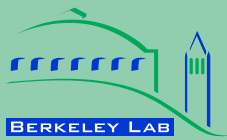


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# **Multizone Age-of-Air Analysis**

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# Multizone Age-of-Air Analysis

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## ABSTRACT

Age of air is a technique for evaluating ventilation that has been actively used for over 20 years. Age of air quantifies the time it takes for an elemental volume of outdoor air to reach a particular location or zone within the indoor environment. Age of air is often also used to quantify the ventilation effectiveness with respect to indoor air quality. In a purely single zone situation this use of age of air is straightforward, but application of age of air techniques in the general multizone environment has not been fully developed. This article looks at expanding those single-zone techniques to the more complicated environment of multizone buildings and in doing so develops further the general concept of age of air. The results of this analysis show that the nominal age of air as often used cannot be directly used for determining ventilation effectiveness unless specific assumptions are made regarding source distributions.

Keywords: Multizone, Ventilation, Air Exchange Rate, Indoor Air Quality,

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## NOMENCLATURE

<u><b>A</b></u> , $A_{i,j}$	Air Change Rate [1/s]
<u><b>Age</b></u> , $Age_i$	Age of air (in zone i) as measured using single-zone technique [s]
<u><b>C</b></u> , $C_i$	(Tracer or contaminant) Concentration (in zone i) [-]
<u><b>D</b></u> , $D_{i,j}$	Distribution Matrix [-]
<u><b>d</b></u> , $d_i$	Relative Dose (in zone i) [-]
<u><b>E</b></u> , $E_{i,j}$	(Matrix made up of the $i^{\text{th}}$ element of $j^{\text{th}}$ ) Eigenvector ( <b>e</b> ) [-]
<u><b>e</b></u> <sub>1</sub>	Ventilation Effectiveness in zone i
<u><b>λ</b></u> , $λ_i$	Eigenvalues [1/s]
<u><b>Q</b></u> , $Q_{i,j}$	Air flow (Matrix) [m <sup>3</sup> /s]
<u><b>S</b></u> , $S_i$	(Tracer) Source (in zone i) [m <sup>3</sup> /s]
<u><b>α</b></u> , $α_{i,j}$	Age matrix (equal to the inverse of the air change rate matrix) [s]
<u><b>V</b></u> , $V_i$	Volume (of zone i) [m <sup>3</sup> ]

### Notes:

- 1 Bold underlined symbols represent matrix (or vector) quantities in the equations. A double headed underline indicates a square matrix; a single-headed underline represents a vector.
- 2 The diagonal elements of the air flow matrix,  $Q_{ii}$  is the total flow into or out of that zone and the  $Q_{ij}$  elements are negative and represent the flow to zone "i" from zone "j". All volumetric flows are at the density of the zone they are going to. Similarly, the diagonal elements of the air change rate matrix are the total air change with all zones and the off diagonal elements are negative and represent the air change to zone "i" from zone "j"

## INTRODUCTION

Ventilation and the transport of both contaminants and clean air is becoming an ever more important issue as we strive to both improve energy efficiency in buildings and the indoor air quality within those buildings. Air motion is a complex interaction of naturally driven pressures and mechanically induced ones interacting with a wide variety of pathways.

Because both the pathways and the motive forces are a combination of known, intentional and unknown ones, it can be very difficult to determine the quality and quantity of airflow in all but the simplest and most controlled building environments. Various methods for measuring and quantifying airflows have been developed.

One of the metrics commonly used to evaluate ventilation is the “age of air” technique. It derives its name from the assumption that its local value is independent of the distribution of contaminant sources and thus represents an unambiguous method for comparing ventilation systems.

The age-of-air metric has also been used in the field, because experimentally it can be measured in a straightforward manner using a single tracer gas. In using the age-of-air metric in anything other than a single-well mixed zone, there are inherent limitations, which are not always recognized, especially when trying to apply age-of-air approaches to ventilation effectiveness or relative dose calculations.

We will herein extend the age-of-air technique more fully to more complex environments and in doing so will clarify how it can be used to define the effectiveness of multizone ventilation.

### Background

The “age of air” technique is a well known approach to determining how “stale” air has become by estimating the time since that air was delivered to the zone of interest. Sutcliffe (1990) has reviewed the various approaches to spatial ventilation effectiveness with some focus on the age of air.

The age of air concept focuses on the how long the air has been in the zone (or sub-zone) of concern. If this length of time is linearly related to how polluted the air has become, then this concept can help to characterize exposure.

Another measure which is used, but less often, is that of contaminant removal effectiveness which focuses not on the delivered air, but the extraction of pollutants. Broun and Waters (1991) have reviewed this concept in some detail.

Either (or neither) of these concepts may be appropriate for a particular application. It is important to evaluate one’s specific objective and compare that to the assumptions of the concepts being employed for suitability. We must then look at the application of the age of air concept to the problem in a multizone environment.

There are many references cited in Sutcliffe (1990) above for how to measure parameters associated with air change efficiency. ASHRAE (2002) has published a standard (129) on *Measuring Air Change Effectiveness*, which clearly defines how to make age of air measurements in a variety of ways. It also lists clearly the limits and limitations of the approach.

There are several equivalent definitions of the age of air. We have chose the one that for practical reasons is most often chosen:

$$3 \quad \text{Age} = \frac{\int_{\text{initial}}^{\infty} C(t)dt}{C_{\text{initial}}}$$

Where

*Age* is the nominal age of air (s)

*C(t)* is the concentration (-) as a function of time (s)

*C<sub>initial</sub>* is the starting concentration for the decay (which is allowed to decay to zero.)

This equation can be applied at any point in any space and is usually called the local mean age of air because it described the average time the air at that point has been “inside” We will use the term nominal age of air.

The concentration referred to in equation 3 could be the concentration of a well mixed contaminant, but it also serves as the experimental definition of age of air when it refers to the concentration of an initially well-mixed tracer gas. Either way that concentration can then be linked to ventilation and the source strength through conservation of mass, which is the governing requirement for dilution problems. This is conventionally written in the form of the continuity equation, which is in volumetric rather than mass terms and presumes Euclidean geometry. For a single-zone case the equation is

$$4 \quad V \cdot \dot{C}(t) + Q \cdot C(t) = S$$

Where

*V* is the volume (m<sup>3</sup>)

*Q* is the airflow (m<sup>3</sup>/s)

*S* is the source strength (m<sup>3</sup>/s) as a function of time (s)

The volume terms are understood to be relative to the density of the marked (i.e indoor) air. (For simplicity, we will generally drop the “(t)” notation and the time dependence of the variables will be understood.) The air flow, source strength (and volume) can also change with time, but are assumed stationary.

According to ASHRAE Standard 129 (2002), this age-of-air approach is only valid for a single isolated zone, but there is no clear definition for what to do in a multizone case. In the well-mixed single zone case, the age of air becomes equal to the ratio of the zone

volume to the ventilation (i.e. the inverse air change rate). Walker (1985), Sherman (1989a) and D'Ottavio (1985) have pointed out some of the issues with using single-zone techniques in a multizone environment.

## MULTIZONE CONTINUITY EQUATION

The age-of-air concept is only really useful in a zone that is not well-mixed. This is typically stated as there is a breathing zone within a space which is different from the average and it is this zone that we care about, but it could in principle be applied to other situations. For example, Rudd (2000) has attempted to apply these techniques to different rooms in a single-family home by assuming the single-zone approach can be used directly.

We will look at the problem a bit more generally. Any space can be broken up into multiple interacting zones. Each zone is assumed to be well mixed. The zones can be of any size: they can be rooms; they can be sections of air within a room, or even smaller. They can be multiple spaces as well as long as they can be assumed to be well-mixed. This approach spans the spread from a single well-mixed zone to incrementally small volumes. The former is addressed by standard single-zone approaches and the latter by numerical techniques such as computation fluid dynamics.

We will look at the case where there are finite (but potentially large) number of internally well mixed, interacting zones filling the entire interior space. A contaminant (or tracer) that leaves the interior space is lost and air coming from outside the interior space is "clean"; thus the "outside" is not treated as an explicit zone. In such a case we can write the multizone continuity equation by adding indices to designate the zone number:

$$5 \quad V_i \cdot \dot{C}_i + \sum_j Q_{i,j} \cdot C_j = S_i$$

The volume, concentration and source strength indices refer to the appropriate value in each zone. With this notation the diagonal elements of the air flow matrix,  $Q_{ii}$  is the total flow into or out of that zone and the  $Q_{ij}$  elements are negative and represent the flow to zone "i" from zone "j". All volumetric flows are at the density of the zone they are going to. See Sherman (1989b) for a more detailed description of the multizone continuity equation.

The continuity equation can be presented in matrix form as follows:

$$6 \quad \mathbf{V} \cdot \dot{\mathbf{C}} + \mathbf{Q} \cdot \mathbf{C} = \mathbf{S}$$

Whichever way this equation is written it represents a set of coupled, inhomogeneous, first order differential equations. How we approach the solution to this problem depends on what our ultimate objective is and what information we have at our disposal. We will analyze this equation in a similar manner to that of Sinden (1978).



The standard first step in the solution is to separate the problem into a solution to the homogenous set of equations:

$$7 \quad \underline{\mathbf{V}} \cdot \dot{\underline{\mathbf{C}}}^h + \underline{\mathbf{Q}} \cdot \underline{\mathbf{C}}^h = 0$$

Where the superscript  $h$  indicates the homogenous solution, which is the transient solution. The superscript,  $p$ , below indicates the particular solution,

$$8 \quad \underline{\mathbf{Q}} \cdot \underline{\mathbf{C}}^p = \underline{\mathbf{S}}$$

This particular solution is also the steady state solution, which can be added to the transient (homogeneous) one:

$$9 \quad \underline{\mathbf{C}} = \underline{\mathbf{C}}^h + \underline{\mathbf{Q}}^{-1} \cdot \underline{\mathbf{S}}$$

Now we are left to solve a set of coupled differential equations to the homogenous equation. Such homogenous equations can be decoupled using eigenvector techniques. To put it more standard form, we need to divide through by the volume<sup>2</sup>

$$10 \quad \underline{\mathbf{A}} = \underline{\mathbf{V}}^{-1} \cdot \underline{\mathbf{Q}}$$

to convert the air flow matrix to the air change rate matrix:

$$11 \quad \dot{\underline{\mathbf{C}}}^h + \underline{\mathbf{A}} \cdot \underline{\mathbf{C}}^h = 0$$

Where

$A$  is the air change rate (1/s), (not to be confused with the age of air.)

In this approach we find a set of vectors such that multiplying them by the air change rate matrix will result in a simple multiple of them. That is for some concentration set:

$$12 \quad \underline{\mathbf{A}} \cdot \underline{\mathbf{C}}_k = \lambda_k \underline{\mathbf{C}}_k$$

Where

$\lambda$  is an eigenvalue with the same units as air change rate (1/s)

$\underline{\mathbf{C}}_k$  is a concentration eigenfunction (of time) yet to be determined (-).

---

<sup>2</sup> In this paper we will assume the volume matrix to be diagonal with the diagonal elements being the physical room volumes. Sherman (1989b), however, has discussed the interpretation of off-diagonal elements .

A solution to this is not possible for any arbitrary matrix, but for any realistic physical matrix of air flows it will be possible to get a unique set of eigenvectors and associated eigenvalues. The solution to the problem decouples for any eigenfunction

$$13 \quad \dot{\underline{C}}_k + \underline{A} \cdot \underline{C}_k = 0 = \dot{\underline{C}}_k + \lambda_k \underline{C}_k$$

and can be solved simply to show the time development from some initial condition:

$$14 \quad \underline{C}_k = c_k \hat{\underline{e}}_k e^{-\lambda_k t}$$

Where  $\hat{\underline{e}}_k$  is the  $k^{\text{th}}$  normalized eigenvector. By convention the eigenvectors are normalized to be of unit length and so  $c_k$  is a set of coefficients required to match the initial conditions.

Because we have a set of independent eigenvectors any arbitrary set of initial concentrations can be represented by a linear combination of the eigenvectors. From that initial condition then the eigenfunction must decay following equation 14 and the homogenous solution<sup>3</sup> becomes:

$$15 \quad \underline{C}^h = \sum_k c_k \hat{\underline{e}}_k e^{-\lambda_k t}$$

The eigenvectors and eigenvalues are found (through standard numerical techniques not repeated here) from the air change rate matrix. The coefficients for each term are those necessary to satisfy the initial conditions of the specific problem.

The set of eigenvectors together are a matrix, the *eigenmatrix*. The basic eigenvalue equation for this problem can then be written in matrix form using the eigenmatrix:

$$16 \quad \underline{A} \cdot \underline{E} = \underline{E} \cdot \underline{\lambda}$$

---

<sup>3</sup> In an idealized situation it is possible to have two identical eigenvalues, typically if some symmetry is operational. This degenerate case requires the special handling, as described in standard texts. Such handling changes the form of the equation somewhat, but not the final conclusion. For typical inverse modeling cases such as we are dealing with, the problem does not usually occur, so we will ignore that special case herein.

Where the matrix,  $\mathbf{E}$ , is the assembled eigenvectors and the *diagonal* matrix,  $\underline{\lambda}$  contains the associated eigenvalues<sup>4</sup>. With the eigenmatrix it is possible to convert from the eigenvalues to the air change rate matrix and vice versa; and similarly for their inverses.

### Age of Air Definition

The age of air measurement technique is a special case of a decay of concentration from some initiation value. This approach is exactly the solution to the homogeneous equation derived above with all initial conditions being the same. That is the concentration in each zone will follow the time series using the eigenmatrix and a set of coefficients determined from initial conditions:

$$17 \quad C_i(t) = \sum_k c_k E_{i,k} e^{-\lambda_k t}$$

Since each zone is well mixed we can use the definition of nominal age of age (Equation 3) for each zone to define the age of air in that zone by inserting this equation therein. Thus the age of air in each zone (and therefore at every point) can be defined in terms of our eigenvector parameters:

$$18 \quad Age_i = \frac{\sum_k \frac{c_k}{\lambda_k} E_{i,k}}{C_{initial}}$$

We can solve for the unknown coefficients by using the boundary conditions that the initial concentration is the same in every zone. The boundary conditions fix all of the concentration coefficients such that in each zone

$$19 \quad C_{initial} = \sum_k c_k E_{i,k}$$

and by inversion

$$20 \quad c_k = C_{initial} \sum_j E_{k,j}^{-1}$$

Which mean that the age of air can be related to the inverse of the air change rate matrix through the eigenmatrix):

---

<sup>4</sup> By convention eigenvalues are ordered in increasing order of eigenvalues. (That is the first eigenvalue is the smallest.) For our problem the real part of all the eigenvalues must be positive. The first eigenvalue must be real, but subsequent eigenvalues may be in complex conjugate pairs. All elements of the first eigenvector must be positive, but subsequent eigenvectors may have negative elements.

$$21 \quad Age_i = \sum_{k,j} \frac{E_{i,k} E_{k,j}^{-1}}{\lambda_k} = \sum_j A_{i,j}^{-1}$$

where we have used the diagonality of the eigenvalue matrix and Equation 16. Looking at this equation suggests that if we want to examine age of air issues it might be convenient to define an age of air matrix as the inverse air change rate matrix:

$$22 \quad \tau \equiv \mathbf{A}^{-1}$$

Where

$\tau$  is the age of air matrix (s)

Equation 21 shows us that the age-of-air approach is able to measure the row sum of the elements of the inverse air change rate matrix, but tells us nothing about the distribution of those individual terms. If the air change rate matrix is diagonal (i.e the zones are isolated as required by Standard 129), the result will be exactly right.

## Clearing Time

For some applications our concern is to clear the space of a contaminant that has built up over time and fills the multizone space. Examples of this would be morning start-up of a building, occupancy after a toxic event such as fumigation, or bake-out/commissioning of new or repaired structure.

In such a case, we can assume that the space is evenly filled with an unacceptable amount of some contaminant and we wish to run the ventilation system until the concentration in all zones meets some acceptability criterion. Age of air is the perfect metric for this application: the decay to zero (and thus the sum of the elements of the inverse air change rate matrix) is then what we need. Thus the nominal age of air approach is both necessary and sufficient to estimate the clearing time of the space.

If one has an initially uniform concentrations of a contaminant that one wants substantially cleared from the interconnected spaces, the concentration in each zone can be approximated as an exponential and the time to flush to a specific level is proportional to the age of air in that zone:

$$23 \quad t_i = -Age_i \ln \left( \frac{C_{initial}}{C_{final}} \right)$$

Where  $C_{final}$  is the desired or acceptable level.

Clearing, however, is only one application for ventilation.

## INDOOR AIR QUALITY

Ventilation can be used for many things: to transport heated or cooled air, to provide oxygen for metabolic activities or combustion or to provide air motion for thermal comfort. Our concern here is in looking at ventilation to control the exposure of occupants to internally generated contaminants – i.e. ventilation for acceptable indoor air quality. Ventilation effectiveness therefore depends on how it impacts one's exposure to a contaminant (i.e. the concentration).

Nominal age of air is often used to measure ventilation effectiveness. Age of air is, just as the name indicates, the time it takes for elemental volume of air from "outside" to reach the location in question. Qualitatively then nominal age of air is a measure of ventilation effectiveness, but quantitatively it may not be. The outside air does not simply "go bad" with time indoors. Rather it picks up indoor contaminants as it traverses the interior space. The rate at which it picks up those contaminants depends on the path and the emission strength in each zone. To see how this relates to ventilation effectiveness we need to link it through exposure to contaminants, which is generally what we are concerned with when looking at indoor air quality issues.

For most of the contaminants of concern in the indoor environment, dose (i.e. integrated concentration over time) can be used as the quantity of interest. Most contaminants of concern in the indoor environment, at least the ones we wish to control through ventilation, are continuously emitted. Therefore we can use the steady state concentration for a contaminant as our indicator of exposure:

$$24 \quad \underline{C} = \underline{Q}^{-1} \cdot \underline{S} = \underline{\tau} \cdot \underline{V}^{-1} \cdot \underline{S}$$

### Perfect Mixing

Consider now the special case where there is good mixing within the entire space. That is, the space we are looking at is really a single-well mixed zone. In this case the concentration will be uniform at

$$25 \quad C_o = S_o / Q_o$$

Where we have used the total source emission and the total air exchange with "outside":

$$26 \quad S_o = \sum_i S_i \quad Q_o = \sum_{i,j} Q_{i,j} \quad V_o = \sum_i V_i$$

### Relative Dose

Using the reference of perfect mixing we can define a relative dose vector that tells us how much exposure one can get in each zone, relative to the perfect mixing case.

$$27 \quad \underline{\mathbf{d}} = \underline{\mathbf{C}} / C_o = Q_o \underline{\mathbf{Q}}^{-1} \cdot \underline{\mathbf{S}} / S_o$$

Where  $\underline{\mathbf{d}}$  is the relative dose vector.

This can allow us to define ventilation effectiveness as ratio of what the dose (i.e. exposure) would have been under perfect mixing to what it is under the actual situation:

$$28 \quad \varepsilon_i = \frac{C_o}{C_i} = \frac{S_o}{Q_o \cdot \sum_j \tau_{i,j} S_j / V_j} = \frac{1}{d_i}$$

Note that the ventilation effectiveness,  $\varepsilon_i$ , is just the inverse of that component of the relative dose vector.

The problem does not simplify further and we cannot decouple the source terms except under some special conditions. The first special condition, of perfect mixing, would lead to unity by definition. The second special condition is that the age of air matrix is diagonal. This corresponds to the situation where a zone is completely isolated and is not of much interest. The special condition that is most relevant is that of volume distributed source strength.

### Volume Distributed Sources

Consider the special case in which the emission rate of the contaminant of concern is strictly proportional to the volume of the zone:

$$29 \quad S_i = S_o \cdot V_i / V_o$$

In this special case the ventilation effectiveness reduces to the form normally used in age of air approaches

$$30 \quad \varepsilon_i = \frac{V_o}{Q_o \cdot Age_i}$$

Or equivalently for the relative dose:

$$31 \quad \underline{\mathbf{d}} = \frac{Q_o}{V_o} \underline{\mathbf{A}} \mathbf{g} \mathbf{e}$$

Note that the ratio of the volume to the total airflow is often termed *nominal turn-over time* in age-of-air approaches. Its inverse is the nominal air change rate.

## DISCUSSION

Age of air techniques are fundamentally associated with the transient problem of clearing a space of contaminants. Analytically this couples age of air with the homogenous solution to the governing equations. In contrast standard indoor air quality problems focus on long-term exposure and steady state. Analytically this couples relative dose issues to the particular solution. Care must be taken when trying to use transient information to solve steady-state problems, or the converse.

Maldonado (1983) stated that the standard definition of the age of air could be used to estimate occupant exposure to contaminants. As derived above, this is only true for a specific distribution of contaminant sources. In some circumstances it may be very reasonable to assume such a distribution of sources, but for many it will not.

Consider the case of an office building in which there may be three vertical zones: the lower zone; the breathing zone and the upper zone. We care about the exposure in the breathing zone, but air may move through each. The upper zone may have few contaminant sources; the breathing zone has the occupants and their activities; the lower zone may have contaminants from the flooring or carpet and other surfaces. Which path the air takes to reach the breathing zone will determine the level of contaminant exposure and the age of air may not be a good indicator.

In a dwelling there are rooms that may have high contaminant loads, such as bathrooms, laundries, kitchens, hobby rooms and toilets, and there are rooms with low contaminant loads such as large open spaces. In such cases nominal age of air will similarly not be a good predictor of exposure.

In situations where the contaminant distribution cannot be reasonably assumed to be proportional to the zone volume, more information is needed than can be supplied by the simple age of air estimates. We can, however, generalize the concept of age of air to include the fact that different paths for the air will contribute differently to exposure.

We seek an effective age of air that allows us to use the simple definition of equation 31, but maintain the full detail of equation 28. We can do that if we define a source weighting for the contaminant of concern in each zone

$$32 \quad \underline{\sigma} = \frac{V_o}{S_o} \underline{V}^{-1} \cdot \underline{S}$$

Where  $\underline{\sigma}$  is the relative intensity of the contaminant of concern compared to the volume-weighted one used in the simple age of air definition. In such as case we can define the effective age of air as follows:

$$33 \quad \underline{Age}_{\text{eff}} = \underline{\tau} \cdot \underline{\sigma}$$

If the weighting elements are all unity then the effective age of air is equal to the simple one. The effective age of air will be different for each contaminant distribution and therefore must be recalculated for every contaminant of concern.

### Distribution Matrix

There are alternative ways to represent the relative dose, which may be more useful when the distribution of sources is quite different from that assumed by the age of air methods. We can, for example, define a distribution matrix that does not depend on the distribution of sources:

$$34 \quad \underline{\mathbf{D}} = \underline{Q}_o \underline{\mathbf{Q}}^{-1}$$

Where  $\mathbf{D}$  is the distribution matrix, which can be used to determine the relative dose:

$$35 \quad \underline{\mathbf{d}} = \frac{Q_o}{V_o} \underline{\mathbf{Age}}_{\text{eff}} = \underline{\mathbf{D}} \cdot \underline{\mathbf{S}} / S_o$$

Thus the distribution matrix is a property of the ventilation system and, unlike the age of air, is independent of the distribution of sources. For many applications this may be a preferable metric.

### Measurement Issues

Traditionally defined age of air can be measured relatively robustly using single-gas decay methods such as that in ASHRAE Standard 129. It can be much more problematic to measure effective age of air or the distribution matrix.

Discussion of measurement techniques is beyond the scope of this report, but Miller (1997) has shown that single-tracer decay approaches are extremely difficult. Sherman (1990) has gotten good results, however, from a steady-state approach using multiple tracers.

## CONCLUSIONS

We have used a theoretical eigenvector analysis of the multizone continuity equation to rigorously define and then evaluate the ventilation metric, *age of air*. This has exposed some fundamental strengths and weaknesses that must be kept in mind when applying it to real buildings.

This nominal age of air is ideally suited to applications where one is interested in the length of time it takes to replace “bad” air with “good” air. These transient approaches include applications such as building start-up, post-event clearing (e.g. from a toxic or scheduled event) or as part of commissioning or cleaning activities.



The nominal age of air approach can be used for a wider variety of approaches when each zone is substantially isolated from other zones. In such cases, the multizone nature of the application is unimportant and all the single-zone benefits to the approach are valid.

The nominal age of air approach can also be used to estimate steady-state exposures under the special assumption that the source of contamination is the same in every unit volume (i.e. emission is proportional to the volume of each zone.) The nominal age of air approach is invalid in any multizone environment when air exchange and/or source strength variations between zones is a contributing factor. For such applications, the nominal multizone age of air approach should *not* be used.

The *nominal* age of air approach can be generalized to account for non-uniform source distributions by defining an *effective* age of air. To account for the general case of source distribution an alternative approach using the *distribution matrix* has been developed. The distribution matrix is independent of source distribution and therefore is a function only of the ventilation pattern; arbitrary source distribution patterns may then be used to evaluate the relative exposure to contaminants.

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