.g. SAIFI, MAIFI, ASIFI)	- Reliability analysis	Exposed system, for example, heavily wooded
Protection [4]	- Symmetrical components - Fault characteristics and locations	- Safety - Coordination requirements	- Short-circuit analysis - Relay coordination	Connection of units in secondary circuits with limited protection equipment (such as, fuse protection and single phase circuit)
Communication and metering [5]	- Network bandwidth - Data flows - Communication failures modes	- Depends on control methods - Reliability (MTBF)	- Simulation (Monte Carlo) models - Statistical/Markov reliability models	System with limited communication infrastructure, that is, some metering and two way communication but not complete

Table 1 — Traditional distribution support objectives

[1] Standard utility practice allows voltage deviations of ± 5% or ± 7% in emergencies. Some utilities also schedule particular voltages outside this range to take advantage of voltage dependent loads but this is less common.

[2] Standard utility practice allows up to 20% voltage dip depending on the frequency of the harmonic. There are specific limits on harmonic content. In principle, it is not allowed for any customer to negatively impact any other customer.

[3] Standard utility practice is to compute the various reliability indices (most commonly SAIFI, MAIFI and ASIFI) and ensure they satisfy limits. Historically, the acceptable levels for those outage rates are determined by agreement between regulatory commissions and utility planners. These traditional measures of reliability will probably remain useful for feeders that do not have sensitive load customers. They are not particularly meaningful measures from an individual customer viewpoint and it is not clear yet whether

public policy will be as directly involved in performance requirements as in the past. Reliability requirements typically used for large units connected to the transmission system, e.g., availability, may need to be modified to be similar to SAIFI type measures.

[4] Utility practice tends to vary regarding protection and relay coordination requirements. The requirements, and design philosophy, tend to be influenced by historical safety and legal considerations at a particular utility.

[5] There does not appear to be a clear consensus yet among utilities concerning the degree to which metering and communication systems will be implemented in the distribution system. There is very limited communication and metering on most feeders today and as a result communication system and information flow models are not well developed.

Ancillary service	**Required model(s) for study**	**NERC or RTO performance criteria**	**Analysis tools**
Voltage support [6]	- Steady-state distribution and transmission network - Static load models	- Security (reliability) criteria	- Three phase distribution load flow - Transmission network load flow - Contingency analysis - Time domain simulation (ETMSP)
Regulation and load following [7]	- Simplified steady-state network - Generator slow dynamics - Load damping and dynamics	- Continuous monitoring of area control error (ACE) - Disturbance conditions on ACE	- Time domain simulation (simple models)
Contingency reserve - spinning [8]	- Steady-state network - Generator slow dynamics	- Security criteria	- Three phase distribution load flow - Transmission network load flow - Contingency analysis
Contingency reserve - non-spinning [8]	- Start-up performance and procedures	- Security criteria - Restoration procedures	- Three phase distribution load flow - Transmission network load flow - Restoration simulation
Communication and metering requirements [9]	- Network bandwidth - Data flows - Communication failures modes	- Must be robust with respect to outages and interconnected with other facilities - Interchanges must be metered	- Simulation models - Statistical/Markov reliability models

Table 2 — Distribution support objectives as ancillary services

[6] Controlling authorities in each market will establish support requirements based on security criteria. Those criteria are too extensive too list here, see for example [6]. Each area must provide adequate support but security requirements are based on system level studies today and do not investigate the distribution system performance. In theory, competitive bidding should establish the market value for voltage support but ancillary service markets for voltage have been slow to develop in most areas. Even slower to develop has been demand side bidding so there is not today a clear market mechanism for voltage service from a consumer viewpoint. Still, demand side bidding must eventually lead to consideration of VAR factor and appropriate payments.

[7] Controlling authorities in each market will establish specific requirements based on performance criteria. Traditionally such criteria require generator units to have particular droop characteristics (typically, 5%). Further, overall system performance must provide a small yearly ACE average as well as average time for ACE to return to zero following a disturbance. These criteria will obviously be more complicated under market conditions. In some areas (e.g., California), consumers are now able to provide their own regulation services within certain constraints. There is no specific provision for these services at the distribution level. Further, the traditional ACE criteria are not directly applicable at the distribution level, as the area interchange concept is not appropriate.

[8] The types of contingency reserves listed here, spinning, non-spinning and replacement, are based on the proposed NERC Policy 10 [7] terminology for convenience and is not intended to imply any specific technical requirement (e.g., that "spinning" reserve must come from rotating machines). The markets for reserves have developed more quickly than that for voltages. Reserves are again established by system level security studies and must be coordinated among areas but those studies have not been applied at the distribution systems level. Consumers are currently able to provide their own reserves within certain constraints in some areas (e.g., California).

[9] NERC has set general guidelines for communication requirements for different ancillary services [7].

Model	Parameters required	Availability	Comment
Three-phase network	- Three phase distribution network data (line impedances, transformer reactances, shunt capacitors, etc.) - Transmission network required for some security studies	- Widely available	Phenomena are well understood and widely accepted models exist up to distribution transformers (i.e., up to transformers with secondary voltage of 480 V). Data and models are not available for these low voltage circuits. Most utilities have the higher voltage data, although there are often large numbers of errors in the databases (e.g., the phase information, if it exists, is more often in error than correct). There is also data published in the literature.
Static load models	- Constant PQ and voltage dependencies, including voltage unbalance	- Section 2	Utilities have limited data but there is representative data published in the literature.
Load damping and dynamics	- Three phase voltage and frequency dependencies	- Section 4	Load models developed on a component basis exist [e.g., 8] as well as on a system wide basis. For studies of regulation, the load at the system level is normally represented by a frequency damping coefficient that may not be adequate at the distribution level. Further, it may be quite different for the three phases.
Generator slow dynamics	- Response to slow frequency changes and power output commands, e.g., ramp rates - Electromechanical or electrochemical models	- Section 3	Manufacturers have published DER performance characteristics and some models are proposed in the literature [e.g., 9]. Still, no generally accepted models for DER units exist. To use existing analysis methods, a simple first order model may be sufficient (e.g., inertia and damping) if the time constants are on the order of several seconds.
Generator start-up performance	- Start-up times and procedures	- Models not well established	- Models needed for proper classification under ancillary services
Transients — fast dynamics	- Three phase inverter models - DER electrical characteristics	Models for newest inverters require development Section 3	- Models needed to determine if units may contribute to system instabilities (oscillations in the 0.2-2 Hz range). For most inverter models, the dynamics of the control is critical, implying fundamental frequency models only. Detailed inverter models are required only for harmonic and some specialized dynamic studies.
Reliability	- Outage statistics - Time to repair statistics	- Network outage rates widely available - DER rates still unknown	Network outage rates widely available and published data exists. Manufacturers have published data on DER reliability but limited field experience is available.
Symmetrical components	- Zero, negative and positive sequence circuits - Fault characteristics and locations - Inverter contributions to fault currents	- Widely available	Short-circuit models available up to distribution transformers. Often assumed DER does not contribute significantly to fault currents if using inverters.
Load growth forecasts	- Load growth patterns for different load types	- Widely available	Load growth patterns depend greatly on the load types, e.g., residential is quite different than commercial
Load duration curves	- Daily load patterns	- Widely available	Utilities have this data on an aggregate basis. Sufficient metering does not exist for a more detailed analysis of different parts of the distribution system.

Table 3 — Support scenario modeling issues

Data	**Data source**	**Special requirements**
Three phase steady-state network	Utility partners or published data [e.g., 10]	- Modify loads to create overloads for various equipment stress relief studies - Allow various placement, phase connection and size of units - Model network up to the distribution transformers - Radial system with both single and three phase circuits
Static load models	Utility partners or published data [e.g., 8]	- Use voltage dependent load models, including three phase unbalance (Section 4)
Load duration curves	Utility partners or published data	- Use mixture of commercial, residential and industrial loading
Load growth forecasts	Utility partners or published data	- Use load growth patterns for different load types and area development
Generator slow dynamics	Manufacturer data and CERTS research projects	- Development of models to allow analysis under NERC type criteria - Development of models to allow study of criteria for new concepts, such as droop for inertia-less units, three phase AGC, etc. (Section 3)
Load damping and slow dynamics	Utility partners or published data, e.g., [8]	- Mixture of load types based on component models to allow analysis under NERC criteria
Transmission and generation system	Standard network data and dynamic models, e.g., IEEE 14 bus.	- Transmission and generation models needed to allow studies of the validity of distribution systems providing certain ancillary services - Unnecessary to use large system but must investigate interaction for security
Generator start-up performance	Generator manufacturer data and CERTS research projects	- Development of models to allow analysis under NERC type criteria
Transients — fast dynamics	Inverter manufacturers and CERTS research projects	- New inverters under development require appropriate three phase models (Section 3) - Data needed to investigate interaction with transmission system oscillations in the 0.2-2 Hz range
Reliability models	Published outage statistics, manufacturer data and CERTS research projects	- Use of published data for network outage rates and repair times for exposed areas - Combination of manufacturer data and research projects to determine failure rates for DER
Symmetrical components	Utility partners and inverter manufacturers	- Identify possible significant contribution of DER to fault currents - Communications systems to coordinate response

Table 4 — Summary of support scenario required data

1.3 Proposed Studies for Sensitive Load Scenario

The modern industrial facility depends on sensitive electronic equipment that can be shut down suddenly by power system disturbances. A large number of these disturbances on the power system are a result of line faults which can cause momentary voltage sags. This results in equipment malfunctioning and high restart cost.

Small distributed resources (DR) can increase reliability and power quality by allowing them to be placed near the load. This provides for a stiffer voltage at the load and uninterruptible power supply functions during loss of grid power. The power electronics interfaces found on most small DRs can also control voltage dips and unbalances. Currently, systems for controlling volt disturbances use a voltage sources inverter which injects reactive power into the system to achieve voltage correction. One method is to inject shunt reactive current, the other is to inject series voltage. These systems are effective in protecting against single phase voltage drops (or swells) due to distant faults or unbalanced loads. These systems are costly, complex and are needed only during voltage events. An alternative to these systems are micro-sources with a more robust inverter to protection against single phase voltage drops and swells.

This section identifies studies of static and dynamic scenarios that would be constructive for understanding the use of DER under this scenario. This subsection details those studies. Tables 5-6 outline the data and performance requirements for the sensitive load studies. For static studies see Sections 2.4, 2.6 2.8. 4.2, 4.3 and 4.4. For dynamic studies see the models in Sections 3.3, 3.5, (Figures 35 and 36) and 3.6.

Sensitive loads

In recent years, various industries are installing precision equipment such as robots, automated machine tools and materials processing equipment to realize increased product quality and productivity. As a result, modern industrial facilities depend on sensitive electronic equipment that can be shut down suddenly by power system disturbances. Although voltage spikes, harmonics and grounding related problems may cause such problems, they can be overcome through appropriate design of robustness into the control circuits. A larger majority of the problems occur due to the fact that processes are not able to maintain precision control due to power outage that lasts a single cycle or voltage sags, which last more than two cycles. A few cycles of disturbance in voltage waveform may cause a motor to slow down and draw additional reactive power. This depresses the voltage even deeper, eventually leading to a process shut down. This results in equipment malfunctioning and high restart cost. The number of outages, voltages dips and duration is an important issue. In the manufacture of computer chips alone, losses from sags amount to $1 million to $4 million per occurrence, accordance to Central Hudson Gas & Electrical Corp. Poor power quality, particularly voltage sags are becoming increasingly unacceptable in competitive industries where product defects can mean dire economic consequences.

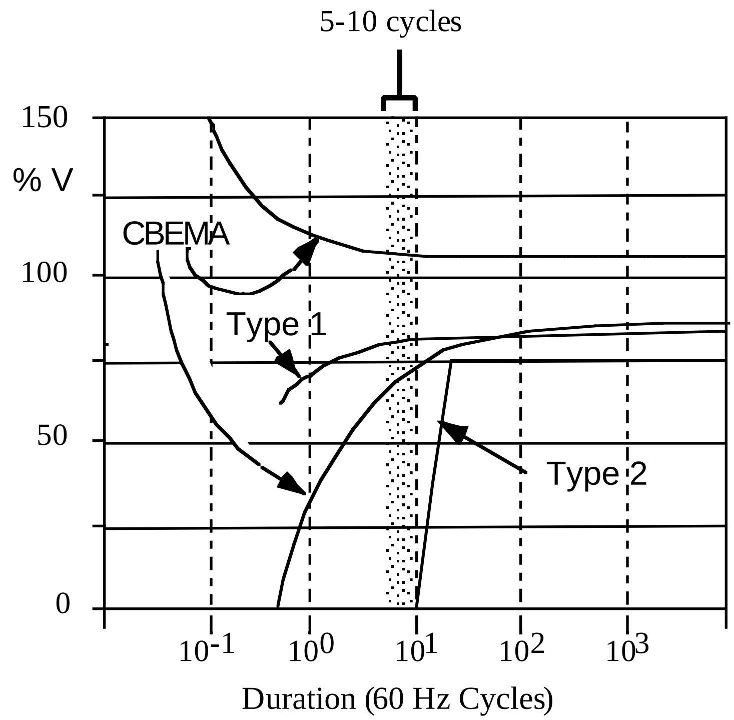


Figure 1. Voltage Sensitive Curve

Figure 1 indicates the sensitive of equipment to voltage dips as a function of duration. For example the CBMEA specifications for computers allow for dips greater than 25% for the first 10 cycles while for longer duration the dips must be less that 15%. Type-1 & 2 represent the behavior of programmable logic controllers (PLC). Type-2 has been replaced by Type-1. Note that Type-2 could withstand 100% voltage drop for 10 cycles while Type-1 will trip with a 20% drop for 1-2 cycles.

Electric utilities have traditionally responded to such needs of the customers and their demands for reliable electric supply to a high degree of satisfaction. This has been achieved through increased capital investment in generation, transmission and distribution infrastructure. Increased investments to maintain a quality infrastructure had been possible in a regulated economic scenario of guaranteed prices and returns. However, in the unfolding deregulated operating environment, electric utilities face a competitive market place, where it is increasingly difficult to commit capital expenses to meet the needs of a selected group of customers. The problem is exacerbated by the fact that increased demands in power quality by customers have coincided with reduced availability of capital for infrastructure investment. In addition, the technologies such as Dynamic Voltage Restorers (DVR) necessary for providing such ultra-reliable power supply for large and sensitive customers is just becoming available.

Faced with such a scenario, several sensitive consumers of electricity have taken to installing large Uninterruptible Power Supply (UPS) systems to meet contingent situations. This is particularly common in the information industry segment. These systems convert utility power into dc, which is stored in large battery banks, and converted back into ac to feed customer equipment. This solution is expensive because the initial cost of the equipment is high and the operating cost of equipment is also high due to losses. The overall demand for UPS and standby power supply equipment has been growing rapidly in the past years, illustrating the severity of the problem.

In order to address this problem, concepts such as Custom Power and Premium Power have been proposed, with modest success, Figure 2. Typically, these solutions do not integrate distributed power generation into developing solutions to the sensitive load problem. Recent investigations have shown that there is a high degree of match between the capabilities of distributed resources and demands of sensitive loads, and that they can be a viable and competitive solution to the problem. This work addresses the control and placement of distributed resources as a solution to the sensitive load problem. In particular, the focus is on systems of distributed resources that can switch from grid connection to island operation without causing problems for critical loads.

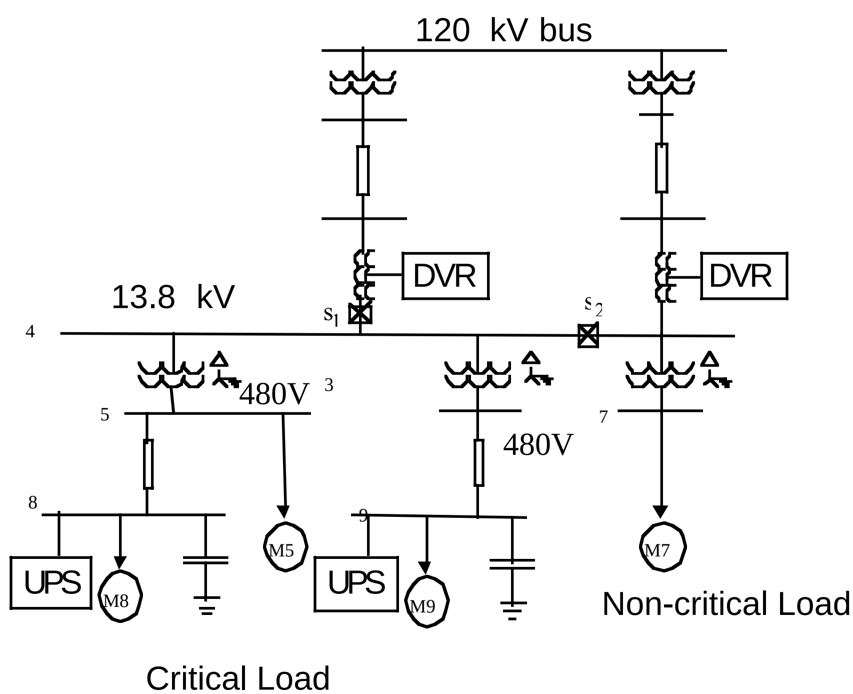


Figure 2. Premium Power Park

Test System

The test system must include a grid connection, some cable lines, distribution transformers, feeders at different voltage levels and loads. These can be office (primarily DC) loads, but must include motors, in particular some induction machines, and some synchronous machines. The machine loads will all have a three phase connection to the network, while smaller loads will be connected on a per phase basis, such as the rectified loads. The test system will also have at least three different locations for micro-sources within the local system, to fully test the islanding operation mode. In additition, each location may have more than one physical unit, depending on the amount of active power immediately needed near the set of micro-sources.

In order to have a fully defined test system, data for all components must be identified. Sources for induction and synchronous machine data can be found in the classical literature. Voltage level across the system must be defined: we expect to operate the network at three voltage levels: the grid connection point at high voltage, typically 230KV, stepped down to the feeder level at 13.8KV and then further reduced to 480V to accommodate load needs. Data is from the literature is also available for all the transformers, with particular care devoted to the three phase terminal connection. It is common practice in the industrial plants to connect the terminals at delta on the high side of the transformer, while a wye connection with the center grounded is typically used on the low side. This is done to eliminate the propagation of the zero sequence due to shorts from one side to the other of a transformer. We will use this choice of terminal connections for the transformers in the network.

Important information regarding the loads is about the sensitivity. Load sensitivity data is very hard to find in the literature because it is considered plant dependent. Each manufacture decides what importance each of the loads has. It is useful to look over all the factors that can impact the sensitivity of the loads.

Load sensitivity

Within an industrial plant, the continuity of production is only as reliable as its electric power delivery. Under this regard, the continuity of service required is dependent on the cost of that operation if it is interrupted. This is the reason why load sensitivity is strictly connected with the needs of each manufacturer. More in general, defining load sensitivity does not only include the identification of parts of the network that can be tripped off during islanding, but mostly it deals with the safety limits that the loads can be reasonably operated within. Such limits may be identified with voltage tolerance around a nominal operating value (usually few percent of the rated value). Frequency tolerances around the typical 60 Hz values may appear as a band few Hertz wide. Another index to identify the sensitivity of a load is defining the maximum amount of time that the load can be tolerated off line, or that the load can tolerate unbalances at its terminals. Ultimately, it will be the customer that will decide the sensitivity of the loads and this decision will be crucial to the choice of the unit ratings and location. In our test system we will make believable assumptions on the sensitivity of the loads based on the considerations just mentioned.

Problems of interest

The goal of the study is to gain an insight on the design of the micro source, allowing it to achieve the desired performance while minimizing the ratings of the unit. A problem that needs to be faced is the voltage control under single and three phase voltage dips due to machine load changes and faults. In particular, the induction machines need to be fully modeled with an electro-mechanical representation to analyze problems such as motor starting and load change tracking.

Another area of interest is investigated when the main grid connection fails and the system moves to an islanding mode of operation. In particular, there are the problems of islanding detection, transfer, operation and reconnection to main grid. The interaction of the harmonics generated by the inverter of the micro-source with the rest of the system needs also to be assessed to individuate possible resonance points within the system.

Steady State Studies

Steady state analysis is intended to be carried on with power flows appropriately modified to capture the behavior of the distributed resources and some key loads such as induction machines. These models are described in Sections 2 and 4. In addition, the power flow formulation may be single or three phase. The goals of steady state analysis are:

- Finding an optimal location for the micro-source, by testing different locations and basing the decision on the sensitivity of the loads across the network.
- Holding down the rating of the micro-source, while maintaining the desired performance in terms of three phase and single phase voltage dips regulation
- Analyzing the sharing strategy that the units have during island, by using the droop characteristic as a constraint added to the power flow equations
- Analyzing the harmonic impact on the rest of the system by representing the inverter with its full model and carrying on a time domain simulation of the steady state behavior.

Unlike the other steady studies performed with power flow analysis, here a time domain solver program (such as EMTP) needs to be used to simulate the inverter bridge behavior.

Dynamic Studies

Dynamic analysis is needed to obtain information on the stability and on the response of the system to typical events that may occur in the local network. These studies are meant to be carried on with a transient analysis program (EMTP) representing all the control details in the micro-source, although the unit is represented with the ideal model. In particular, dynamic analysis is aimed to obtain information on:

- Removal of single phase voltage dips due to unbalanced loads or to distant faults. Unbalances are simulated by loads with single phase connection on the network (usually rectified loads) and faults are applied reasonably far from the units, since a near short would trip the units off line or would require too much negative sequence current to achieve cancellation.

- Removal of three phase voltage dips due to motor loads. In this case machinery has to be represented with its full electro-mechanical model as to capture the dynamics subsequent to a start-up, and a mechanical load change. Besides voltage regulation, active power tracking is also important when operation in island mode is considered.
- Failure in the grid connection and island mode transfer. It is important to define the sensitivity of the loads so that it is possible to identify which loads it is possible to shed during islanding. Detection of grid connection failure is only relevant to the extent of coordinating the breakers that are responsible for shedding part of the network. Load tracking and sharing between the distributed resources present in the isolated micro-grid is studied when motor loading is changed. Reconnection to main grid after islanding is also important for assessing stability and meeting the requirements in sensitive loads.

Performance requirement	Required model(s) for study	Performance criteria	Analysis tools	System description
High quality voltage	- Transients — fast dynamics - Micro-source steady state and slow dynamics - Storage steady state and slow dynamics - Load steady state, damping and dynamics - Small steady-state network models (Sections 2-3)	- Control of voltage droop - Control of voltage flicker - Harmonic content - Local voltage stability	-Time domain simulation (EMTP type) for both fast and slow dynamics -3 phase steady state solutions tools with voltage unbalance	Need detailed information on targeted industrial manufacturing processes
High reliability	- Reliability models - Slow dynamics of sources - Load damping and dynamics -Small network models -Section 3-4	-Voltage stability during transfer from satellite to island mode. - Instantaneous power balance in island mode -Typically defined as outage events and duration - System performance criteria	- Reliability studies including capacity - Start-up performance and procedures for resynchronization - Capacity studies - Relevant security criteria	Islanding of several customers with variable load types (microgrid) Section3
Economic benefits of premium power	-Availability & reliability models - Power quality cost models - Cost of NSE, (not supplied energy)	- SAIDI & SAIFI power quality indexes - Customers requirements		Highly dependent on customers viewpoint related to power quality. Details of customers cost and process are required.
Protection & safety	- Symmetrical components - Short circuit load, generator & storage characteristics.	-Various system safety and equipment standards - micro-source and storage protection	- Short-circuit studies - Relay coordination	Single and three phase circuit descriptions with bi-directional fault current flows

Table 5 — Sensitive load performance requirements

Performance characteristics	**Criteria**	**Comments**
Control of voltage droop	Allowable variation set by customers	Standard utility practice allows up to 20% voltage dip depending on the duration, frequency and phase of the dips. MicroGrids do not need to follow this standard, but in most cases the requirements of the sensitive equipment more stringent than utility practice.
Control of voltage flicker	Standards exists, but allowable variation can be set by customers	Standards exists based on the of sensitivity of the equipment plus worker s annoyance provoked by flickering lights.
Harmonic content	Allowable variation set by customers	There are specific limits on harmonic content. In principle, it is not allowed for any customer to negatively impact any other customer. There are important issues of equipment heating, resonance over voltages and harmonic instabilities due to harmonics.
Local voltage stability	Must be stable and within voltage range set by customer	
Voltage stability during transfer from satellite to island mode	No change in voltage criteria during transfer to island mode	
Instantaneous power balance in island mode	Requires adequate storage to preserve voltage criteria during fast load changes.	
Acceptable outage events and duration	Variation and level set by customers	
System performance criteria	Variation and level set by customers	
SAIDI & SAIFI power quality indexes	Variation and level set by customers	
Various system safety and equipment protection	Various system safety and equipment protection standards	
Micro-source and storage protection	Need of more robust equipment standers to prevent unnecessary protection trips	

Table6 — Sensitive load performance assessment

2. General Load Flow for Balanced Case

2.1 Overview

The evaluation of applications of micro-grids requires extensive use of steady state techniques. For example the placement of micro-sources to reduce losses and/or enhance voltage along with determining their ratings can be achieved using load flow methods. Small micro-sources are normally applied at 480 volts or less. At these voltage levels the distribution network is a mix of three phase and single phase loads with different power flows in each phase along with unbalance voltages between the phases. In such cases three phase loads, such as induction machine, need to be modeled in more detail that a constant PQ injection model. The issues of three phase load models during voltage unbalance is important, but is not taken in account during this phase of work due to lack of adequate commercial tools.

This section looks at solutions to the steady state problem when there is balanced phase voltage with the possibility of unbalance load flows in each phase. Micro sources are assumed to be in one of two control modes; one, regulate real power and local voltage, or regulate real and reactive power. It is import in such studies to include PQ limits based on the rating of the micro-source and the interface inverter. For isolated application the formalism of the load flow is modified to allow for the proportional load sharing among micro-sources.

2.2 Load Flow Formulation

The classic Power Flow problem formulation assumes that the network is known in its power generation, transmission lines and load details. This information has to be enough to evaluate the bus admittance matrix for the network and to properly define active power injections at the generators and complex power used by the loads. In general, Power Flow variables are the magnitudes and the angles of the voltages at all the buses of the network. Once these key variables are known, it is possible to find the complex power injection at all buses by means of the following relation:

Here λ is the complex vector, its entries are voltage magnitude in increasing order of bus, and the angles for each of the voltages. Voltage magnitudes and angles are real numbers, the voltage of each bus in the network is represented in the vector λ regardless if their value is know or not.

If we assume that we ordered the bus number so that the first k buses are all generation, then we will have the following active generation vector:

$$P_{gen} = \begin{bmatrix} P_1 \\ \dots \\ P_k \\ 0 \\ \dots \\ 0 \end{bmatrix} \qquad \text{Eq. 2}$$

The buses where the generators are located are defined as P-V from bus 2 to n and $V-\delta$ for the bus 1. Bus number 1 is chosen as the slack bus, the bus that takes all the losses of the network and all the remaining power that is needed to the loads, but is not provided by the remaining generators. All generator buses regulate voltage at their terminals by injecting a proper amount of reactive power.

Actual generators have Q limits that they cannot exceed, and the model takes care of this fact by switching from a P-V model to a P-Q model for the generator. After the solution with the P-V setup has been obtained, the value of needed reactive power is checked against the limit that the unit can provide at the most. Only if the limit is exceeded the bus is converted to $P-Q_{\max}$ and the solution is found again. Now, the voltage at the terminals of the machine will have a reduced value compared to the earlier solution.

The overall Power Flow equation can be written as:

$$S(\lambda) - P_{gen} + S_{load} = 0 \qquad \text{Eq. 3}$$

Here, S_{load} represents the complex power taken by the loads at each of the buses. Clearly, where no load is present, the entry relative to that bus has a null value.

2.3 Per Unit System on the AC Network

When studying the steady state solution of a network, the representation of the AC system usually takes place in per unit quantities. The grid connection down to the loads through the feeders may have different voltage levels due to the presence of transformers. Figure 3 shows a typical AC distribution network.

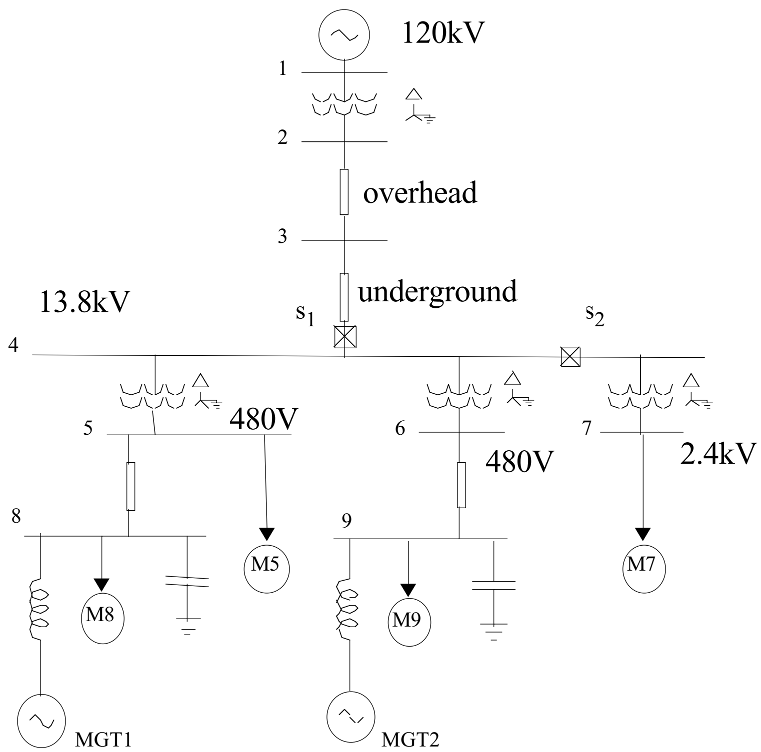


Figure 3. Voltage AC Distribution Network

The base power for the system, S_B, could assume any value but for distribution systems 1 or 10 MVA is used. The base voltage is assumed to be the rated line to line rms value that the voltage has on that feeder. The network of Figure 3 therefore has 4 different base voltages. After the pu conversion the transformers will disappear and voltages will be uniform and near the rated value of 1 pu. across the whole network. The base impedance is chosen as:

$$Z_B = \frac{V_B^2}{S_B} \qquad \text{Eq. 4}$$

Due to the choice of base impedance, the actual three phase power is given by the P and Q in per unit multiplied by the base power S_B.

The base current is chosen as:

$$I_B = \frac{S_B}{\sqrt{3}V_B}$$

Table I shows an example with $S_B = 1\,MVA$ and with the voltage levels from Figure 3. Base impedances and currents for all parts of the network can be found in this table.

S_B [MVA]	V_B [KV]	Z_B [Ω]	I_B [A]
1	120	14400	4.811
1	13.8	190.44	41.836
1	2.4	5.75	240.5
1	0.48	0.2304	1202.8

Table I. Example of Base Representation for AC Network

2.4 Micro-Sources Models and Operation

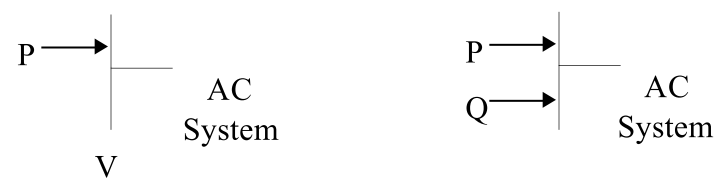
The steady state model of the Micro-Source does not take into consideration the type of power source. Indeed, the task of the prime mover is to sustain the DC bus voltage of Figure 1 at a desired value. Although during transients this voltage may vary a little, the micro-turbine and the fuel cell will bring its value to the desired DC quantity in steady state. The dynamics of the DC bus voltage are strongly dependent on the kind of prime mover, but the steady state behavior is not affected by it. Therefore, the models that are developed in this section can be applied to both micro-turbine and fuel cells.

The steady state behavior of the micro-source is largely dominated by the capability of the unit that come under the form of limits. These limits arise from device ratings, mainly the prime mover and inverter ratings. These ratings are correlated with each other: the inverter must be able at the minimum to deliver the peak power from the prime mover. The prime mover is rated for its maximum output power, while the inverter ratings limit the amount of P and Q that can be delivered into the AC system.

The locally distributed resource is best modeled using a P-V representation assuming active power and voltage regulation and also assuming that there are adequate P and Q ratings. The Micro-Source can also be represented using a P-Q representation assuming active and reactive power regulation. These two representations of the inverter in the Power Flow formulation arise from the need to capture the steady state behavior from the three modes of operation for a micro-source.

i) Control of active power P injected into the AC system and regulation of bus voltage magnitude V.
ii) Control of active power P injection and control of reactive power Q injection, usually set to a limit due to device rating constraints, limit that typically depends on actual power setting.
iii) Control of active power P injection and regulation of reactive power Q to a fixed value.

Figure 4 shows the single phase representation of the inverter to be used in the Power Flow problem. Since there is no active power loss in the inductor, the active power injected from the inverter terminals is the same that will reach the AC system. In addition to this, the voltage regulation takes place at the AC system bus, and not at the inverter terminals.



Control of P and V

Control of P and Q
P and power factor

Figure 4. Power Flow Models for Micro-Sources

Therefore, when the control of power and voltage magnitude is enforced, the unit will look like a P-V bus and the inductance that connects the inverter terminals with the AC system can be completely ignored without affecting the solution.The micro-source is able to sustain the displaced voltage magnitude by injecting a proper amount of reactive power. If this power exceeds a maximum allowed amount, then the unit will behave like a $P - Q_{\max}$ source, with both quantities being enforced at the bus where the inverter terminals are located. In the case of power factor control, then reactive power injection will be related to active power, here we can look at the unit as a P-Q bus directly connected to the grid.

Dealing with Prime Mover and Inverter Ratings

When considering limits on the reactive power output that a unit can yield it is very important to understand what are the reasons that impose those limits. In this way it is possible to define a model of when the non-linear switching from the P-V model to the $P - Q_{\max}$ takes place.

Limits are always a consequence of the fact that the ratings of the inverter and the prime mover are bounded. The desired active power output of the micro-source can never exceed the prime mover ratings. The desired reactive power output cannot grow indefinitely due to the inverter ratings. In general, due to the specification of the manufacturer, these ratings may appear under the form of power factor limits, volt-ampere limits, current magnitude limits. The following limits are to be expressed in per unit using the base of the system as explained in Section 2.1. More in detail we have:

- Power Factor limits

Due to the nature of the power electronic devices that are in the inverter, the manufacturer may express the limit by setting a minimum power factor that cannot be exceeded when the maximum rated active power of the machine is injected in the network. Suppose for example $|pf| \geq 0.75$. Then, it is possible to define the maximum volt-ampere at rated active power output as:

$$VA = \frac{P_{rated}}{0.75} \qquad \text{Eq. 5}$$

and from the complex power relationship

$$VA^2 = Q^2 + P^2 \qquad \text{Eq. 6}$$

it is possible to find the value of the maximum reactive power, as a function of the current active power output:

$$Q_{\max} = \sqrt{\left(\frac{P_{rated}}{0.75}\right)^2 - P_{output}^2} \qquad \text{Eq. 7}$$

- Volt-Ampere limits

The ratings of the inverter usually exceed the active power ratings of the prime source, this to allow to be able to inject some reactive power amount even during the times when the active power output equals the rated peak value.

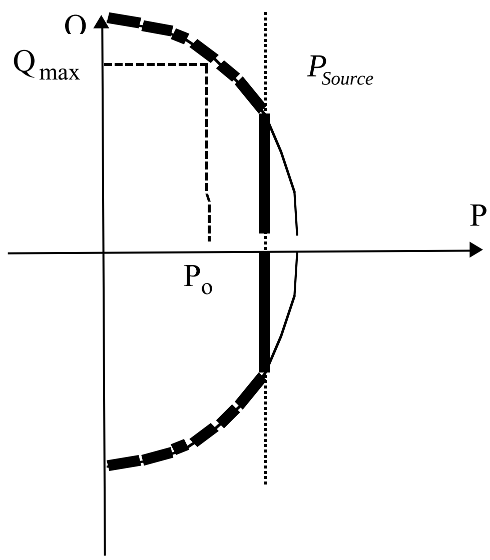


Figure 5. Capability Curve from Device Ratings

As Figure 5 helps to understand, when the active power output is lower than the maximum rated value, then the allowable maximum reactive power injection increases. The locus is a sector of a circle and is described by Eq. 6. Therefore, the maximum allowed amount of reactive power when a Volt-Ampere limit is specified becomes:

$$Q_{\max} = \sqrt{VA_{rated}^2 - P_{output}^2} \qquad \text{Eq. 8}$$

Figure 5 also shows how to find graphically the maximum reactive power from a given output power P_o .

- Current Magnitude Limits

Limits are related to the inverter s current. The valves have a maximum current that they can safely carry before reaching their thermal runaway process. Current limits appear to be very similar to the VA limits, but the latter includes information on the voltage. When a reduced voltage appears at the terminals of the unit, the same VA implies greater current flows. A current limit is a more direct index for device protection. The consequence of this definition of the limit is that the value for the maximum reactive power admissible depends not only on the active power output, but also on the magnitude of the inverter terminal voltage. Figure 6 shows the voltage and the admissible range for the current vector represented on the complex plane.

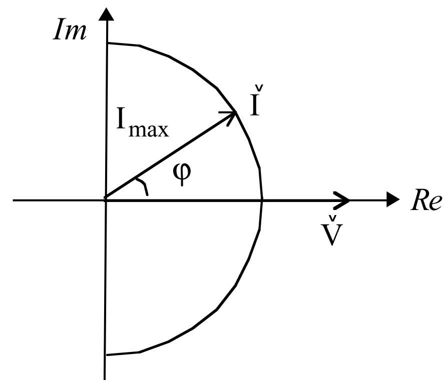


Figure 6. Inverter Voltage and Current in the Complex Plane

Then the maximum reactive power as a function of local voltage and power angle is given by:

$$Q_{\max} = \sqrt{V^2 I_{\max}^2 - V^2 I_{\max}^2 \cos^2(\varphi)} = V I_{\max} \sin(\varphi) \qquad \text{Eq. 9}$$

As Figure 6 suggests, the angle φ is the displacement between the voltage and current vectors at the inverter terminals. The expression for the reactive power limit indicates that the lower is the voltage magnitude, the lower is the resulting limit. In addition to this, it is important to notice that the requirement that limits the maximum current constraints also the active power that the unit can provide at the time the current limit is reached. At that time, the maximum active power is determined by:

$$P_{\max} = V I_{\max} \cos(\varphi)$$

The main goal of the unit is to enforce desired active power injection, and secondarily, to support voltage magnitude. Even in the case when the maximum current is reached, we would still like to be able to provide the desired active power request. If the value of the voltage V falls to values small enough, then the active power request cannot be satisfied. There are three possible cases:

1.)

$$P_{request} < V I_{max}$$

then there exist a value for φ such that:

$$\cos(\varphi) = \frac{P_{request}}{V I_{\max}} \qquad \text{Eq. 10}$$

with

$$P_{request} \leq P_{\text{Power Source}}$$

Substituting into Eq.9 yields:

$$Q_{\max} = \sqrt{V^2 I_{\max}^2 - P_{request}^2} \qquad \text{Eq. 11}$$

2.)

$$P_{request} = VI_{max},$$

then it is still possible to deliver the active power request by setting

$$\mathrm{Q}_{\max} = 0$$

3.)

$$P_{request} > VI_{max},$$

then from Eq. 10 it results that there is no value for the angle φ that allows us to reach the needed requested value for the active power. This condition can be reached if the voltage drops to small values. In this case, the maximum active power injection that can be reached is $P_{\max} = VI_{\max}$ and contemporarily having $Q_{\max} = 0$.

Therefore, the correct Power Flow solution that enforces a limit on the maximum current magnitude that can be achieved is the following. First the solution with a P-V representation of the micro-source is obtained. Then the resulting current injected by the micro-source is compared against the maximum limit. If the actual current is larger than the limit and we are in the either case 1) or 2), then a P-Q representation with $P = P_{requested}$ and $Q = Q_{\max}$ is chosen. If condition for case c) is verified, then a P-Q representation with $P = P_{\max}$ and $Q = 0$ is chosen.

Inverter Ratings	Power Source Rating	Limits
Power Factor Constraints	$P_{\max}$	$P_{request} \le P_{\max}$ $Q_{\max} = \sqrt{\frac{P_{\max r}^2}{pf^2} - P_{request}^2}$
Volt-Ampere Constraints	$P_{\max}$	$P_{request} \le P_{\max}$ $Q_{\max} = \sqrt{VA_{\max}^2 - P_{request}^2}$
Maximum Current Constraints	$P_{\max}$	$P_{request} \le P_{\max}$ $Q_{\max} = \sqrt{(VI_{\max})^2 - P_{request}^2}$

Table II. Different Forms of Inverter Ratings

Table II summarizes the different forms of inverter ratings that may be specified by the manufacturer. The table is meant to be enforced in its per unit values, evaluated under the system base defined in Section 2.1. For each case, the limits for active and reactive power are indicated.

2.5 Model of Micro-Sources in Isolated Network

When the grid connection is missing due to geographical constraints or to a malfunction, micro-sources in the isolated network have problems sharing load, as well as regulating the voltages at their desired setpoints. For load flow analysis the slack bus of the network needs to be replaced by a slack scaling factor to insure load sharing.

Micro-sources operating in parallel cannot rely on an explicit data network that links them to perform their functions. This would create new problems of reliability for the new data network. It is important that the units have a way to communicate with each other in order to ensure proper power sharing. The interaction between units is achieved by means of a frequency droop in the isolated network.(See Section 3.43)

The micro-sources are seen as P-V buses. The active powers for the micro-sources are no longer chosen independently, but rather they are all linked together. The equations that describes the output active power for all the micro-sources are:

$$P_i = P_{0i} + g\,(P_{\max\ \mathrm{i}} - P_{0i})$$
$$\mathrm{i} = 1,\ \text{all the micro-sources} \qquad \text{Eq. 12}$$

In this equation, P_{oi} is the output active power before the redispatching takes place, while $P_{\max\ i}$ is the maximum power that the i-th current unit can give. The slack parameter g can assume any value between 0 and 1. This equation tells us that all units are ramped up together from their current operating point to each one's maximum output power with the same scaling factor g. The slack power is provided by all micro-sources through frequency droop.

2.6 Micro-Sources Connected to Grid

Now that the models for the micro-sources have been defined, we can look back at the Section 2, where the classic Power Flow problem was described. We need to modify the classic case adding the micro-source models just defined. The most important change in the Power Flow formulation as of Eq. 3 is that the vector of active generation will include the powers from all the micro-sources connected to the local grid. Assuming that to order the micro-sources so that they appear on the buses from k+1 to m, then the power generation vector is:

$$P_{gen} = \begin{bmatrix} P_1 \\ \dots \\ P_k \\ P_{k+1} \\ \dots \\ P_m \\ 0 \\ \dots \\ 0 \end{bmatrix} \begin{array}{l} \mathrm{P}_1 \ \text{- Grid Slack} \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} P_{2\to k} \ \text{Grid generation} \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} P_{k+1\to m} \ \text{Micro - Sources} \\ \\ \\ \\ \end{array} \qquad \text{Eq. 13}$$

Here, the first k components represent generations from the grid, while from k+1 to m they represent micro-sources.

The following Table summarizes the buses and their characteristics. For each bus it is specified what is imposed and what will be found after the problem solution:

BUS	V	δ	P	Q	
1	fixed	fixed	open-slack	open	
$2 \to k$	fixed	open	fixed	open	
$k+1 \to m$	fixed	open	fixed	open	inverter bus
i	open	open	fixed	fixed - $Q_{\max}$	inverter bus

Table III. Bus Characteristics for Micro-Sources Connected to the Grid

The last two rows of this Table show the case of a distributed resource (hence $i \in [k+1, m]$) that has reached its maximum reactive power output and is therefore operating as a $P - Q_{\max}$ bus.

2.7 Micro-Sources in an Isolated Network

Here, the power generation vector has to be defined again. The first k entries are zero, since the grid generation is no longer connected and cannot provide active power injection. The vector for the generation can be redefined as follows:

$$P_{gen} = \begin{bmatrix} 0 \\ \cdots \\ 0 \\ P_{k+1} \\ \cdots \\ P_m \\ 0 \\ \cdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \cdots \\ 0 \\ P_{0\ k+1} \\ \cdots \\ P_{0\ m} \\ 0 \\ \cdots \\ 0 \end{bmatrix} + g \left(\begin{bmatrix} 0 \\ \cdots \\ 0 \\ P_{\max\ k+1} \\ \cdots \\ P_{\max\ m} \\ 0 \\ \cdots \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \cdots \\ 0 \\ P_{0\ k+1} \\ \cdots \\ P_{0\ m} \\ 0 \\ \cdots \\ 0 \end{bmatrix} \right) = P_{0s} + g\,(P_{\max\ s} - P_{0s}) \qquad \text{Eq. 14}$$

It is clear now that there is no longer an explicit slack bus, as all the units will pick up the amount of slack power together. It remains only to rewrite the Power Flow problem considering the active power output just defined. Notice that g, being the slack coefficient, is the same for all the machines and is a scalar. At this point we have defined all the quantities to completely describe the new Power Flow equation:

$$\mathrm{S}\,(\lambda) - \left[\mathrm{P}_{0\mathrm{s}} + \mathrm{g}\left(\mathrm{P}_{\max\ \mathrm{s}} - \mathrm{P}_{0\mathrm{s}}\right)\right] + \mathrm{S}_{\mathrm{load}} = 0 \qquad \text{Eq. 15}$$

The resulting powers form the units will satisfy the condition of Eq. 12 while voltages will be regulated as long as the reactive power needed to support them does not exceed the maximum amount, else a $\mathrm{P} - \mathrm{Q}_{\max}$ model for the machine is chosen.

BUS	V	δ	P	Q	
$1 \to m$	open	open	fixed - 0	fixed - 0	
[illegible]	fixed	fixed - 0	open - slack	open	inverter bus
$k+2 \to m$	fixed	open	open - slack	open	inverter bus

Table IV. Bus Characteristics for Micro-Sources in Islanded Mode

Table IV shows the buses during the island mode. It is important to notice that the reference bus for the angles is the angle at the inverter side of the first micro-source, while all the units participate in picking up the slack power of the network.

2.8 Micro-Sources Connected to Grid Subject to Contract Constraints

In addition to the operation with and without grid connection, it is also possible to envision a further mode when the connection with the grid is subject to contract constraints that set the amount of active power that it is possible to purchase from the external generations. This implies that in the Power Flow problem it is no longer possible to designate a generator from the grid as the slack bus.

One of the micro-sources has to be designated as a reference for the angles, and the amount of active slack power has to be pick up by all the units by means of the slack coefficient as explained in Eq. 12 The new active power generation vector is:

$$P_{gen} = \begin{bmatrix} P_1 \\ \dots \\ P_k \\ P_{k+1} \\ \dots \\ P_m \\ 0 \\ \dots \\ 0 \end{bmatrix} = \begin{bmatrix} P_1 \\ \dots \\ P_k \\ P_{0\ k+1} \\ \dots \\ P_{0\ m} \\ 0 \\ \dots \\ 0 \end{bmatrix} + g \left(\begin{bmatrix} 0 \\ \dots \\ 0 \\ P_{\max\ k+1} \\ \dots \\ P_{\max\ m} \\ 0 \\ \dots \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \dots \\ 0 \\ P_{0\ k+1} \\ \dots \\ P_{0\ m} \\ 0 \\ \dots \\ 0 \end{bmatrix} \right) = P_{0s} + g\,(P_{\max\ s} - P_{0s}) \qquad \text{Eq. 16}$$

The entries from k+1 to m, relative to the micro-sources, behave in the same way as described in the earlier section: they have a fixed setpoint and a scalar slack coefficient that allows all the powers of the distributed resources to increase contemporarily. The entries from 1 to m, relative to the grid generation are fixed to the desired value. These units do not participate in providing for the slack power, as their output level is not affected by the value of the slack coefficient g.

BUS	V	δ	P	Q	
$1 \to m$	fixed	open	fixed	open	
[illegible]	fixed	fixed - 0	open - slack	open	inverter bus
$k+2 \to m$	fixed	open	open - slack	open	inverter bus

Table V. Bus Characteristics for Micro-Sources Subject to Contract Constraint

Table V summarizes the characteristics of the generation buses in the Power Flow formulation when a contract constraints the active power injection from the grid supply to some defined value. All grid generations behave as P-V buses. The first micro-source (labeled as k+1) provides a reference for the angles, while all distributed resources contribute to providing the remaining needed active power for the local network.

3. Dynamics of Power Electronics Based Micro-Sources [Inertia-Less Generation]

3.1 Overview

Micro-sources dynamics are fundamentally different from conventional central station generation technologies. For instance, fuel cells and battery storage devices have no moving parts and are linked to the system through electronic interfaces. Micro-turbines have extremely lightweight moving parts and also use electronic system interfaces. The dynamic performance of such inertia-less devices cannot be modeled simply as if they were scaled down central station units

Virtually all micro-sources require power electronics to interface with the power network and its loads. In all cases there is a D.C. voltage source, which must be converted to an ac voltage source at the required frequency, magnitude and phase angle. In most cases the conversion will be performed using a voltage source inverter. Hence the inverter and its control is a key component of the systems dynamics. However, the properties of the primary power source represent an important limitation on micro-sources device with respect to power and energy flow. For instance, a micro-turbine requires about 10-15 seconds for a 50% change in power output. A fuel cell requires about few seconds interval for a 15% change in power output, but requires a recovery period of a few minutes to establish equilibrium before it can provide another step change in power output. Hence it is rather important to include such limitations in the dynamics of the power sources.

To remove such limitations some form of storage is necessary at the ac or dc bus to cope with instantaneous changes in power demand. This is critical in the case of sensitive loads. In an island mode micro-grids will be incapable of meeting load requirements if storage is not included.

3.2 Micro-turbines and Fuel Cells

Emerging technologies have made the use of Fuel Cells and Micro Turbines economically attractive in electric power production. Their relative small size, typically below 300 kW, allows them to be placed at key locations within an existing electric distribution network. This can increasing the locally available power and enhancing the overall system reliability.

In all cases, the electricity is available as a dc voltage. The interface with the electrical grid is achieved by means of power electronics. An inverter is responsible for the conversion from DC to the desired AC voltage magnitude and angle. Fuel Cells produced dc voltage while the electric power from Micro Turbines comes from the high speed permanent magnet generator coaxially installed on the same shaft of the turbine. The result is an ac voltage at few kHz. This subsequently rectified. Figure 7 gives the broad view of a micro sourced generator. The power source sustains the voltage at the DC bus, with the inverter as the interface to the electrical grid.

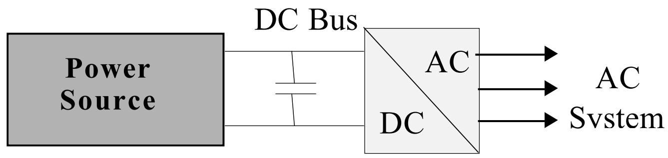


Figure 7. Micro-Source Overview

The slow dynamic model of the micro-source can capture the behavior of the unit during load transient. Typically such events occur when new loads are brought either on line or off line. The model assumes a balanced mode of operation where loads and voltages are distributed equally on the three phase system. While the steady state model was not dependent on the type of micro-source, dynamic behavior is dominated by this factor. Micro-turbine and fuel cells have different responses and the model needs to keep this into account. For voltage source inverters the basic model is a controlled voltage connected to the system through and inductance. The single phase representation of the dynamic model of the micro-source is represented in Figure 8.

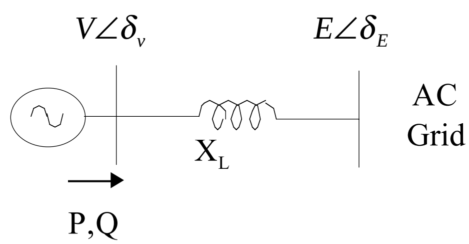


Figure 8. One Line Inverter Diagram

The inverter is represented as an ideal voltage source that operates at a voltage magnitude of V and with an angular displacement of δ_v. These values are controlled to insure the desired amount of active and reactive power injection. The reactive power is injected to sustain the voltage magnitude E of the bus where the connection with the AC grid takes place. The magnitude V and angle δ_v are provided by the controller that regulates P and E. From the errors between measurements and desired setpoints for active power P and voltage magnitude E, the inverter generates a voltage of magnitude V and phase δ_v that are responsible for the overall system behavior.

The units used in this model are the same as in the Power Flow formulation. A per unit representation of the network is used, leading to a value of the power in per unit that when multiplied by the base power will yield the overall three phase power. This representation is valid under the assumption made earlier of balanced operation mode.

The formula that give the active and reactive power are reported below:

$$\delta_p = \delta_v - \delta_E$$

$$P = \frac{VE}{X_L}\sin(\delta_p) \qquad \text{Eq. 17}$$

$$Q = \frac{V^2 - VE\cos(\delta_p)}{X_L}$$

As V and δ_v generated by the inverter regulate the P and Q. It is clear that the series inductance, X_L, plays a critical role.

3.3 Sizing of Inductor

Inductor size is critical to insure full range of P & Q operation with out excessive inverter voltage. Measurements are taken from both sides of the inductor and a controller generates desired values for V and δ_v to provide requested active power injection P, and voltage magnitude E.

Micro-source limits and regulated bus voltage E constraints determine the size of the inductor. The value E =1 pu is normal. The inverter has limits on its output quantities:

- Limit on V (such as $V_{\max} \le 1.2\ pu$). This condition is dictated by the value of the voltage at the DC bus and by the voltage stress on power electronic devices.

- Limit on δ (such as $\delta_{\max} \le 30\ \deg rees$). This condition provides linear power control. P(δ_p) in Figure 9 shows that 30 degrees is a reasonable choice of $\delta_{p,\max}$.

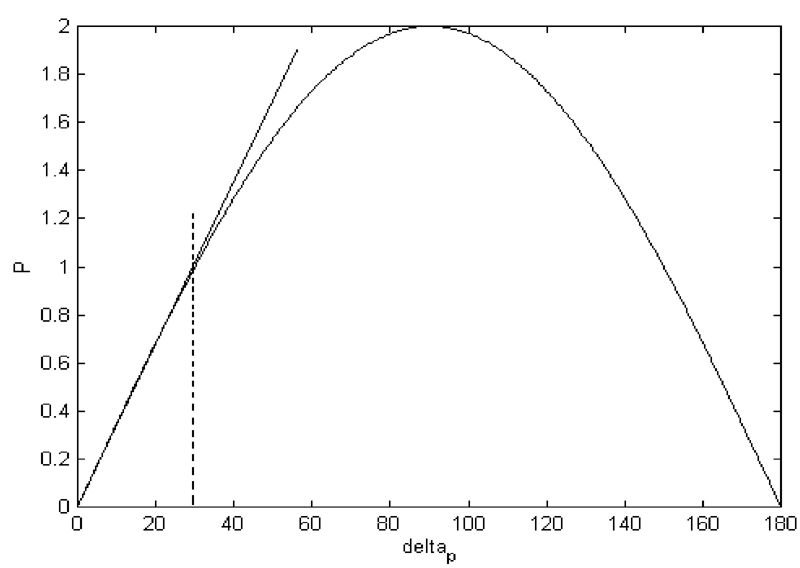


Figure 9. Power-Angle Characteristic

Inverter Operation Area (IOA)

When designing the inverter, the choice of ratings of the devices plays a key role. The inverter, at the minimum, has to be rated to provide maximum power. It is possible to match the ratings of the inverter and the maximum power only if we force reactive power to zero at peak output power. More in general, each inverter has limits dictated by the ratings of the silicon devices. These limits appear in the form of maximum voltage that the power electronic device can safely support, and the maximum current that it can carry. Based on these considerations, the inverter can be designed to provide a range of active and reactive powers.

The size of the inductor must be such that it is possible to deliver the rated active and reactive power.

In turn, the active and reactive power that the inverter can provide are determined by the limits on the inverter. There are four major families of limits, each one determining a different Inverter Operation Area (IOA):

a) Limit on the power factor

Typically, it comes in the form of $pf \geq |pf_{\min}|$ when the active power is at its maximum output (i.e. the power source rating). For example for $pf_{\min} = 0.75$. is:

Then the max rating in VA for the machine can be seen as:

$$P_{\max} / 0.75 = VA_{\max} = \sqrt{P_{\max}^2 + Q_{\max}^2}$$

In the $P-Q$ plane such a constraint can be represented by Figure 10.

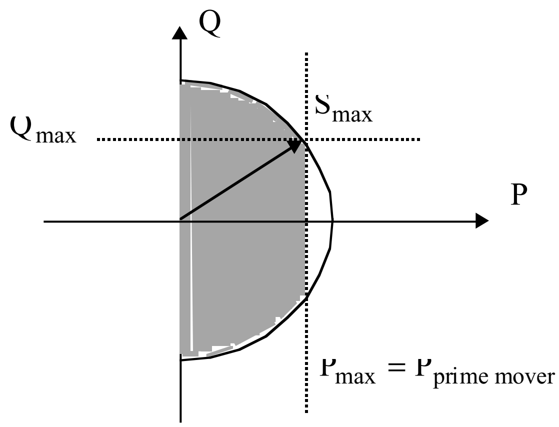


Figure 10. Constraint on P-Q Plane from Power Factor

The shaded area represents the values of P and Q that, given the ratings on the prime mover, the inverter has to be able to provide to the AC system.

b) Fixed setting on Q

The condition Q=0 over the operating whole range is sufficient for the minimum inverter ratings. The manufacturer may also constrain the operation of the inverter as constant Q injection, with Q not necessarily equal to zero. On the P-Q plane, the IOA constraint appears as a line as shown in Figure 11.

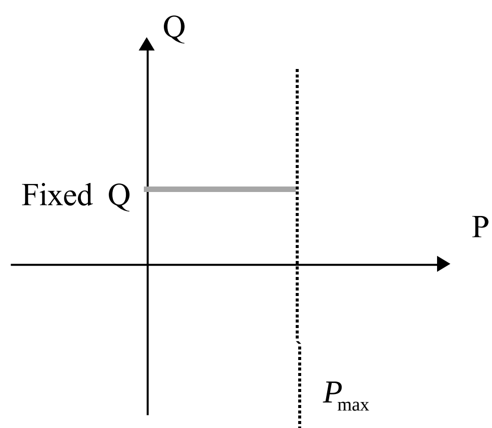


Figure 11. Constraint on the P-Q Plane for Fixed Q Setting

c) Maximum Volt-Ampere ratings

Frequently, limits on the reactive and active power come in the combined form of a limit on the maximum Volt-Ampere that the unit can withstand. Assuming that we want to maintain the possibility of delivering reactive power during peak active power output, then the IOA for this case is shown as the shaded region in Figure 12.

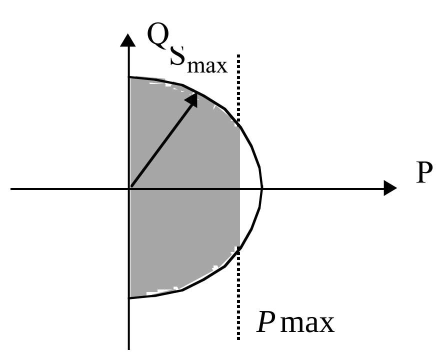


Figure 12. Constraint on the P-Q Plane for VA Rating

d) Maximum current magnitude

Conceptually, if the voltage at the inverter terminals was constant, then it would be identical to case c). Maximum current condition directly enforces the limits of the devices keeping into consideration that if the voltage is lower the IOA region is smaller. Here, the IOA is identical to the one of Figure 12. The radius of the circle is given by:

$$S_{max} = VI_{max}$$

It is clear that if the voltage V falls below the value:

$$V \leq \frac{P_{max}}{I_{max}}$$

then the IOA becomes a full circle and the maximum amount of deliverable power is lower than $P_{prime\ mover}$ and is exactly:

$$P_{max} = VI_{max} < P_{rated}$$

Criterion to Obtain Maximum Inductor Size

Equation 17 spells out the formulas to obtain P and Q given the network parameters. It is clear that a larger inductance allows for a lower power delivery, while a smaller inductance allows larger power flows. It follows that there is an optimal size for the inductor. If the inductor is smaller than this quantity, then we are still guaranteed to make it to deliver the specified amounts of power. Then, the optimal inductor size can be seen as the maximum size that the inductor can have, and still being able to satisfy the delivery requirements.

To develop some intuition on how to solve the problem it is useful to build some maps of the power that it is possible to deliver given the limits that need to be satisfied.

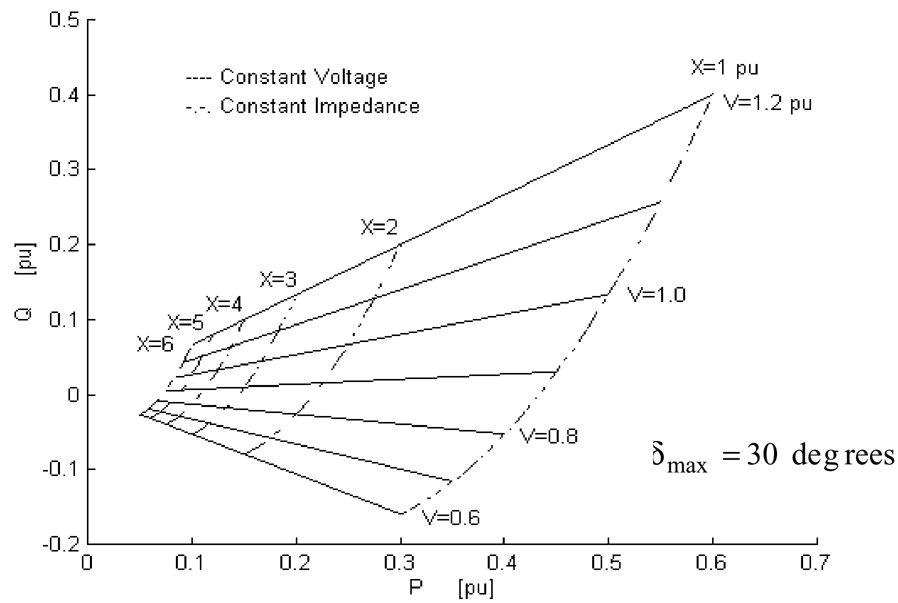


Figure 13. P and Q Plane Capability with Constant Voltage and Impedance

Figure 13 plots P and Q from equation 17 assuming that $\delta_p = \delta_{p\max} = 30$ degrees and E=1pu. Then, the values for $V \in [0.6,\ 1.2]$ and $X \in [1,\ 6]$ pu are separately spanned. The result of overlapping the plots yields a map. Now it is clear what effect the size of the inductor has on the active and reactive power that one can deliver. This map and the following ones are evaluated assuming a per unit system representation. The system base defined in Section 2.1 is used to plot these graphs. Assuming $S_B = 1\,\text{MVA}$ and assuming that the rated power of the prime mover is 100 KVA, then quantities P and Q around 0.1 per unit should be achievable by the inverter.

Each value of the inductance defines a certain region in the space that can be reached: Figure 14 shows this region for X=3 pu and δ spanning from 0^o to 30^o.

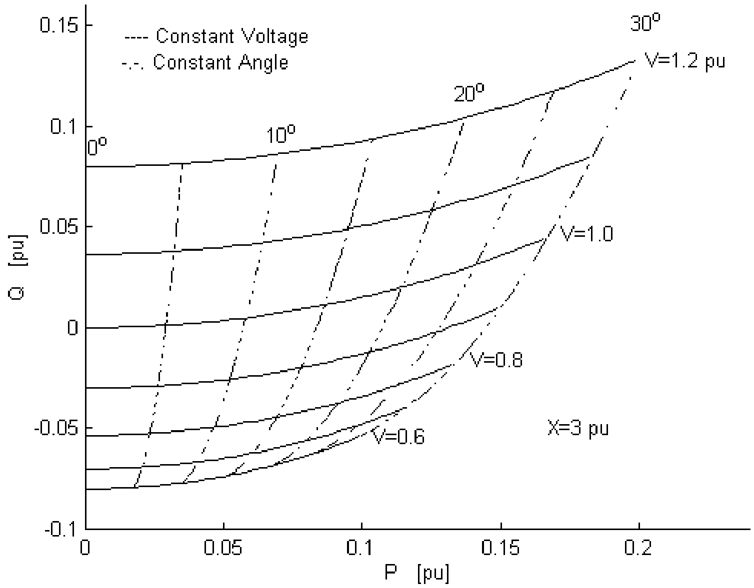


Figure 14. P and Q Plane Capability with Constant Voltage and Angle X=3 pu

Figure 15 for X=2 pu shows that with this smaller impedance it is possible to reach a larger region in the plane P-Q, infact in addition to the points that we were able to reach before, now we have the added capability of reaching the point P=0 and $Q = \pm 0.1$ pu.

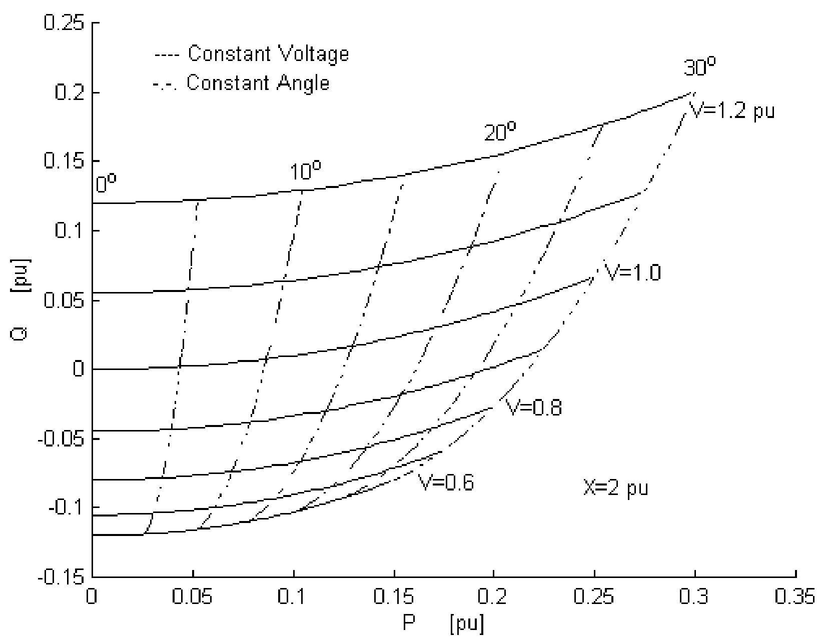


Figure 15. P and Q Plane Capability with Constant Voltage and Angle X=2 pu

Figure 16 drawn for X=1 pu shows that it is yet possible to reach more operating points in the P-Q plane. The region of space in the P-Q plane shown in Figures 14, 15 and 16 for different impedances, will be labeled as region Ω throughout the rest of this work.

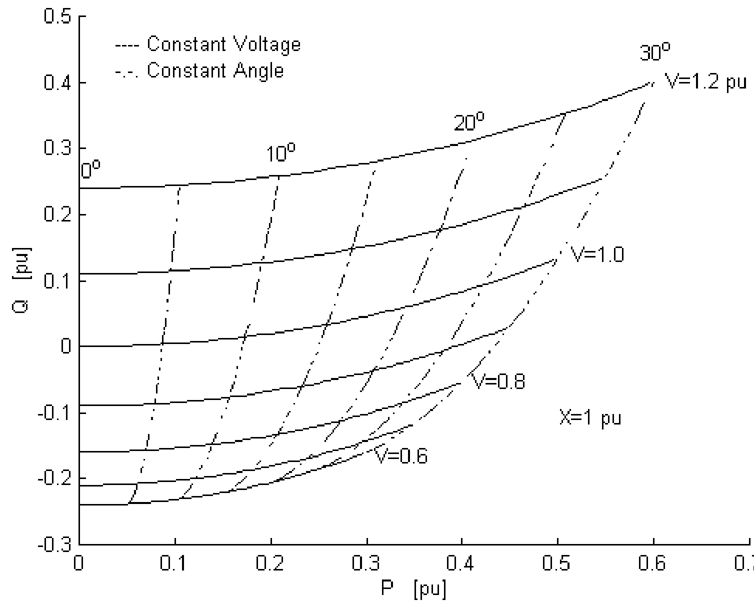


Figure 16. P and Q Plane Capability with Constant Voltage and Angle X=1 pu

It is important to notice that as the size of the inductor decreases, the region Ω increases in size, while, in good approximation, preserving its shape. Now it is possible to refine the definition of ideal inductance size with the following:

Let Ω be the region of space in the P-Q plane reachable given the requirement for E, V, δ and with a given X.
Let IOA (Inverter Area of Operation) be the region of space in the P-Q plane needed to be reached as specified by the manufacturer.
The ideal size for X is the maximum impedance such that the condition $IAO \in \Omega$ *is satisfied.*

With this definition in mind it is possible to look at each of the specifications that the manufacturers impose on the inverter (defining the size and shape for the IOA as of Section 3.1.1) and find the ideal inductor size.

<u>Ideal Inductor Size</u>

Now it is possible to revisit inductor sizing and for each of the typical operation areas define the optimal inductor size.

a) Limit on the power factor

As Figure 17 shows, enforcing the maximum active power alone may not be enough to satisfy the condition $IOA \in \Omega$. Figure 17(a), $X = 3pu$, shows that the rated power can be delivered but there are some regions around P=0 and $Q = \pm 0.1\, pu$ that are not reachable. Figure 17(b) , $X = 1pu$, shows that by enforcing points P=0 and $Q = \pm 0.1\, pu$, the condition $IOA \in \Omega$ is surely satisfied.

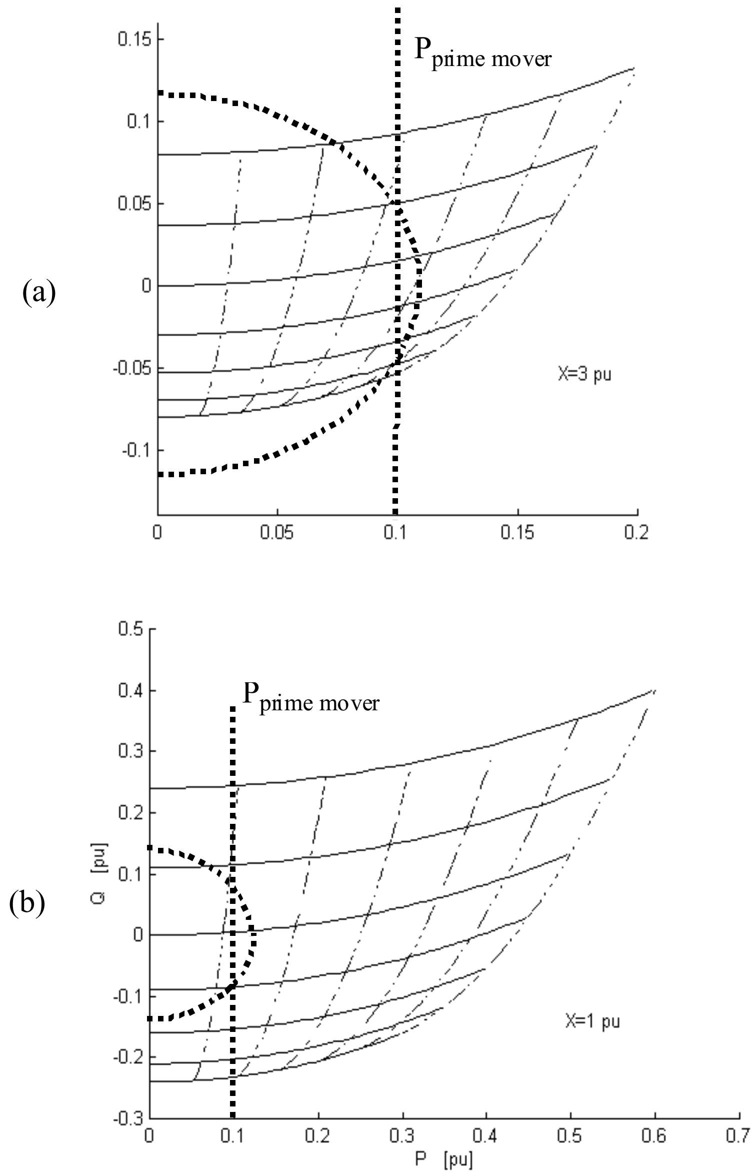


Figure 17. Enforcing Constraints with Power Factor Limits

b) Fixed setting on Q

Figure 18 helps to understand the condition that allows for the best inductance.

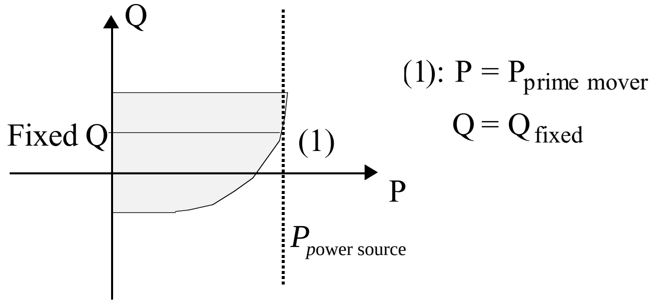


Figure 18. Enforcing Constraints with Fixed Q Setting

Here X is such that satisfies the equations for P and Q at point (1) with E specified, V within the limits and δ at its maximum value.

c) Maximum Volt-Ampere ratings

Looking at Figure 14 it is possible to notice that the intersection of Ω with the axis of P = 0 is such that the maximum positive Q is just slightly larger than the maximum negative Q. This is due to the choice of V_{max} . Another choice for the maximum voltage may yield opposite results.

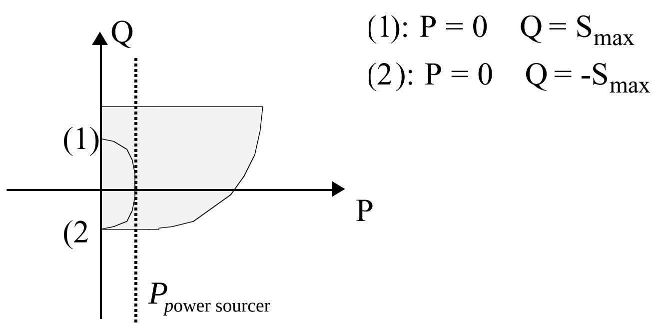


Figure 19. Enforcing Constraints for VA Ratings

Figure 19 has been drawn according to the same choice of maximum voltage that led to Figure 13. From Figure 19, it seems that it is enough to guarantee that the point (2) is reachable in order to satisfy the condition $IOA \in \Omega$. As discussed above, this conclusion can be deceiving. The inductance size is values of the inductance that satisfy the P and Q equations at the points (1) and (2) with V within the specified limits and with and δ at its maximum value.

d) Maximum current magnitude

The approach is very similar to the case c), with the only exception that the value for the maximum Volt-Ampere must be defined depending on the maximum current limit. Since a current magnitude limit is enforced, the largest amount of Volt-Ampere that need to be delivered through the inductance happens at the condition of maximum voltage:

$$S_{\max} = V_{\max} I_{\max}$$

With this value for $S_{\max}$ it is possible to repeat the same logic of case c) to find the optimal inductor size.

3.4 Micro-Sources / Power Source

Fuel Cells and Micro Turbines provide a dc voltage source. Each controls the output power by fuel rate. This control, in turn, is regulated by a governor whose goal is to maintain the DC bus voltage constant. This governor is a single input, single output block. Its input is the measure of the DC voltage, while the output is the position of the fuel valve. This position is limited by an upper (full throttle) and lower (shut off) limits. The fact that the power source is somewhat limited in its output performance will result in constraints in the response of the prime mover to step changes in output power requests. It is worth spending some time to understand the way a change in the command propagates through the power source, depending on the nature of the prime mover.

Micro-Turbine

The full diagram for the power source based on Micro Turbine technology is represented in Figure 20. Here, it is possible to follow the air mass flow as it enters the compressor after having gone through the permanent magnet generator casing. This provides cooling of the stator. The rise in pressure is obtained by one stage of a centrifugal compressor After this stage, the air goes though a heat exchanger called recuperator that rises its temperature. This feature dramatically improves the efficiency of the unit. At this point the air enters the combustion chamber, where it is mixed with the fuel (typically natural gas) and burned in a continuous process. The high temperature gases are then expanded in the turbine (here again, of radial design) where the useful work is extracted and converted into mechanical form.

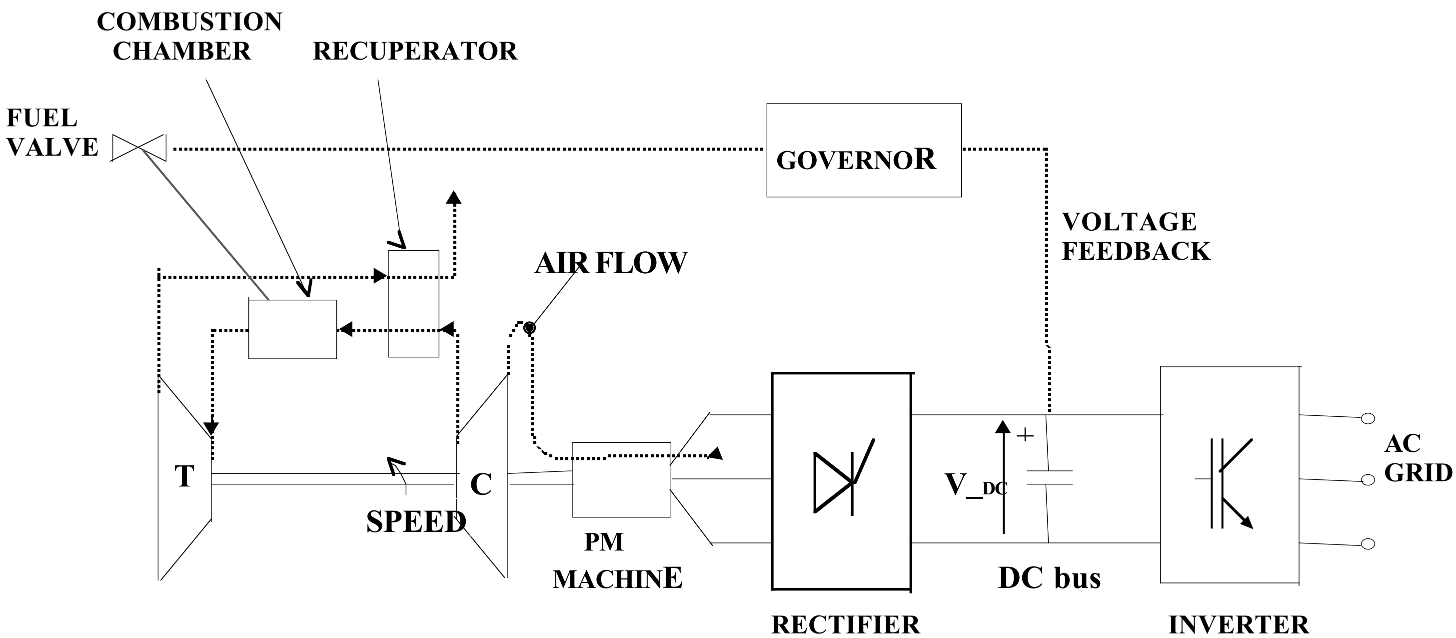


Figure 20. Micro-Turbine Block Diagram

The output mechanical power is regulated solely by the fuel mass flow. As the more fuel enters the combustion chamber, more heat is generated and more power is converted in mechanical form. It is important to notice that a step change in the fuel valve position will not result in an instantaneous increase of power. Actually, at the time of the change, the output power decreases. This is due to the fact that the speed of the turbine had not time to change, and the consequently the air flow entering the combustion chamber didn't have time to increase. The increased fuel rate generates a non ideal stechiometric ratio between fuel and air, instantaneously lowering the power available at the shaft. As the turbine-compressor block increases speed, more air is drawn into the combustion chamber, allowing for all the extra fuel to be burned, and in turn for an higher power output.

Figure 21 shows the Micro Turbine response to a step change in the fuel valve. Time constants may be as high as 10 seconds. This property of a prime mover presents problems for micro-grids with changing loads. See test report on 28 kW MTG behavior during step changes in load conducted by: Southern California Edison and Paragon Consulting, June 22, 1999 as part of this project.

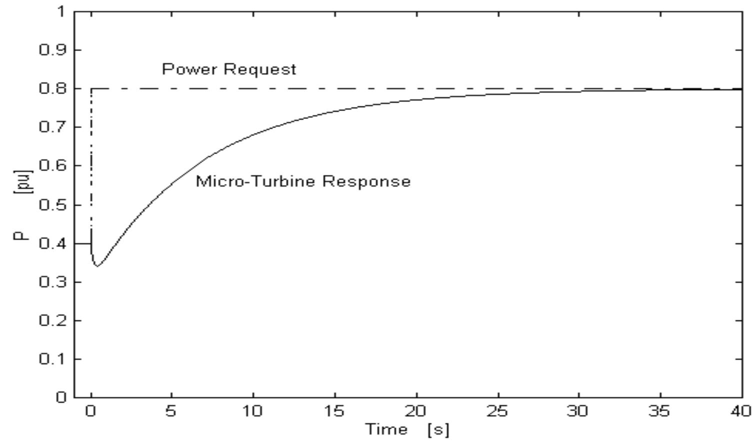


Figure 21. Micro-Turbine Step Change Response

The only remaining detail that is worth mentioning is that the electric power available at the terminals of the permanent magnet machine is produced at a frequency that is directly proportional to the Micro-Turbine speed. Generally, there is no control loop to regulate the speed and therefore there is no mean to set the output frequency to a desired value. Furthermore, the thermodynamics of the compressor and turbine block require the mechanical speed of the shaft to be some tens of thousand of revolutions per minute, which makes the frequency of the output electric quantities to be in the range of few kHz. Network interface requires a tight control of the frequency, therefore the conversion to an intermediate DC stage before conversion to the desired constant frequency is mandatory.

Fuel Cell

The Fuel Cell technology-based prime mover diagram is shown in Figure 22. As in the micro-turbine, we can recognize the DC bus voltage measurement that is fed in a controller, whose output regulates the position of a fuel valve. Fuel Cells are capable to produce low voltage at the sides of a membrane.

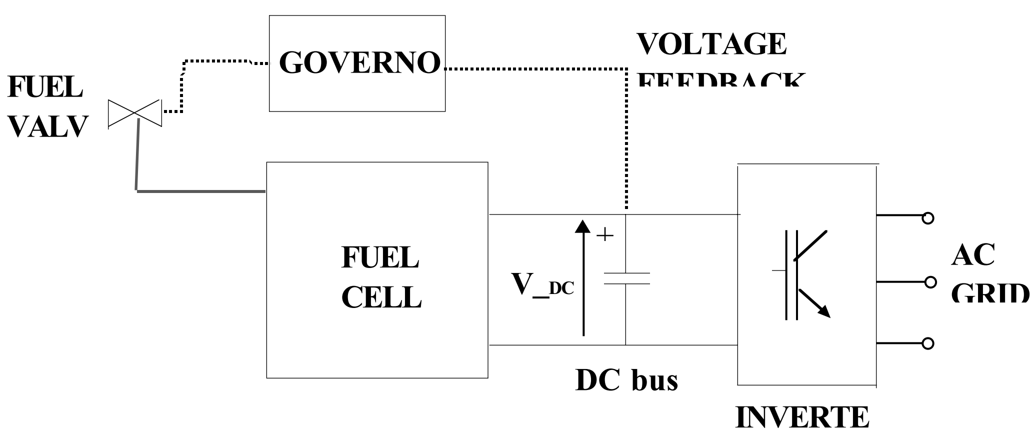


Figure 22. Fuel Cell Block Diagram

For example, the Proton Exchange Membrane (PEM) can be crossed only by ions charged with one sign, while it is totally impermeable to the charges of the other sign. The sides of the membrane result charged like the plates of a condenser. The charges can reach the other side of the membrane by flowing through the external circuit, which is the electrical load. Once the charges arrive from the load, they neutralize the ones already existing on the other side of the membrane.

The result of this process is that charges of opposite sign must be constantly created at either side of the membrane. The accumulation of charges comes from a ionization process driven by a chemical reaction. This reaction is highly sensitive to parameters such as concentration of the reactants and products of the reactions, temperature and pressure.

Each membrane has a determined maximum voltage that it can produce given by the charges per unit of surface that are present, while the current is determined by the total amount of charges, which is determined by the total surface of the membrane. Typically, one membrane can produce less than a Volt between its sides. It is required to stack hundres of these membranes to obtain an output voltage level that can be suited for AC system interface.

A step change in the input reactant element will not be noticed as a sudden increase in charges at the sides of the membrane. Typically, there is a sudden but contained rise in output power that takes place in 10-20 milli-seconds, while the newly desired output power level is reached limited until the membrane reaches its thermal equilibrium, which may 10s of seconds. Figure 23 shows the response of the Fuel Cell to a step change command in the desired output power.

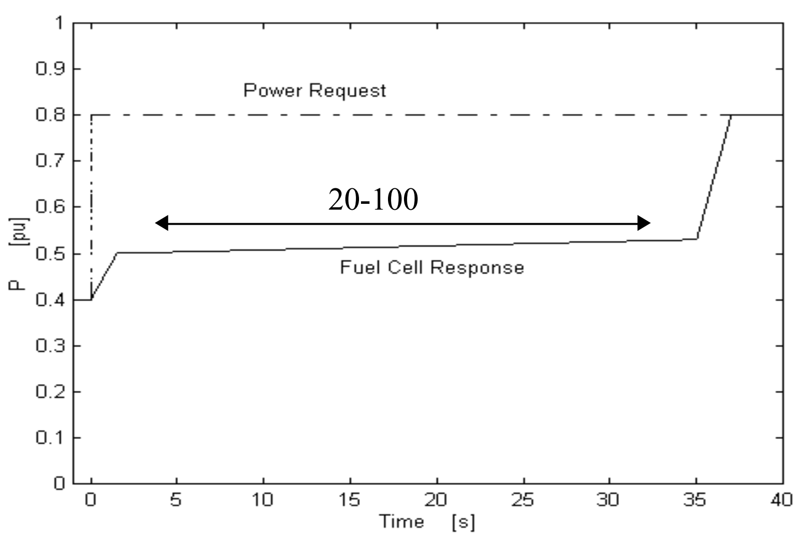


Figure 23. Fuel Cell Step Change Response

This characteristic makes the Fuel Cell a poor candidate for isolated systems, where loads require instantaneous power.

Instantaneous Power Issue

The introduction of a micro-source within an existing power system assumes that there may be a cluster of small generators located in electrical proximity, interconnected through on-site feeders and operation can be isolated or connected to a transmission/distribution grid.

Micro sources have a slow response to changes in commands and also do not provide any kind of internal form of energy storage. These inertia-less systems are not well suited to handle step changes in the requested output power. It must be remembered that the current power systems have storage in the mechanical energy of the inertia of the generators. When a new load is applied the initial energy balance is satisfied by the inertia of the system: this results in a slight reduction in system frequency.

The power electronics interface between the DC bus and the AC grid has fast response and is inertia-less, this due to the fact that the matrix of switches of the inverter does not have any sort of energy storage capability. The step sized power demand in the AC system should be instantaneously matched by an identical supply of power from the micro source power source.

Distributed resources have a problem in instantaneous power tracking. Figure 24 shows that a poor load tracking is penalized with a difficulty in holding the local voltage to the desired value. Here a load is applied and the micro-source ramps up to pick up the whole quota of extra power request. The missing transient power that the micro-source is not fast enough to provide is taken from the connection with the grid that supplies only a part of the local requested power during steady state. Voltage is maintained by the power grid

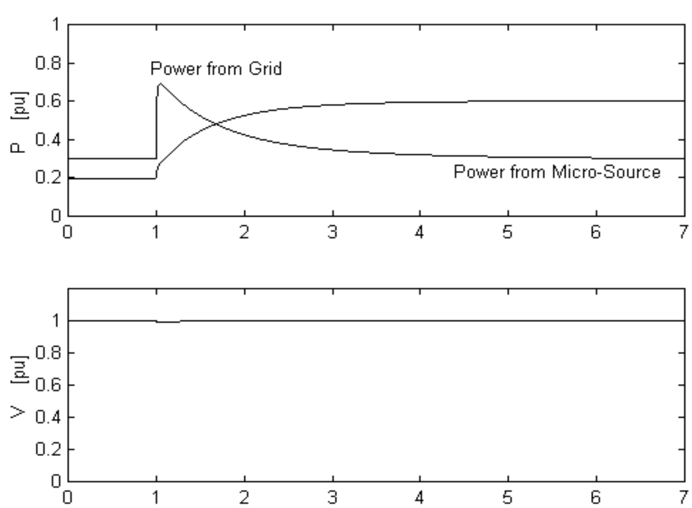


Figure 24. Load Coming On-Line with Grid Connection

The requested power from the load coming on-line is a step function, while the inertia-less micro source always takes a finite amount of time to ramp up to the newly requested value. Figure 25 shows an applied load while in an islanded mode. Since the unit cannot change its output power instantaneously, the power is balanced by voltage reduction. As the power injected from the micro source increases to supply the needed load the voltage is restored.

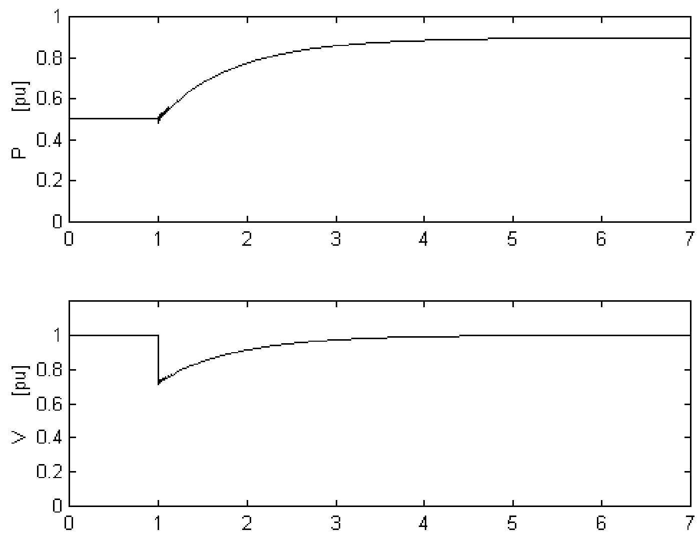


Figure 25. Load Coming On-Line without Grid Connection

Figure 26 shows that there is a missing quota of energy from the difference of the power that the load is requesting and the power that the unit is able to provide shown by the shaded area.

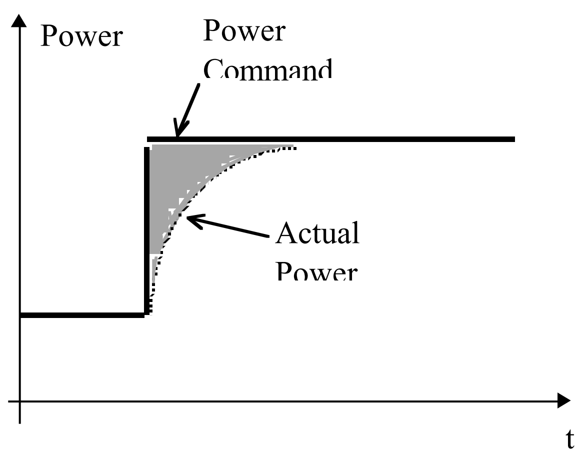


Figure 26. Missing Transient Energy

If the connection to the main grid is active, then the missing quota can be provided by the utility. It will be seen from the grid terminals as a temporary, pulse-like power request. Utilities do not like these topology of events, since they are hard to service.

If the connection to the grid is missing due to a malfunction or a to geographical constraint that makes it impracticable, then the need for some sort of storage is manifest. Voltage drop could be large even for small loads coming on line. Storage is requested to satisfy the instantaneous power balance as a new load comes on-line without penalizing the quality of other network quantities, such as bus voltage magnitudes. Storage can be provided on the DC as well on the AC side or by including rotating machines in the Micro-grid.

The capacitor bank that we see in Figure 7 located on the DC bus is only sized to smoothen the voltage ripple, but by all means it is not able to store and provide the transient amount of energy that is missing as of Figure 26. Storage can take place in different forms, but the most practical is the DC battery placed in parallel to the already existing DC bus terminals of the distributed resource.

Customers may indicate the sensitivity of each of their load clusters, setting the boundaries for the largest voltage dip that they can tolerate. It is possible to optimize the size of the battery knowing the physical location and sensitivity of the local loads, setting the stage for the hybrid system constituted by the combination of the distributed resource with the storage capability of the battery bank. Such an hybrid system allows for many degrees of freedom providing a solution to customers with different system needs and priorities in their loads.

To complete the picture it is important to notice that at a given time, there may be more distributed resources located in electrical proximity to each other. Power sharing during island mode must be addressed since it is critical to system performance and survival in such times. Control of the inverters that perform the grid interface should be based only on information available locally. In a system with many micro-sources, communication of information between systems is impractical. Communication of information may be used to enhance system performance, but must not be critical for system operation. Essentially this implies that the inverter control should be based on terminal quantities only.

To summarize, the introduction of micro-source within the context of an already existing grid can provide control of local bus voltage, control of base power flow thus reducing the power demand from the main grid feeder, and ultimately frequency control associated with load sharing within units located in the micro-grid.

3.5 Power Electronics Models

When interfaced to the grid with the auxiliary help of a battery to supply the transient need of power, the details on the distributed source behavior tend to lose importance when looking at the performance of the unit as seen from the grid terminals. Indeed, the inverter will be connected to the terminals of the battery, and will only see a relatively stiff DC voltage. The power source is no longer required to have a dramatically fast transient response, since its task will only be to keep the battery charged.

The fact that we can look at the system as an inverter connected to a battery is the simplification that makes the design and analysis of the power electronic interface less burdensome. The nature of the power source (Fuel Cell or Micro Turbine) is no longer crucial and its details during transient response no longer needed. These details, along with the load sensitivities during island mode provided by the customer, are only needed to size the battery.

The most basic requirements for the inverter operations are the capability of delivering a preset amount of active power to the grid along with the ability to hold the voltage magnitude at the point of connection with the AC system to a desired value. To study the dynamic behavior of the power electronics interface, measurements and control block structure must be defined.

Full Model

Figure 27 shows the details of the DC/AC interface. The inverter terminals are connected to the grid through an inductance. It is this reactance that allows the dispatching of the desired amount of active power into the grid.

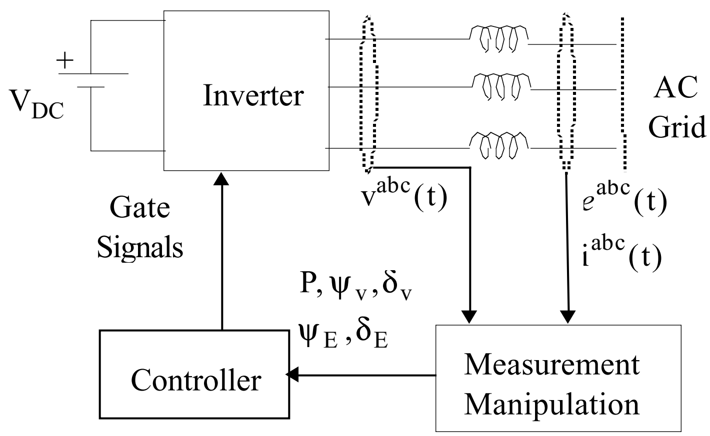


Figure 27. General Inverter Scheme

Figure 27 shows the flow of information from the measurements, to the control blocks, to the gating signals that command the power electronics in the inverter. Measurements consist of phase voltage and currents taken at the regulated bus, and the voltage at the inverter terminals. The governor is composed of blocks that are responsible for holding the delivered power and regulated bus voltage to the desired values. The gating signals are generated depending on the type of bridge used. Here, we use a six switches bridge combined with a voltage flux control strategy.

The model starts with the manipulation of the time domain quantities as measured from the sensors. These quantities are converted to the inverter and regulated bus voltage flux magnitudes and angles (respectively ψ_v, δ_v and ψ_E, δ_E), and the active power injection at the regulated bus, P.

These quantities are fed into the governor block that generates the gating signals. The overall picture is given in Figure 28, where it is possible to follow the flow of information as it is processed, from left to right.

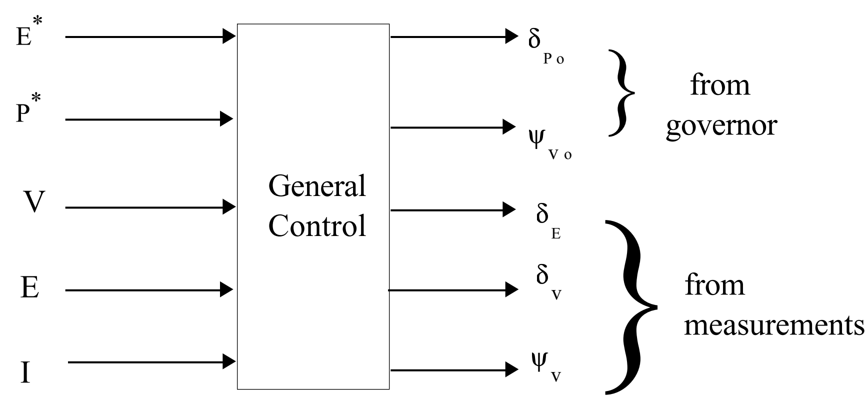


Figure 28. Detailed Inverter Control Scheme

More in detail, the following expressions describe how to obtain the voltage flux magnitude and angle for the inverter terminals. The procedure is identically applied to the regulated bus voltage measurements.

$$v_{ds} = \frac{v_c - v_b}{\sqrt{3}} \qquad e_{ds} = \frac{e_c - e_b}{\sqrt{3}}$$

$$v_{qs} = \left(\frac{2}{3}\right)(v_a - \frac{1}{2}v_b - \frac{1}{2}v_c) \qquad e_{qs} = \left(\frac{2}{3}\right)(e_a - \frac{1}{2}e_b - \frac{1}{2}e_c)$$

$$\Psi_{vd,vq} = \int_{-\infty}^{t} v_{ds,qs}(\tau)d\tau \qquad \Psi_{ed,eq} = \int_{-\infty}^{t} e_{ds,qs}(\tau)d\tau \qquad \text{Eq. 18}$$

$$\Psi_v = \sqrt{\Psi_{vd}^2 + \Psi_{vq}^2} \qquad \Psi_e = \sqrt{\Psi_{ed}^2 + \Psi_{eq}^2}$$

$$\delta_v = -a\tan\left(\frac{\Psi_{vd}}{\Psi_{vq}}\right) \qquad \delta_e = -a\tan\left(\frac{\Psi_{ed}}{\Psi_{eq}}\right)$$

Where $v_{a,b,c}$ is the magnitude of each phase voltage generated by the inverter, and $e_{a,b,c}$ is the system phase voltages. In Figure 28 it is possible to notice that there are two externally defined setpoints: the requested power injection P^*, the desired regulated bus voltage magnitude E^*.

The gate pulse generator can be understood by the following diagram of Figure 29.

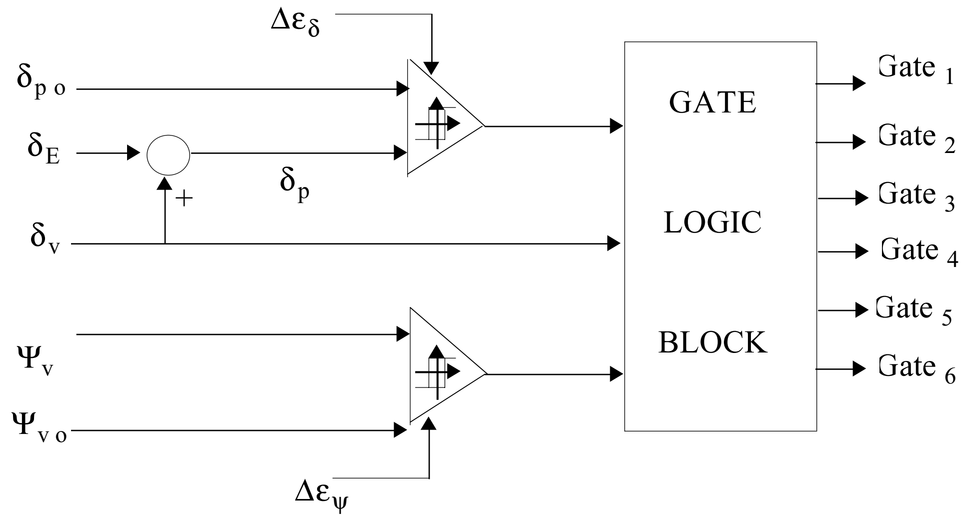


Figure 29. Hysteresis Control for Full Model

The errors between actual and desired amounts activate the remainder of the firing scheme only if they exceed a threshold value. If the error is larger than the hysteresis band (whose widths are indicated by $\Delta\varepsilon_{\delta}$ and $\Delta\varepsilon_{\psi}$) then a decision towards a new switching sequence is made. If the errors are within their hysteresis band, the switches will hold their current status.

The new switching position is chosen according to the current angular position of the vector at the inverter terminals defined by δ_v, and depending on the fact that we need to increase/decrease the flux magnitude or its angle. Angle changes have priority on flux magnitude changes. Once the hysteresis threshold is crossed and the direction of the next switching is selected, a lookup table will give the gating sequence for the switches.

More in detail, Figure 30 shows the inverter output only possible values, six active and one zero voltages. The inverter terminal voltage can only switch discretely between these values. To reduce δ_p the zero vector is chosen, otherwise one of the six states is chosen based on need to increase or decease the flux magnitude. A lookup table indexed by sector number and the need to increase/decrease ψ_v, provides switching logic. Table VI summarizes the choices for the active vectors. Sectors are also shown in Figure 32. Sector "I" is centered about voltage vector "1".

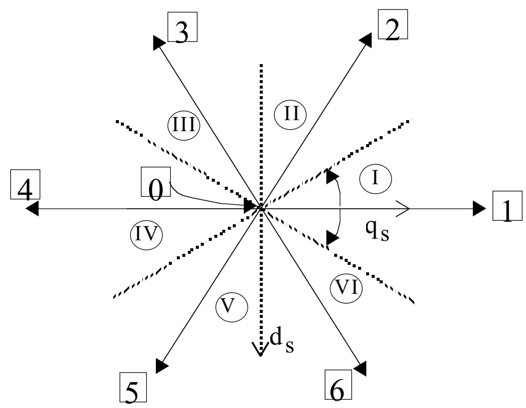


Figure 30. Inverter Output Voltages

	Sector number					
	I	II	III	IV	V	VI
Increase Ψ_v	2	3	4	5	6	1
Decrease Ψ_v	3	4	5	6	1	2

Table VI. Choice of Switching Vector

Ideal Model

Along with this full detailed model, it is possible to define a simpler phasor model for the power electronic interface. This model takes the same outputs quantities coming from the controller, and uses them to create a set of ideal voltage sources. The magnitude and phase angle of the ideal sources are created starting from the output signals of the governor as Figure 31 indicates:

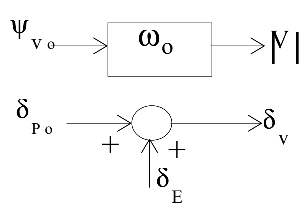


Figure 31. Ideal Inverter Model

The value that the sources have is given by:

$$V^a(t) = |V|\cos(\omega t + \delta_v)$$
$$V^b(t) = |V|\cos(\omega t + \delta_v - \frac{2}{3}\pi)$$
$$V^c(t) = |V|\cos(\omega t + \delta_v + \frac{2}{3}\pi)$$

These are line to ground values for the sources, but in reality the sources are connected to delta as Figure 32 shows.

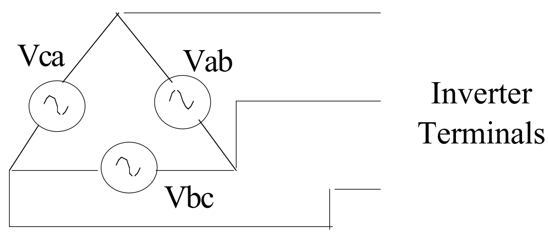


Figure 32. Ideal Inverter Model Sources

Then, the values that the actual ideal voltage sources have can be obtained from the line to ground in the following way:

$$V^{ab}(t) = V^a(t) - V^b(\mathrm{t}) = \sqrt{3}|V|\cos(\omega t + \delta_v + \frac{\pi}{6})$$
$$V^{bc}(t) = V^b(t) - V^c(t) = \sqrt{3}|V|\cos(\omega t + \delta_v - \frac{\pi}{2})$$
$$V^{ca}(t) = V^c(t) - V^a(t) = \sqrt{3}|V|\cos(\omega t + \delta_v + \frac{5}{6}\pi)$$

The line to ground sources can be used to represent the inverter, and in particular they are suited for a single phase model of the machine. If a balanced operation is assumed, then one single phase will be enough to correctly track the response of the micro-source. When representing the micro-source in single phase and in per unit values with the choice of base power and impedance as indicated in Eq. 4, then the measured power flowing into the single phase will be the three phase power. If an unbalanced mode of operation needs to be studied then only the three phase line to line model is accurate. The delta connection of the equivalent ideal sources ensures that no zero sequence current has a path to flow.

3.6 Detailed Control for Use with Ideal Inverter

Control details assume different form depending on the quantities that the unit is required to control. The case with no power-frequency droop regulation has a separate controller for the active power and voltage regulation along with the active and reactive power regulation. Then there is the case with active power and voltage regulation and a droop characteristic embedded in the controller. The following section will show details for each on of the mentioned classes of controls.

Every block will have some coefficient depending on the quantity that they are regulating. Since it is desirable to obtain a general form for the controller, a per unit system is used in the governor. The measurements are translated from the per unit value in the system base to the per unit value of the unit before being passed to the governor. This is done to be able to fit the same controller to units having different ratings. The control always assumes an input of power in per unit based on its own power ratings and an input of voltage in per unit based on the system base voltage. The base power for the unit is the prime mover maximum output power.

$$S_{B\,\text{Micro-Source}} = P_{\text{Pr}\,ime\ Mover}$$

The base power of the system was chosen in Eq. 4, then the conversion to obtain the values to pass to the controller from the measured values is the following:

$$P_{\text{governor}} = \frac{S_{\text{B system}}}{S_{\text{B Micro-Source}}} P_{\text{measured}} \qquad \text{Eq. 19}$$

Same conversion applies to the reactive power. This conversion is made necessary since the system base is chosen independently from the micro-sources, and multiple distributed resources with different ratings can have a controller with the same gains.

<u>Fuel Cell and Micro-Turbine P and E Control without Droop</u>

Figure 33 shows the details when the control is regulating active power injection and supporting the adjacent bus voltage magnitude E. Setpoints for both quantities (i.e. P^* *and* E^*) are provided externally. To regulate the voltage, the setpoint is compared with the measured voltage E and a P-I block is responsible to generate the adequate voltage magnitude V. The P-I is a block with a proportional and integral gain as indicated below:

$$P - I: \quad \mathrm{K_p} + \frac{K_i}{s}$$

The gains for the block that regulates the voltage differ from the ones that control the power.

The voltage V is limited by some minimum and maximum value. The values for these voltages can be estimated from the reactive powers that the unit may be required to inject in its normal operation. When the voltage E dips, the inverter needs to inject reactive power by rising the value of V. The maximum value for this voltage is required when injecting maximum Q and P = 0.

$$Q_{\max} = \frac{V_{\max}^2 - V_{\max}E^*}{X} \qquad \Rightarrow \qquad \mathrm{V}_{\max} = \frac{E^* \pm \sqrt{E^{*^2} + 4Q_{\max}X}}{2}$$

The condition P = 0 implies $\cos(\delta_p) = 1$ in the expression for $Q_{\max}$. The maximum voltage shall never exceed the value of the maximum voltage ratings of the inverter.

The value for $\mathrm{V}_{\min}$ is similarly found as of:

$$\mathrm{V}_{\min} = \frac{E^* \pm \sqrt{E^{*^2} + 4Q_{\min}X}}{2}$$

Where here $Q_{\min}$ stands to represent the minimum value that the unit has when operating as an inductor. Such operation may be needed to depress the voltage E swells, i.e. when E tends to be larger than its required value.

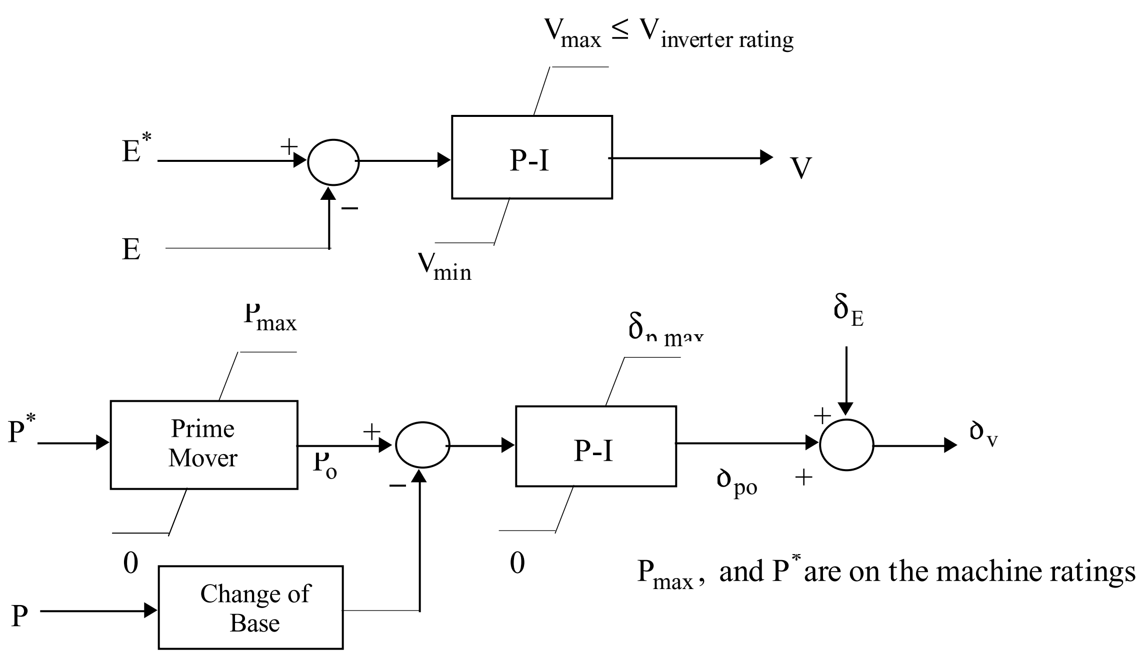


Figure 33. Detailed Control for P and E Regulation without Droop

The active power regulation differs from the fact that a step change in the command for the requested power P^* cannot be instantaneously matched by the response of the prime mover. Indeed, micro-turbine and fuel cells have different ways to ramp up their output power following a change in the power command. The prime mover rate block is responsible to keep into account of this delay. This block is justified by the fact that the governor cannot inject into the AC system power at a faster rate that the prime mover can possibly provide. The new power reference point, P_o, is bound by the fact that it cannot become negative (that would imply that the unit behaves like a load) and it cannot exceed a maximum quota. The value of this maximum power expressed in per unit of the machine base, is discussed in a following sections when describing the prime mover rate separately for the micro-turbine and fuel cell.

P_o is then compared against the measure of power and the error is fed into a P-I block. The measurement of the power is converted from the value in the system base to the micro-source base, while the conversion coefficient is given in Eq. 19. The output of the P-I block is the desired angle between the V and E voltages, δ_p. From this angle and the angle of the voltage E, it is possible to finally obtain the value for the angle of the voltage V: δ_v. The angle δ_p is limited between the values of zero and a maximum value. It is not allowed to be smaller than zero to satisfy the requirement that the unit cannot behave like an active power load, and it cannot be larger than a specified amount (usually 30 degrees) to satisfy the requirement of linearity of the P(δ_p) characteristic.

Fuel Cell and Micro-Turbine P and Q Control without Droop

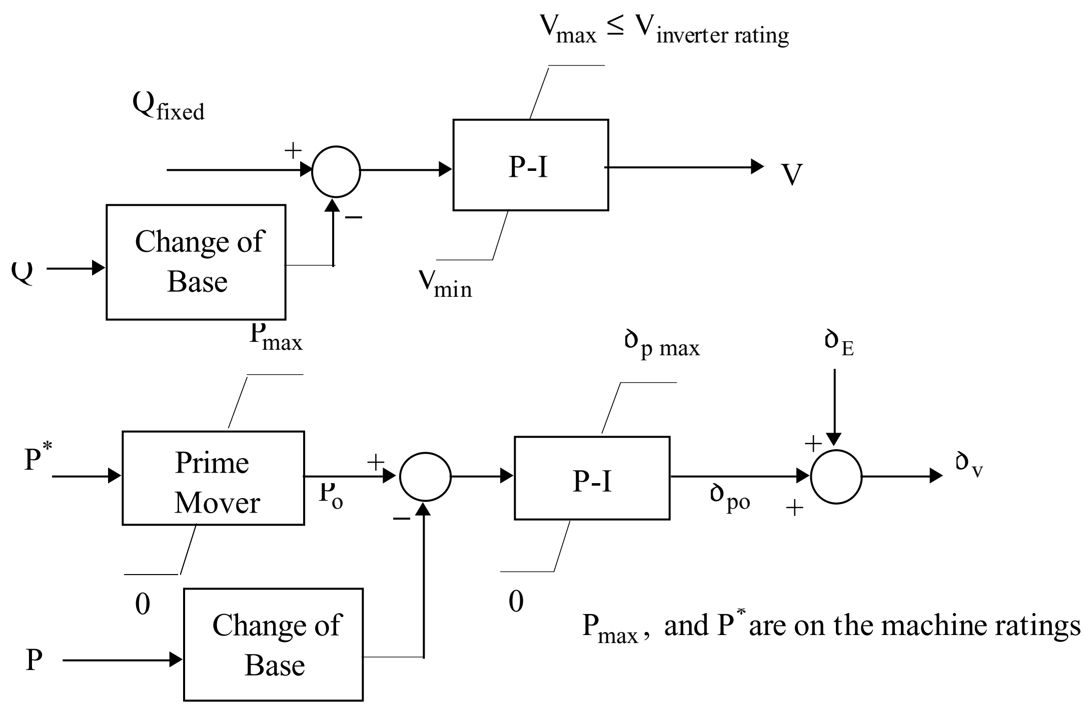


Figure 34. Detailed Control for P and Q Regulation without Droop

When the unit is regulated as a requested active power and fixed reactive power injection, then the active channel part of the controller behaves as described in Figure 33. The part of the controller responsible to regulate the reactive power to the fixed value is described in Figure 34. The fixed value is compared against the measure of reactive power and the error is fed into a P-I block to generate the adequate voltage magnitude V. The limits at which the voltage is subject are the same outlined in the earlier section.

<u>Fuel Cell and Micro-Turbine P and E Control with Droop</u>

When micro-sources operate in parallel, they use a power frequency droop to achieve the function of sharing power during island mode operation. Figure 35 shows the overall controller when a droop characteristic is enforced in the governor.

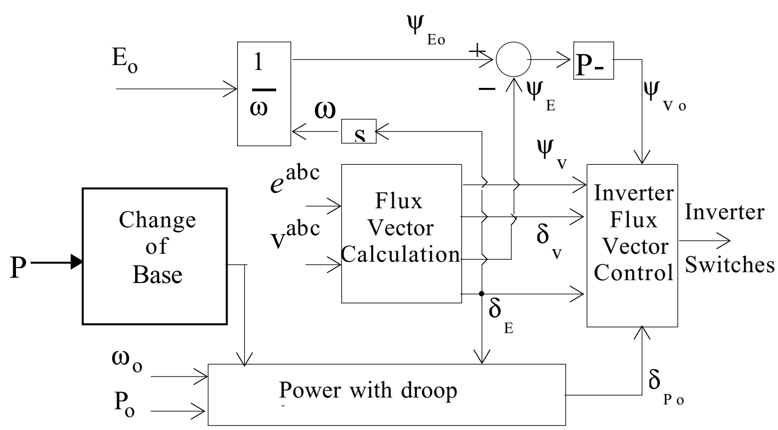


Figure 35. Detailed Control for P and E Regulation with Droop

This is an implementation of the flux vector control. The details of the calculation of the fluxes is given in Eq. 18. The droop block is responsible for automatically change the output power from the desired value anytime the frequency is free to swing, situation that takes place when the main grid connection is missing. The details for the droop block are given in Figure 36.

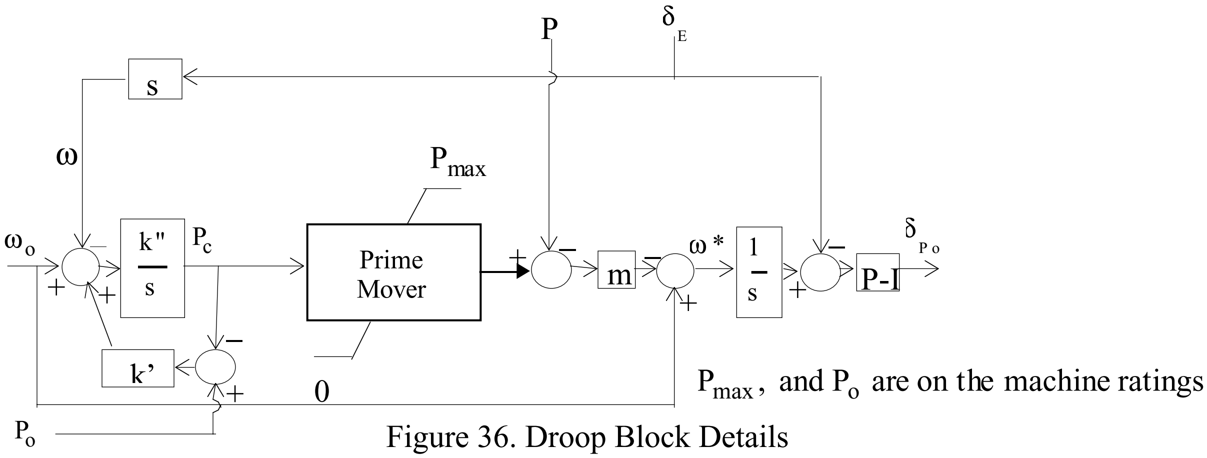


Figure 36. Droop Block Details

This block is also responsible to restore the frequency into the network at the customary value of ω_o after the power regulation has taken place. Power sharing between the units usually takes place very quickly, while frequency restoration is a process regulated by slower time constants, since we want this loop to act only after the active power outputs have reached their new steady state values. The active power injection is limited in its changes by the prime mover rate block, to make sure that the inverter does not try to inject into the network power that it is not yet available from the prime mover.

3.7 Prime Mover Model: Fuel Cell and Micro-Turbine with Large DC Storage

In this section there are the details on the prime mover rate blocks used in Figure 33 and 36. The block has to capture the fact that none of the two prime movers can track an instantaneous change in power since there is no form of storage. Should this storage exist, then it would be seen from the inverter as a very stiff DC bus voltage. In the large storage case, the DC bus can be intended as a battery that is charged by the prime mover as needed. With this form of energy storage it is reasonable to assume that we can instantaneously track the desired power. As the inverter tries to inject more power, the new quota will always be available because the energy is stored in the battery. Figure 37 shows the details on the prime mover rate block in case of large DC storage.

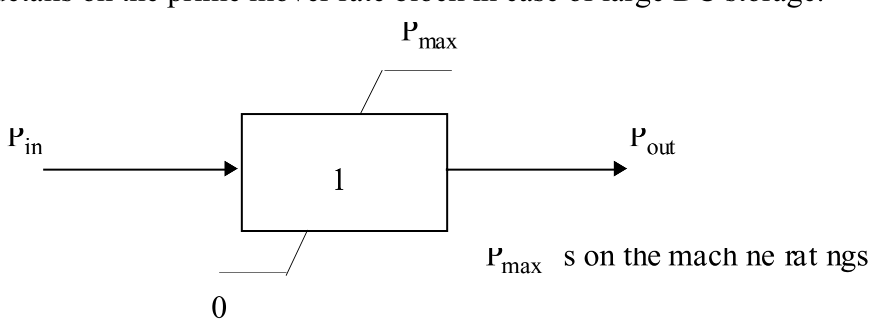


Figure 37. Prime Mover Model with Large DC Storage

The block is represented with an unitary gain that represents the ideal ability of supplying enough power as needed from the command. The block is limited by the fact that the unit cannot look like a load with the zero lower limit, and by the fact that the unit cannot exceed its maximum rated power since $P_{max} = P_{prime\ mover\ rating}$. This value is always intended to be in per unit of the machine base.

To verify the validity of the ideal model, a direct comparison with the full model is carried on. The inverter is connected to a local feeder with a load at the regulated bus, and with a line and transformer connecting with an ideal voltage source. Figure 38 gives a diagram of the circuit where the unit is installed.

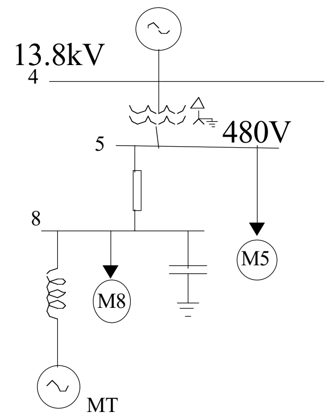


Figure 38. Test Circuit for Model Comparison

The unit is turned on, and the voltage and power reach their desired values. Both models are used for the inverter and then compared. Figure 39 shows the comparison for the active power injected at the regulated bus, while Figure 40 shows the model comparison for the regulated bus voltage magnitude. The match is excellent since responses from either model overlap.

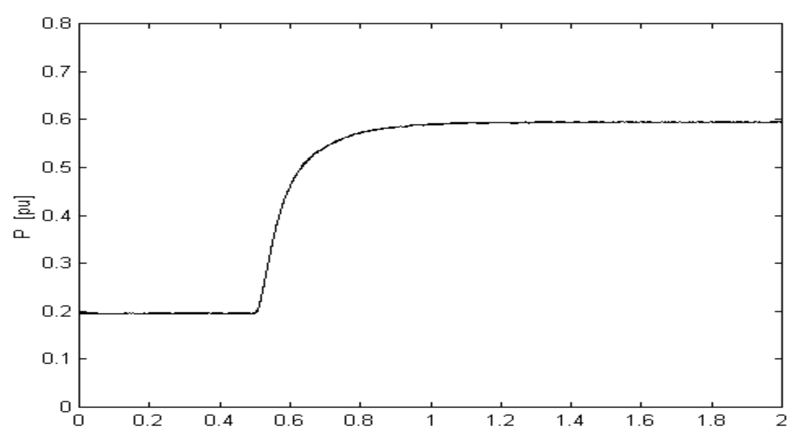


Figure 39. Output Power Model Comparison

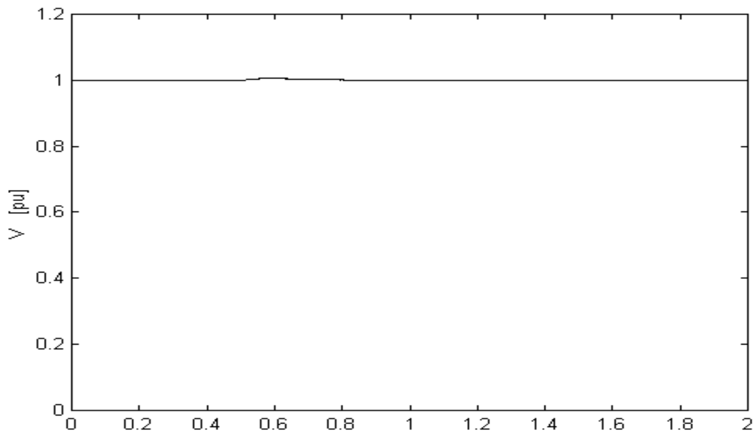


Figure 40. Regulated Bus Voltage Comparison

Figure 41 shows the inverter line to line voltages. The ideal model has a purely sinusoidal controlled ideal voltage source at the inverter terminals. Figure 42 shows the line to ground voltage at the regulated bus. Due to the small width of the hysteresis band, the voltage generated by the full model is nearly sinusoidal when seen at this bus. Figure 43 shows the line current injected by the inverter. Due to the switchings, the full model current has a high harmonic content, although it is apparent how the fundamental plays a dominant role. Finally, Figure 44 shows few cycles for the current comparison regarding the transient determined by the ramp up command given to the inverter output desired power.

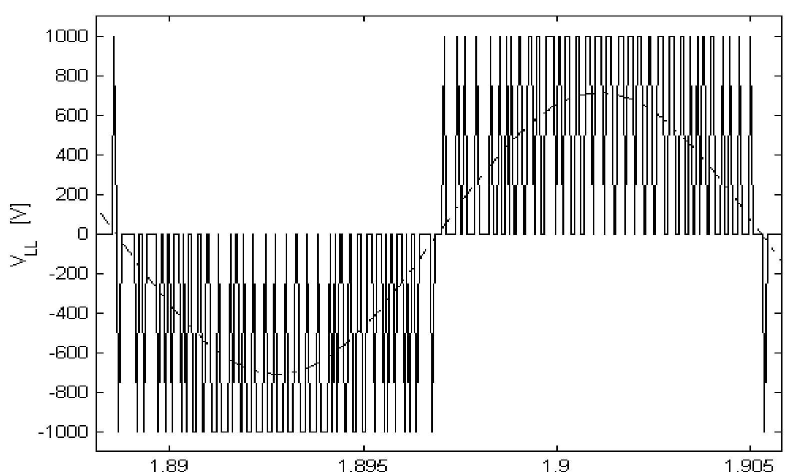


Figure 41. Line to Line Inverter Terminal Voltage Comparison

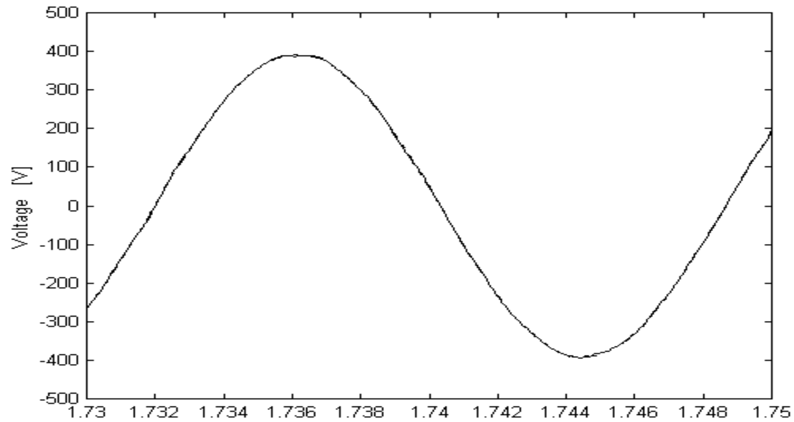


Figure 42. Line to Ground Regulated Bus Voltage Comparison

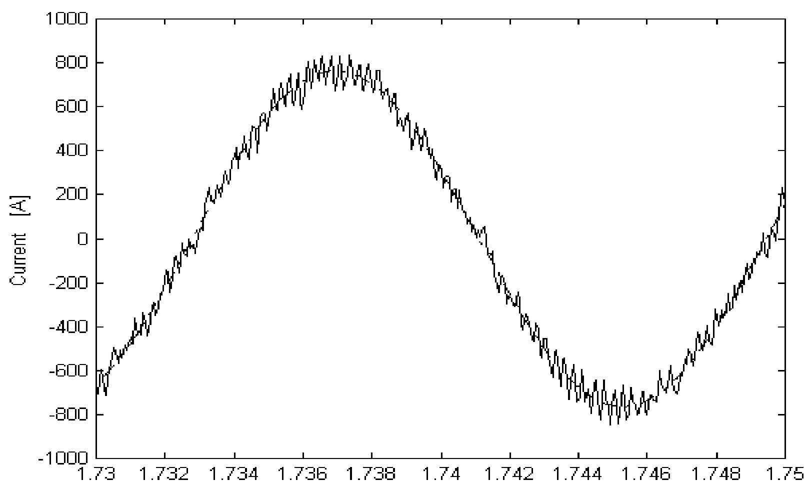


Figure 43. Inverter Output Line Current Comparison

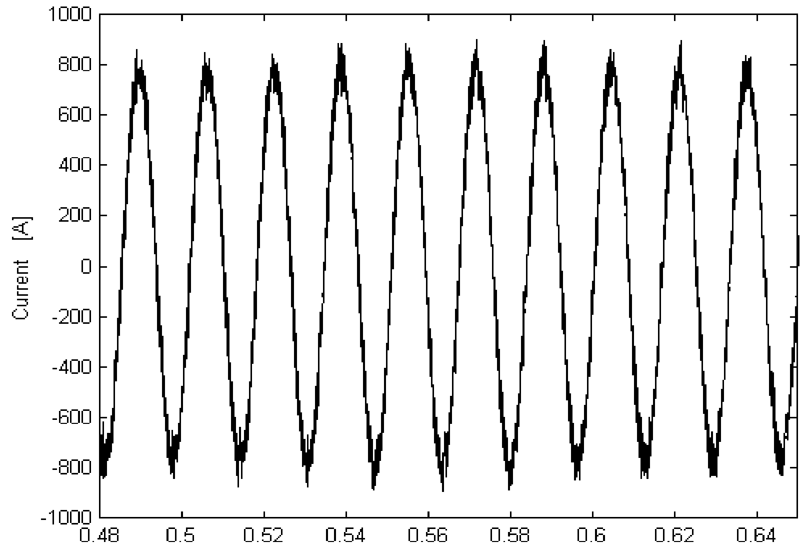


Figure 44. Current Comparison after Step Change

Prime Mover Model: Micro-Turbine without Storage

When no storage is present, the form of the prime mover rate block depends on the type of prime mover: this section deals with the micro-turbine. As described in the earlier sections, the response of a micro-turbine output power to step changes in the power request is well approximated by a time single time constant block. The micro-turbine block needs a ramp rate to model this response.

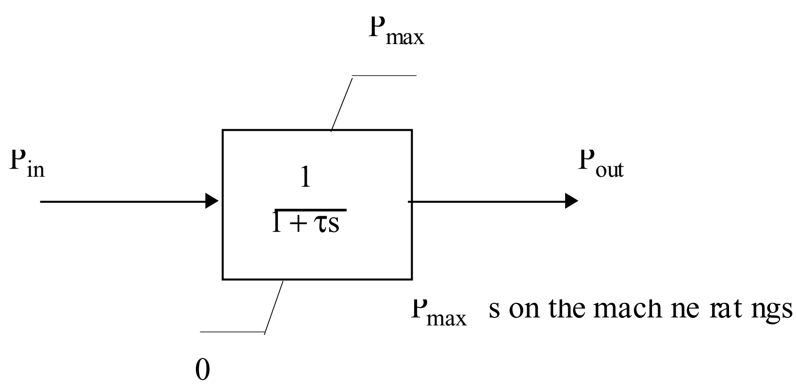


Figure 45. Micro-Turbine Model without Storage

Figure 45 explicitly gives the form of the block. The time constant is chosen as to closely reproduce the actual characteristic as of Figure 21. Figure 46 shows the inputs and outputs of the block of Figure 45.

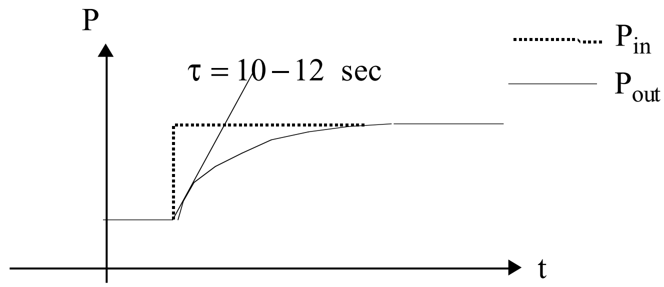


Figure 46. Micro-Turbine Response

This block is limited by the zero value and by the maximum output power that cannot exceed the ratings of the prime mover: $P_{max} = P_{prime\ mover\ rating}$.

Prime Mover Model: Fuel Cell without Storage

The fuel cell rate block has to keep into account of the actual response for this type of prime mover. Each time that there is a new power request, there is a portion of the requested power that is provided instantaneously. This portion of power that the unit can instantaneously provide is indicated with $P_s \in [0,1]$ and depends on the design of the fuel cell. Figure 47 shows that the response is almost instantaneous, while then it takes a long period of time t_τ to reach its thermal equilibrium, when it is possible to give another command for power increase.

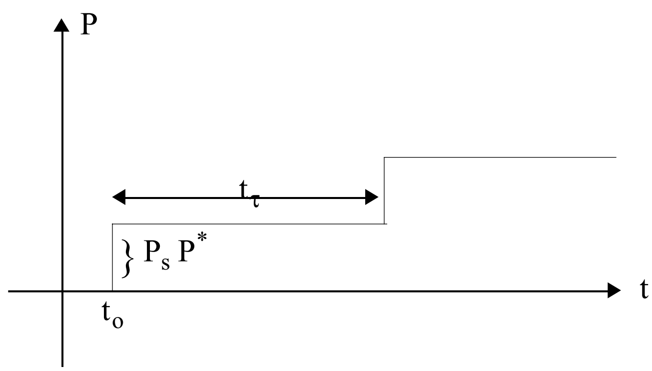


Figure 47. Fuel Cell Response

The prime mover rate block is therefore identical to the case of large storage on the DC bus, as Figure 48 shows.

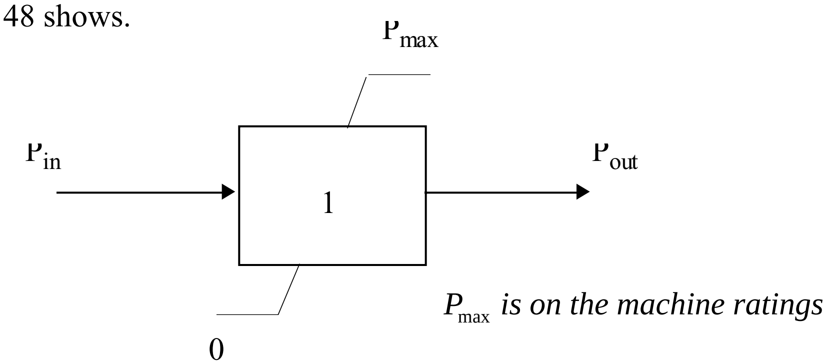


Figure 48. Fuel Cell Model without Storage

The characteristic of the block reflecting the behavior of Figure 47 are achieved by a careful enforcing of the limit of the block. Assuming that t_o is the instant in time when the first power increase command P^* is given and assuming that the power is ramped up each time when possible (i.e. as the unit reaches its next thermal equilibrium), then the maximum power requirement can be derived from the following set of equations:

$$
\begin{array}{ll}
t_o \le t \le t_o + t_\tau & P_{max} = P_s\, P^* \\
t_o + t_\tau \le t \le t_o + 2\, t_\tau & P_{max} = 2\, P_s\, P^* \\
\quad \ldots & \quad \ldots \\
t_o + (n-1)\, t_\tau \le t \le t_o + n\, t_\tau & P_{max} = n\, P_s\, P^*
\end{array}
$$

In all cases the condition $P_{\max} \le P_{prime\ \mathrm{mover\ rating}}$ overrides the definition of $P_{\max}$ as given in the above expressions.

4. Induction Machine Models

Overview

Induction machines represent large quotas of power consumption in many manufacturing plants. It is not uncommon to have many machines operating in electrical proximity of each other in a local grid. When analyzing a micro-grid with distributed resources, it is very important to be able to model the behavior of the induction machines for both steady state and dynamic studies in order to include their non linear behavior. The mechanical load is modeled as a constant torque. In the following section steady state and slow dynamics models of the induction machine will be described. The steady state model can be either balanced or unbalanced and is developed under the assumption that the surrounding network can have either a phase or a sequence representation. The dynamic model is suited only for slow dynamics. Both single phase, balanced voltages, and three phase general models are described.

4.1 Induction Machine Steady State Model

In the steady state machine model the electrical and mechanical constraints are taken into account, and the full coupling between the phases of the machine is considered. The electrical relations consider the constraints that the external network places on the terminal quantities such as voltages and currents. Inside the machine, the torque equation will constrain mechanical and electrical quantities.

For modeling purposes, we considered two distinct approaches for the steady state representation of the induction machine. Both models have a common backbone structure, while they differ in the way the equations are laid out. The common structure comes from seeing the machine as a combination of sequence domain circuits. The first model retains the sequence structure in the variable layout, and also the network connected to the machine is to be intended to be represented in sequence domain. The second model deals with sequence quantities only as the basis to develop the model, but ultimately variables and equations are all expressed in a phase domain and the external network to the machine has to be represented with its phase model.

4.2 Model in Sequence domain

When representing the machine, we look at its sequence domain circuits. That is, we transform the three phase voltages at the terminals of the machine in its sequence components, and electric equations are determined as a consequence of this choice. The reason why we considered this model is because it embodies the full three phase model of the machine including mutual coupling, but still preserving the easiness of study of three distinctly decoupled circuits. The three sequence circuit indeed are decoupled one from the other and any sequence may be solved independently from the others.

We made the choice to represent the network, described in a following section, also in sequence components. This implies that we never have to convert during the computation back and forth between phase and sequence representation of the electrical equations, since only sequence domain analysis is carried on. Phase domain conversion may be performed once the steady state solution of the problem is obtained, and a change of output format may be carried on.

First of all, we need to create the sequence circuits for the induction machine. The positive sequence circuit is identical to the phase circuit. The negative sequence circuit is substantially identical to the positive sequence. The only difference comes from the slip. The magnetic rotating field created by the rotor quantities is seen by the negative sequence quantities impressed in the rotor as rotating twice the speed and in the opposite direction: therefore the negative slip will be:

$$s_n = 2 - s \qquad \text{Eq. 20}$$

The zero sequence circuit is an open circuit. Indeed, the induction machine centerpoint is left floating, and not grounded, which means that the terminal currents have always to sum up to zero, which translates in a null current for the zero sequence at all times. The only reason why there may be some zero sequence impedance is a fault inside the machine. In our analysis the machine is supposed to be in proper conditions at all the times. Figure 49 summarizes the three circuits.

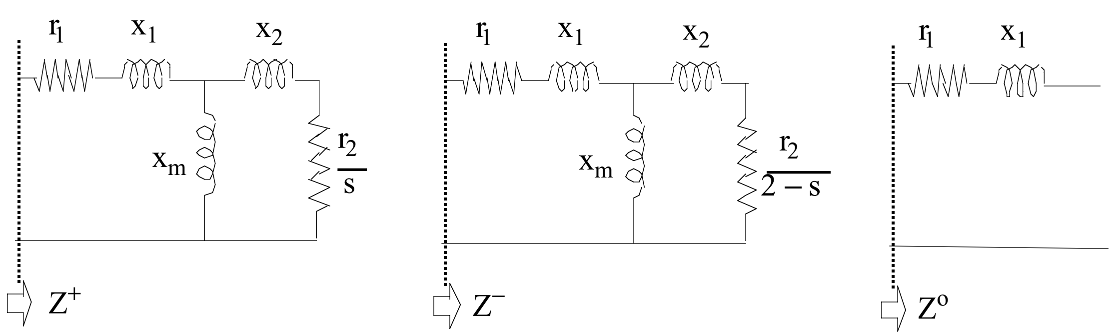


Figure 49. Sequence Circuits for the Induction Machine

Defining the sequence admittance as:

$$Y_{IM}^{+-o} = \begin{bmatrix} \frac{1}{Z^+} & 0 & 0 \\ 0 & \frac{1}{Z^-} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{Eq. 21}$$

Then it is possible to write a matrix relationship between sequence components voltage and current:

$$Y_{IM}^{+-o} V_t^{+-o} = I^{+-o} \qquad \text{Eq. 22}$$

Once transformed back to phase quantities the current I^{+-o} will represent the phase currents at the machine terminals.

Torque Model

Looking at the circuit for the rotor as shown in Figure 50, it is possible to see how the resistance term can be split in two components, one directly connected to the winding physical resistance, the other related to the energy conversion term.

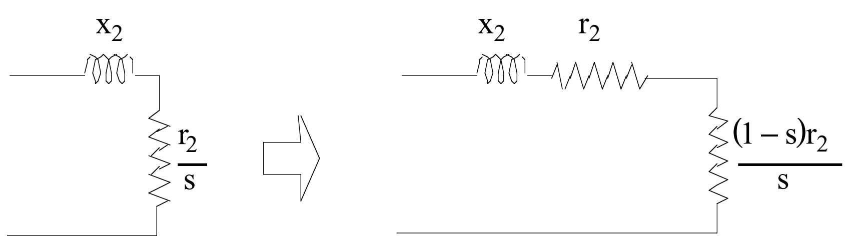


Figure 50. Rotor Circuit Models for the Induction Machine

The power in the positive sequence resistance responsible for the mechanical conversion of the energy is:

$$P^{+} = \frac{3r_2(1-s)}{s}\left|I_{Rot}^{+}\right|^2 \qquad \text{Eq.23}$$

The torque is easily expressed by dividing the power by the rotational speed of the shaft, ω_m:

$$T^{+} = \frac{P^{+}}{\omega_m} \qquad \text{Eq.24}$$

In turn, the mechanical speed at which the squirrel cage rotates, ω_m, can be expressed as a function of the speed of the electrical quantities of the network, ω_e, and the number of poles p, and the slip in the following relation:

$$\omega_m = \omega_e \frac{(1-s)}{\left(\frac{p}{2}\right)} \qquad \text{Eq. 25}$$

The torque equation can be rewritten at the light of the above expression as:

$$T^+ = \frac{3\,\mathrm{p}\,r_2}{2\omega_e}\frac{\left|I_{Rot}^+\right|^2}{s} \qquad \text{Eq. 26}$$

The negative sequence torque is very similar to this expression, with the only change being in the current component and the slip. Negative sequence quantities generate a torque that tends to slow down the machine, therefore it appears with a negative sign in front of the overall torque calculation:

$$T^- = \frac{3\,\mathrm{p}\,r_2}{2\omega_e}\frac{\left|I_{Rot}^-\right|^2}{2-s} \qquad \text{Eq. 27}$$

$$T = T^+ - T^- \qquad \text{Eq. 28}$$

Equation 28 is the other equation that defines the induction machine. This equation connects the electrical and mechanical behavior of the machine. Notice that the currents that appear in the torque equation are not the sequence currents at the machine terminals, but rather the sequence current in the rotor circuits. Looking at Figure 49 it is not hard to see that the sequence rotor currents can be calculated starting from the terminal sequence current by means of a current partitioner:

$$\begin{bmatrix} I_{Rot}^+ \\ I_{Rot}^- \end{bmatrix} = \begin{bmatrix} \dfrac{jx_m}{\dfrac{r_2}{s} + j(x_2 + x_m)} & 0 \\ 0 & \dfrac{jx_m}{\dfrac{r_2}{2-s} + j(x_2 + x_m)} \end{bmatrix} \begin{bmatrix} I^+ \\ I^- \end{bmatrix} = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} I^+ \\ I^- \end{bmatrix} \qquad \text{Eq. 29}$$

Notice also that since there is no zero sequence current, $T^o = 0$ at all times. Therefore, the electrical and mechanical interaction is completely defined with the sets of equations 21, 22, 26 ,27, 28 and 29.

Before looking at a more general case, when a machine is connected to an external network, we summarize the complete set of electrical and mechanical equations that define the induction machine.

$$Y_{IM}^{+-o} V_t^{+-o} = I^{+-o}$$

$$\begin{bmatrix} I_{Rot}^{+} \\ I_{Rot}^{-} \end{bmatrix} = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} I^{+} \\ I^{-} \end{bmatrix}$$

$$T = \frac{3\,p\,r_2}{2\omega_e} \left(\frac{\left|I_{Rot}^{+}\right|^2}{s} - \frac{\left|I_{Rot}^{-}\right|^2}{2-s} \right) = T_L$$

Inclusion of an External Network

In general, the source feeding the circuit is not located exactly at the terminals of the induction machine, but usually is somewhere in the network. To make the problem a little bit more general, we added an electrical network between the ideal sources and the machine terminals as Figure 51 points out:

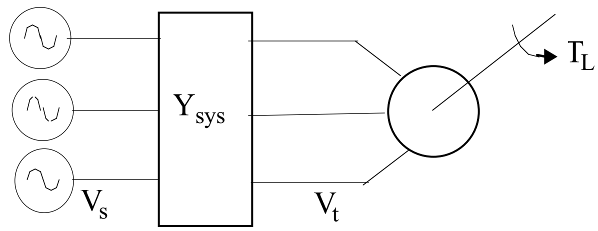


Figure 51. Induction Machine Connected to an External Network

The network impedance can be a Thevenin admittance matrix with the self terms on the diagonal and the mutual terms off diagonal. At this point we have two choices: the first is to transform the quantities of the machine, as of Eq. 22 in phase quantities and retain the network as it is. The other choice is to convert the network into its sequence domain representation and directly interface its quantities with the ones already available from the machine analysis. This would be the same system model as used in short circuit analysis.. We are aware that those calculations are performed already in the sequence domain, therefore a set of equations that describes the network in terms of sequence quantities is a direct way to represent the steady state behavior of the induction machine.

The network as of Figure 51 can be converted to sequence domain by the following simple considerations. First of all, let's define α the complex operator that rotates of 120 degrees counterclock wise any vector in the complex plane:

$$\alpha = e^{j\frac{2\pi}{3}} \qquad \text{Eq. 30}$$

Then we can define the transformation matrix T such that it is possible to covert from sequence to phase quantities (and vice versa):

$$T = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix}$$

Eq. 31

$$\begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} = T \begin{bmatrix} X^+ \\ X^- \\ X^o \end{bmatrix}$$

Here, X represents a generic electric quantity, such as voltage or current. At this point, it is possible to write for the network:

$$\mathrm{Y_{sys}}\left(\mathrm{V_s^{abc}} - \mathrm{V_t^{abc}}\right) = \mathrm{I^{abc}}$$

and in sequence components:

$$\mathrm{Y_{sys}^{+-o}}\left(\mathrm{V_s^{+-0}} - \mathrm{V_t^{+-o}}\right) = \mathrm{I^{+-o}} \qquad \text{Eq. 32}$$

where:

$$\mathrm{Y_{sys}^{+-o}} = \mathrm{T^{-1}\, Y_{sys}\, T}$$

Here, the term I^{+-o} represents the same currents already encountered in Eq. 22, and V_t^{+-o} is the sequence voltage representation of the terminal voltages of the machine, as of Eq. 3 and V_s^{abc} are the phase domain (the format they usually are available) representation of the ideal source voltages.

The simultaneous solution of the system

So far, for the system described in Figure 51, we have a total of nine equations and nine unknowns. Before proceeding in the description of the solution of the set of equations, we summarize the constraints and the unknowns below. First the constraints:

$$Y_{sys}^{+-o}\left(V_s^{+-o}-V_t^{+-o}\right)=I^{+-o} \qquad \text{3 constraints}$$

$$Y_{IM}^{+-o}V_t^{+-o}=I^{+-o} \qquad \text{3 constraints}$$

$$\begin{bmatrix} I_{Rot}^{+} \\ I_{Rot}^{-} \end{bmatrix}=\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}\begin{bmatrix} I^{+} \\ I^{-} \end{bmatrix} \qquad \text{2 constraints} \qquad \text{Eq .33}$$

$$T_L=\frac{3\,p\,r_2}{2\omega_e}\left(\frac{\left|I_{Rot}^{+}\right|^2}{s}-\frac{\left|I_{Rot}^{-}\right|^2}{2-s}\right) \qquad \text{1 constraint}$$

The last equation comes from the consideration that we are in steady state, and therefore the machine torque must be balanced by the load torque to have a net accelerating torque that is null.

The unknowns are as follows:

$$\mathrm{V_t^+},\mathrm{V_t^-},\mathrm{V_t^o},\ \mathrm{I^+},\mathrm{I^-},\mathrm{I^o},\mathrm{I_{Rot}^+},\mathrm{I_{Rot}^-},\mathrm{s} \qquad \text{Eq. 34}$$

The non linear system of equations so far described lends itself to be solved using the Newton-Raphson iterative method. This algorithm allows to find the values of unknown terms that simultaneously force to zero a particular set of equations. A starting guess is used to generate subsequent solutions, and convergence is reached when two subsequent solutions differ by an amount that is smaller than a specified tolerance. Since all quantities have to be real numbers, it is convenient to split the complex numbers representing voltages or currents as magnitudes and angles. We define the vector of the unknown quantities as:

$$x=\left[\left|V_t^+\right|,\left|V_t^-\right|,\left|V_t^o\right|,\delta_t^+,\delta_t^-,\delta_t^o,\left|I^+\right|,\left|I^-\right|,\left|I^o\right|,\delta_i^+,\delta_i^-,\delta_i^o,\left|I_{Rot}^+\right|,\left|I_{Rot}^-\right|,\delta_{Rot}^+,\delta_{Rot}^-,s\right]^T \qquad \text{Eq. 35}$$

The equations that represents the constraints need also to be real, and here the choice has been to consider real and imaginary part for each of the complex equations. The resulting system of equations is:

$$
\begin{aligned}
g_{1,2,3} &= Re\left\{Y_{sys}^{+-o}\left(V_s^{+-o} - V_t^{+-o}\right) - I^{+-o} = 0\right\} \\
g_{4,5,6} &= Im\left\{Y_{sys}^{+-o}\left(V_s^{+-o} - V_t^{+-o}\right) - I^{+-o} = 0\right\} \\
g_{7,8,9} &= Re\left\{Y_{IM}^{+-o} V_t^{+-o} - I^{+-o} = 0\right\} \\
g_{10,11,12} &= Im\left\{Y_{IM}^{+-o} V_t^{+-o} - I^{+-o} = 0\right\} \\
g_{13,14} &= Re\left\{I_{Rot}^{+-}\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} - I^{+-} = 0\right\} \\
g_{15,16} &= Im\left\{I_{Rot}^{+-}\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} - I^{+-} = 0\right\} \\
g_{17} &= \frac{3\,p\,r_2}{2\omega_e}\left(\frac{\left|I_{Rot}^{+}\right|^2}{s} - \frac{\left|I_{Rot}^{-}\right|^2}{2-s}\right) - T_L = 0
\end{aligned}
\qquad \text{Eq. 36}
$$

The iterative method generates the next solution with the following scheme:

$$
x(k+1) = x(k) - \left[J_g\big|_{x(k)}\right]^{-1} g\big(x(k)\big) \qquad \text{Eq. 37}
$$

That is, the jacobian of the system of equations is required. By itself the jacobian is defined as a matrix whose entries are the derivatives of all the equations of Eq. 36 taken with respect to all the variables included in the unknown vector of Eq. 35.

$$
J_g = \begin{bmatrix} \dfrac{\partial g_1}{\partial x_1} & \cdots & \dfrac{\partial g_1}{\partial x_{17}} \\ \cdots & & \cdots \\ \dfrac{\partial g_{17}}{\partial x_1} & \cdots & \dfrac{\partial g_{17}}{\partial x_{17}} \end{bmatrix} \qquad \text{Eq. 38}
$$

The derivatives of the g functions with respect the set of parameters of Eq. 35 are straightforward, where only the derivatives with respect to the slip variable of the terms $Z^{+}(s)$ and $Z^{-}(s)$ that appear in Y_{IM}^{+-o} through Eq. 21 may require some attention. Same care must be made when taking the derivative with respect to s of the partitioning terms K_1 and K_2 appearing in Eq. 36

With this setup, it is possible now to solve for the system of equations. The value of the load torque is assumed to be the value that the torque has at rated conditions. That is, rated voltage and rated slip. The rated slip is taken as being equal to the rotor resistance when expressed in per unit of the machine ratings.

4.3 Model in Phase domain

The machine is always internally represented with its sequence domain circuits, as of Figure 51, and consequently, the electrical quantities are correlated with the machine admittance of Eq. 21. The variables that represent the steady state behavior of the machine, are now phase quantities. Now terminal voltages and currents are represented in the phase domain and the equations that correlates them have to change accordingly. The admittance matrix has to be transformed from sequence to phase domain and according to the definition of transformation matrix as given in Eq. 31, the new admittance matrix can be found to be:

$$\mathrm{Y_{IM}^{abc}} = \mathrm{T\,Y_{IM}^{+-o}\,T^{-1}} \qquad \text{Eq. 39}$$

Now we can write the main equation that relates the terminal voltage and currents of the machine as:

$$\mathrm{Y_{IM}^{abc} V_t^{abc}} = \mathrm{I^{abc}} \qquad \text{Eq. 40}$$

This admittance matrix satisfies the basic behavior of the induction machine, that is, even when the phase voltages have a non zero sequence, the phase currents always sum up to zero. This is a consequence of the fact that the zero sequence admittance is worth zero, where this is due to the fact that the neutral point of the induction machine is not grounded, requiring the phase currents to sum up to zero under any kind of operation.

Torque Model

The torque equation is identical to the one of the previous model, but the equation that allows to obtain the rotor sequence currents, needed in the torque equation, has to be changed. In the previous model, Eq. 29 allows to calculate the rotor quantities starting from the already available terminal sequence quantities. Here the terminal quantities are available only in phase format. Therefore we need to transform first the terminal quantities in sequence domain, and then feed them in Eq. 29. It turns out that it is possible to merge these two operations in a single expression:

$$\begin{bmatrix} I_{Rot}^{+} \\ I_{Rot}^{-} \end{bmatrix} = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} I^{+} \\ I^{-} \end{bmatrix} = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} T_p \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad \text{Eq. 41}$$

Where the matrix T_p is a partition of the inverse of the transformation matrix T. More precisely the partition extracts the first two rows of the inverse of T. This is because we only need the positive and negative sequence of the terminal currents to calculate the corresponding rotor currents. More in particular, we have:

$$T_p = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \qquad \text{Eq. 42}$$

With this in mind, it is possible to summarize the electrical and mechanical equations for this model of the machine, where the quantities used to describe the steady state behavior are in phase domain:

$$Y_{IM}^{abc}\begin{bmatrix} V_s^a \\ V_s^b \\ V_s^c \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_{Rot}^{+} \\ I_{Rot}^{-} \end{bmatrix} = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} T_p \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad \text{Eq. 43}$$

$$T = \frac{3\,p\,r_2}{2\omega_e}\left(\frac{\left|I_{Rot}^{+}\right|^2}{s} - \frac{\left|I_{Rot}^{-}\right|^2}{2-s}\right) = T_L$$

Inclusion of an External Network

The inclusion of an external network is simplified using this model. Here the readily available admittance matrix of the system in phase quantities is already suited to represent the behavior of the network when interfaced with the quantities that describe the machine. The resulting network constraining equation is:

$$Y_{sys}^{abc}\left(\begin{bmatrix} V_s^a \\ V_s^b \\ V_s^c \end{bmatrix} - \begin{bmatrix} V_t^a \\ V_t^b \\ V_t^c \end{bmatrix}\right) = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad \text{Eq. 44}$$

In this equation, the admittance matrix is the inverse of the impedance matrix that appears in Figure 51.

The simultaneous solution of the system

The overall system equations are given by the set of Eq. 43 and of Eq. 44. The system of equations forms a consistent set with the unknown terms. Expressed as complex quantities, the variables that we need to solve for are:

$$\mathrm{V_t^a},\ \mathrm{V_{t,}^b}\ \mathrm{V_t^c},\ \mathrm{I_a},\ \mathrm{I_b},\ \mathrm{I_c},\ \mathrm{I_{Rot}^{+}},\ \mathrm{I_{Rot}^{-}},\ \mathrm{s} \qquad \text{Eq. 45}$$

We have already seen that the equations and the variables have to appear inside the Newton-Raphson iterative method as real quantities. All quantities of Eq. 45 but the slip need to be converted in magnitude an phase, obtaining two real variables from one complex. In total we will have 16 real variables. All the equations, besides the torque expression, are to be divided into real and imaginary part, ending up with 16 real constraints that can now be easily fit into the algorithm.

4.4 General Expression of Machine Parameters

The choice of the base quantities for the induction machine uses the rms of the rated phase voltage for the base voltage and the rated output mechanical power as an output. This is a classical choice for induction machine. For other types of machines, the choice may be different. With the base quantities for the induction machine as described, the rated current will exceed the one per unit value because the base current does not incorporate the effects of power factor or efficiency since it is based on a base power that is the active power delivered to the shaft.

The value for the parameters as used in this section are relative to the ratings of the induction machine. Therefore when interfacing the quantities from the motor with the network per unit representation it is necessary to perform a conversion like Eq. 19 for the micro-sources, changing the base from the system units to the induction machine base.

With the choice of per unit parameters done in this base, it is possible to make reasonable guesses on the machine pu parameters once its size is known. Particularly, a work in the literature [13] describes how to obtain the physical relationship between machine rating and pu parameters. We will report here only the main outline of the derivation process.

The most important requirement on the per unit parameters of an induction machine is on the leakage reactance. For induction machines where primary and secondary structure is quite similar the leakage is usually equally divided between the two: $x_1 = x_2$. Since the per unit pull out (peak) torque can be simplified to:

$$T_{pk} \approx \frac{V^2}{2\left(x_1 + x_2\right)} \qquad \text{Eq. 46}$$

then, to obtain a peak torque in the normally required range of 2-2.5 pu requires a value of $x_1 + x_2$ around 0.2-0.25 pu regardless the size of the machine. This means that as other parameters such as r_1, r_2, x_m will change with the machine ratings, the leakage will remain constant, due to design considerations. The way manufacturers use to maintain leakage constant can be understood by looking at the following expression for the per unit slot leakage inductance:

$$x_{1s} = \frac{\left(\frac{x_s}{l}\right)}{\mathrm{K}_{\mathrm{W}}^2 N\left(\tau_P\right)^{\frac{1}{2}}} \qquad \text{Eq. 47}$$

where:

x_s is the leakage reactance of a slot with one only conductor filling the slot

l is the length of the slot

K_W is the winding factor that is about constant at any machine size

$N = \frac{Ns}{2\pi r}$ number of slots per unit of circumferential length

$\tau_P = \frac{\pi r}{\left(\frac{P}{2}\right)}$ pole pitch, depends on size and on pole number

If everything were to be maintained constant in the design of the machine, as the size increases, the leakage in pu would become smaller. In order to keep it constant one could play with the $\frac{x_s}{l}$ (slot reactance per unit length), but for a variety or reasons such as slot fill factor, tooth width / slot width ratio and structural factors, it is not practicable to vary this ratio very much. The control of the leakage is primarily in terms of N, the number of slots per unit of length. It can be proven that by enforcing:

$$N = 25\,(\tau_P)^{-\frac{1}{2}} \qquad \text{Eq. 48}$$

then the total leakage is locked to 0.2 per unit regardless the size of the machine. The slots per pole are given by:

$$N\,\tau_P = 25\,(\tau_P)^{\frac{1}{2}} \qquad \text{Eq. 49}$$

which translates in more slots per pole in larger machines.

Now it is possible to obtain the values of the stator and rotor resistances in per unit as the size of the machine changes. We have:

$$\begin{aligned} r_1 &= 0.0033\,(\tau_P)^{-1} \\ r_2 &= 0.004\,(\tau_P)^{-1} \end{aligned} \qquad \text{Eq. 50}$$

The reason why r_2 is larger than r_1 comes from the compromise that one has to make between having a very small r_2, and high efficiency, and a big r_2 and large starting torques. The value of r_1 is purely parasitic and therefore is kept as small as possible at all times.

The value of the magnetizing inductance should be as large as possible to reduce the magnetizing current requirements. Inserting the design constraining relation of Eq. 48 in the general expression of the magnetizing inductance, it is possible to obtain:

$$x_m = 10\left(\frac{\tau_P}{\left(\frac{P}{2}\right)}\right)^{\frac{1}{2}} \qquad \text{Eq. 51}$$

In turn, through the expression of the air gap power per unit of area it is possible to relate the pole pitch to the size of the machine in terms of horse power and its pole number:

$$\tau_P = 0.095\left(\frac{HP}{\left(\frac{P}{2}\right)^2}\right)^{\frac{6}{23}} \qquad \text{Eq. 52}$$

Plotting the following equations for a range of powers spanning from 1 to 1000HP, it is possible to obtain the following figures. Figure 52 shows the values of the stator and rotor resistance in per unit versus the size of the machine (in logarithmic scale). Figure 53 shows the magnetizing inductance versus the size. For all sizes, $x_1 = x_2 = 0.1$ pu by construction.

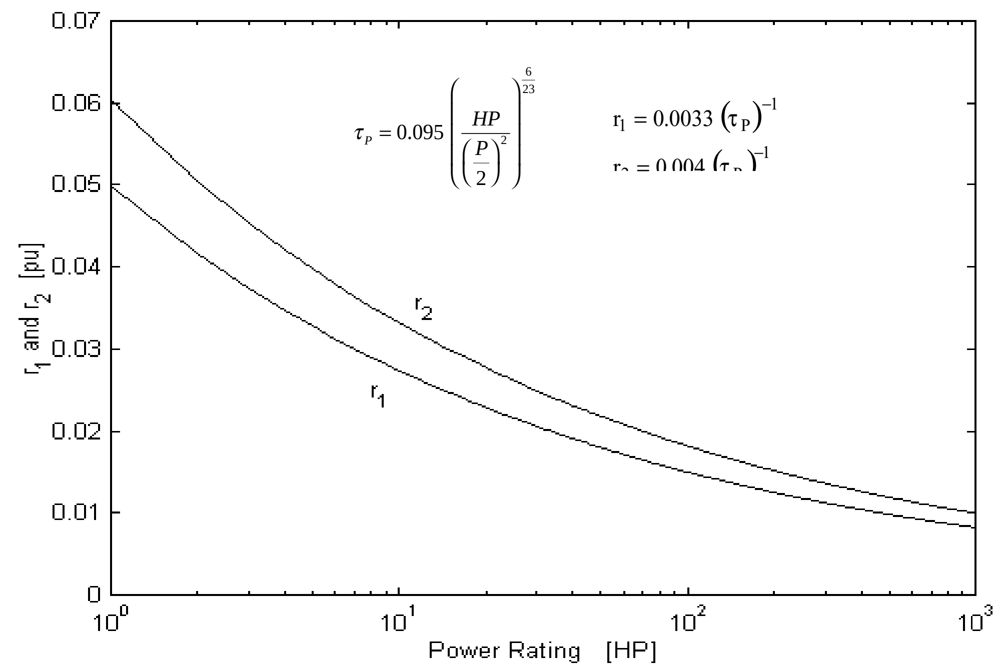


Figure 52. Per Unit Values of Resistances Versus Machine Size

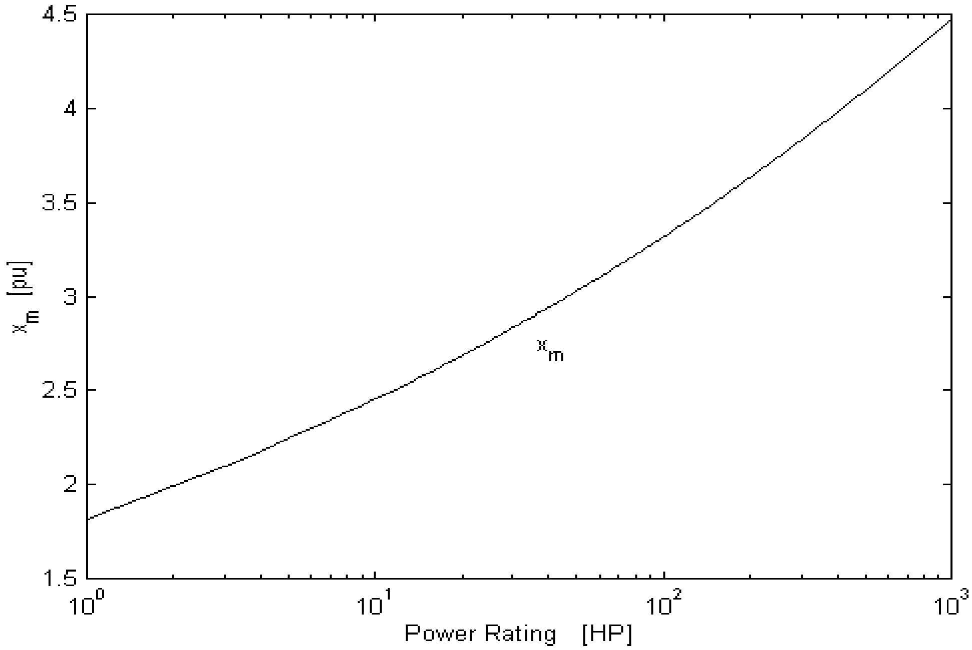


Figure 53. Per Unit Magnetizing Inductance Versus Machine Size

4.5 Induction Machine Slow Dynamics Single Phase Balanced Model

This section describes the model of an induction machine that is operating under balanced mode. The model only describes the machine when used in slow dynamics studies, but it captures the non linear behavior of active and reactive power adsorbed from the network as a function of given mechanical load and network-dependent AC bus voltage.

In a local network where induction machines are present, micro-sources will have dynamics that are slow compared to the induction machine transients. Slow dynamics study means that what we are studying (i.e. the micro-turbine response) is slower compared to the speed of the network transient. Under this assumption, the AC system can be seen as if moving from a steady state point to another. The induction machine as modeled in the following sections lends to this slow dynamics study. Indeed, the machine is represented with an electric circuit, where the mechanical constraint is enforced with the introduction of the slip variable.

For this purpose we suggest to represent the machine not as a mere P, Q load, but rather with its detailed circuit model shown in Figure 54. We recall that from electric machine theory, an induction machine can be represented in the following way:

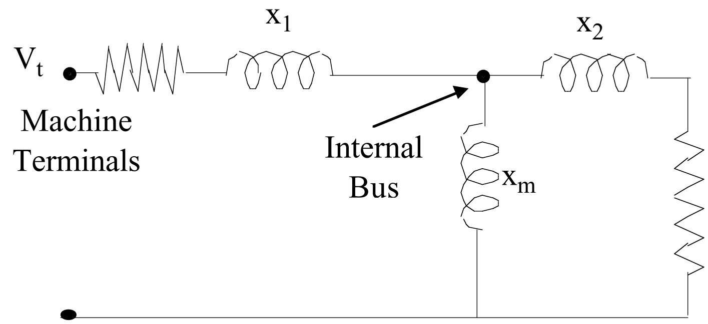


Figure 54. Induction Machine One Line Diagram

The terminal bus is the point at which the machine is physically attached to the AC network , where internal bus is a fictitious bus. r_1, x_1 represent stator active and reactive losses, x_m represents the magnetizing inductance, while terms $\frac{r_2}{s}, x_2$ represent the total active and reactive power delivered to the rotor. Notice that s is the slip of the machine, and will serve as a state variable in dynamic analysis. With this setup, we see that the internal bus gets treated as a P, Q bus with both terms equal to zero. Now we still have to specify how we make sure that the amount of power converted to mechanical form drawn from the terminals can be specified as an external input. When active power demand has been set, we can equate that amount to the active (mechanic) power drawn by the circuit representing the motor, that is

$$P_{mech} = T_{mech}\,\omega_m = P^+ = \frac{3r_2(1-s)}{s}\left|I^+_{Rot}\right|^2 = \frac{3r_2(1-s)}{s} I^+_{Rot}\left(I^+_{Rot}\right)^* \qquad \text{Eq. 53}$$

That is, since only balanced mode is assumed, the active power adsorbed by the circuit rotors is equal to the positive sequence power as derived in Eq. 26. In this equation, it is possible to identify the positive sequence rotor current from Eq. 29 as:

$$I^+_{Rot} = \frac{jx_m}{\frac{r_2}{s} + j(x_2 + x_m)} I^+ = \frac{jx_m}{\frac{r_2}{s} + j(x_2 + x_m)} I^a$$

From the circuit of Figure 54, defining V_t as the complex voltage at the machine terminals, it is possible to obtain:

$$I_a = \left[r_1 + jx_1 + \left(jx_m // \left(\frac{r_2}{s} + jx_2\right)\right)\right]^{-1} V_t \qquad \text{where } V_t = |V_t| e^{j\delta_t}$$

Therefore:

$$I_{Rot}^{+} = \frac{jx_m}{\frac{r_2}{s} + j(x_2 + x_m)} \left[r_1 + jx_1 + \left(jx_m // \left(\frac{r_2}{s} + jx_2 \right) \right) \right]^{-1} V_t \qquad \text{Eq. 54}$$

It is possible to obtain the torque constraint by substituting Eq. 54 in Eq. 53. Solved for the slip, the equation is of the form $s = h(T_{mech}, V_t)$ and may yield three different kind of solutions: 2 independent real solutions, 2 coincident real solutions, 2 complex conjugate solutions. Last condition is met when power demand at the shaft is above the amount the motor can supply, while the second is just a particular case of the first. When obtaining two real solutions it is important to recognize which of them represents a stable steady state operating point. From machine theory, a graphical display illustrates that only the solution with a smaller slip is stable. To better understand these issues, refer to Figure 57.

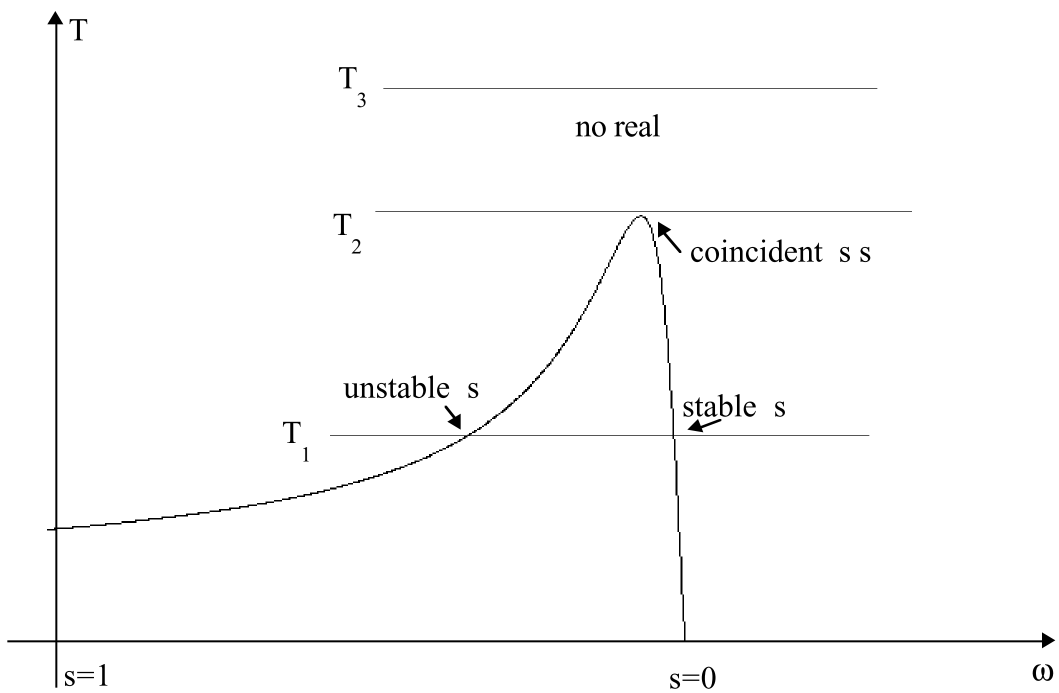


Figure 55. Torque-Angle Characteristic for an Induction Machine

Taking a closer look of Eq. 53 , that substituted for the expression of Eq. 54 and solved for *s* is plotted in Figure 56, we can see how the slip is also a function of the terminal voltage.

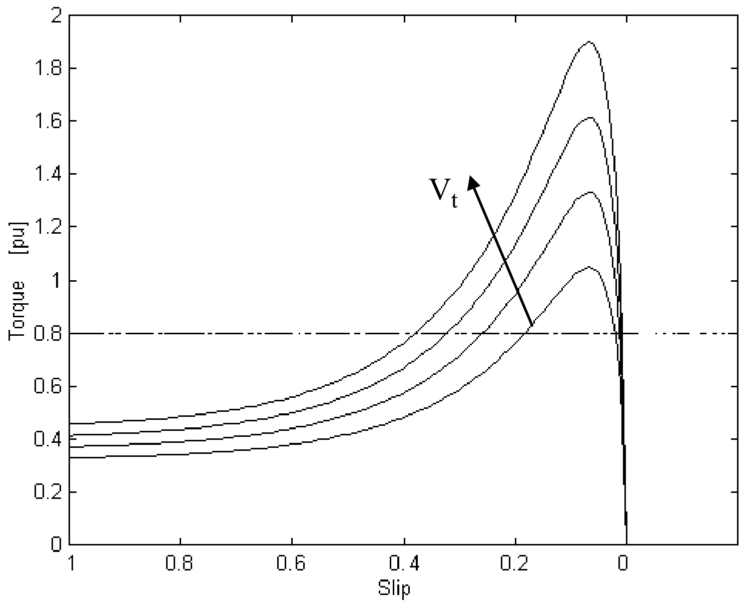


Figure 56. Torque-Angle Characteristic with Different Voltages

PF solving technique is iterative, that is, starting from a flat start guess, it moves, iteration after iteration towards the solution, and the process is stopped when the error condition falls below a given threshold. The problem is that we don t know what is the value of the voltage at the machine terminals before solving, therefore we can t properly assign a value to the rotor resistance. In order to solve this problem we incorporate the evaluation of steady state slip with our iterative power flow solution.

The equations that represent the classic power flow formulations, Eq. 1 and 3 are reported below:

$$\lambda = \left[\left| V_{1,\ldots,n} \right|, \delta_{1,\ldots,n} \right]$$

$$S(\lambda) = V I^* = V (Y_{bus} V)^*$$

$$S(\lambda) - P_{gen} + S_{load} = 0$$

The network with induction machines incorporates the terms of rotor resistances that are function of the slip as the equivalent one line circuit diagram in Figure 54 explains. For each machine installed it is possible to write an equation like Eq 53 with I_{rot}^{+} as of Eq. 54. This set of equations can be seen as a constraint of the form:

$$P_{mech} = f(V_t, s)$$

Then, the new set of power flow equations augmented with constraints coming from the induction machines are:

$$S(\lambda, s_{1,\dots,k}) - P_{gen} + S_{load} = 0$$

$$P_{mech\,1} = f\left(V_{t\,1}, s_1\right)$$

$$\dots \qquad \text{k = number of induction machines in the network}$$

$$P_{mech\,\mathrm{k}} = f\left(V_{t\,\mathrm{k}}, s_k\right)$$

With this setup it is possible to solve for the quantities keeping into account of the mechanical power requirement by simply adding the extra variable of the slip. The coupling of the power constraint equation with the rest of the system takes place through the complex terminal voltage.

An alternative formulation to the problem is when the slip dependent resistance is not incorporated in the system admittance and the rotor circuit branch is substituted with an appropriate complex power injection. The circuit for the induction machine as seen from the network is the following:

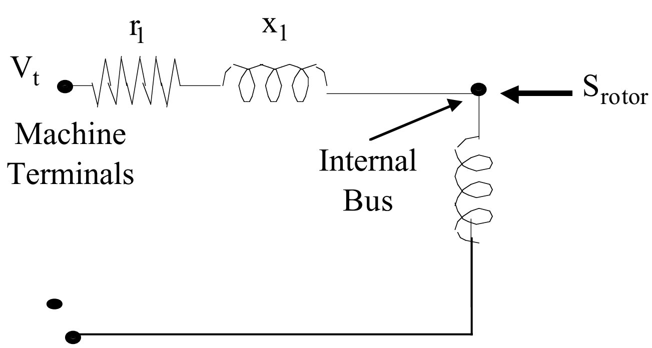


Figure 57. Equivalent Circuit of Induction Machine with Load Model of Rotor

The complex power adsorbed from the rotor branch as a function of the internal bus voltage and the slip is given by:

$$S_{rotor} = V_i \left(\frac{V_i}{\frac{R_2}{s} + jX_2} \right)^* = P + jQ \qquad \text{where} \quad \mathrm{V_i} = |V_i| e^{j\delta_i} \quad \text{internal bus voltage}$$

Eq. 55

The new power flow formulation assumes the form:

$$S(\lambda) - P_{gen} + S_{load}\left(s_1,\ldots,s_k\right) = 0$$

$$P_{mech\ 1} = f\left(V_{t\ 1}, s_1\right)$$

$$\ldots \qquad\qquad k = \text{number of induction machines in the network}$$

$$P_{mech\ k} = f\left(V_{t\ k}, s_k\right)$$

Here we keep into account that the load vector at each of the entries corresponding to an internal bus of an induction machine will be replaced by Eq. 55. The constraint equations remain the same as defined for the previous power flow formulation.

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