Window Optics

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1. Abstract

Optical and radiative properties of glazing materials are primary inputs for determination of energy performance in buildings. This paper revisits the problem and reformulates the calculations to encompass a variety of solutions to practical problems in window optics. Properties of composite systems such as flexible films applied to rigid glazing and laminated glazing can be predicted from measurements on isolated components in air or other gas. Properties of a series of structures can be generated from those of a base structure. For example, the measured properties of a coated or uncoated substrate can be extended to a range of available substrate thickness without the need to measure each thickness. Similarly, a coating type could be transferred by calculation to any other substrate. A simple monolithic model for extrapolating from normal properties to oblique properties is shown to have sufficient accuracy for the purpose of annual energy performance calculations. A process is initiated to develop a reliable method for determination of effective indices suitable for more detailed spectral and directional optical calculations.

2. Introduction

Optical and radiative properties of glazing materials are primary inputs for determination of energy performance in buildings. Direct measurement of all relevant quantities is not a general option. Those properties that can be most easily and commonly measured, using standard spectroradiometers, are the spectral transmittance and reflectance at normal incidence. Procedures written a generation ago for optical combination of multiple pane windows are now standardized.

Other properties are needed, however, than can be determined from normal radiometric measurements alone. Prediction of the properties of composite systems such as applied flexible films and laminated glazing can not be directly obtained from measurements on isolated components with air on each side. Properties of a series of structures can be generated from those of the base structure. For example, the properties of a coated or uncoated substrate could be extended to a range of available substrate thickness without the need to measure each thickness. Similarly, a coating type could be transferred by calculation to any other substrate. Sunlight does not generally fall at normal incidence to a window and in fact often strikes at angles for which the transmittance and reflectance are significantly different from their values at normal incidence. A reliable procedure for extrapolating from normal properties to oblique properties is thus needed for accurate annual energy performance calculations.

Any of the above operations could be performed in a straightforward and unified way if we knew the optical indices at every point within the system. Fresnel’s Equations and analogous equations for thin-film structures provide solutions for any plane-parallel window system. Although not considered in this paper, we could extend the solutions to arbitrary geometry using ray-tracing. One obstacle to this fundamental approach is that manufacturers may be unwilling to reveal structure and properties of their products or they may not know the optical indices or even layer thickness well enough to perform the calculation. On the other extreme, we could measure the properties that we need for each angle and polarization or for any combination of layers and coatings. Angle-dependent properties can presently be measured by a handful of laboratories, but the amount of data generated for all products would be impractical. Furthermore, oblique measurements do not yet fall within the scope of existing standards.

Although the fundamental optical parameters of coated window materials are not readily available, this is not to say that they cannot be determined. Some of the most complex coated glazing structures have been successfully analyzed by both ellipsometric and radiometric techniques. In general, however, the determination of the optical constants is a time-consuming and specialized operation. Furthermore, the numerical process is not fail-safe and will not automatically converge to a correct or satisfactory solution. It is possible, however, to arrive at approximate solutions that meet most of the required criteria without necessarily resulting in the true values for the optical indices. Another possibility is to program an “expert system” that will always find a way to an acceptable solution.

This paper will begin with a brief review and reformulation of the relevant theory of multilayer calculations. Various levels of detail are considered, depending on the available information. New solution methods will be presented for applied films, laminates and other problems that do not require the numerical determination of optical indices. A simple approach is given for extrapolation of angle dependent optical properties from data at normal incidence. Other situations require the determination of the optical indices of coatings with varying levels of accuracy. A general approach to this problem is proposed.

3. Experimental

The optical data used for inputs to and verification of the calculations described throughout this paper were measured by a variety of experimental techniques. Transmittance and reflectance measurements are made over the solar spectrum from 300-2500 nm. All measurements were made at normal incidence using either a Perkin-Elmer Lambda 9 or Lambda 19 spectrophotometer. The radiometric measurements and spectral averages were made in accordance with the standard practice of ASTM 1585[1] and ISO 9050[2]. A variable-angle spectroscopic ellipsometer by J.A Woollam Co. provided supplemental data over the visible spectrum and part of the solar infrared (290-1700 nm). As a further check, directional measurements were made at 670 nm using a feedback-stabilized laser diode with a short coherence length and a large-area silicon detector. The error for this system is typically 0.1%. For the numerical analysis of optical indices, radiometric and ellipsometric data were fitted together, weighting both data types according to their experimental standard deviations.

4. General Multiple Layer Models

This entire section serves as a review of the calculations that are used to obtain the properties of a plane-parallel layered system such as a typical window from the properties of the component parts. When all of the required component properties are available, then these methods give a straightforward determination of the total properties. Since knowledge of the detailed properties is not usually the case, then these “forward” calculations also serve as the engines needed to search “backwards” for the component properties from the measured data.

Depending on the level of information available, three different models are considered based on 1) “external” radiometric properties, 2) ”internal” radiometric properties, and 3) optical indices. Pfrommer[3] and van Nijnatten[4] recently presented unified models for a window structure that might consist of thick substrates, thin-film stacks and air gaps. The solutions were cast in the form of transformation matrices for each layer analogous to those of the thin-film solution presented in Section 4.3. These solutions are very appealing, both for their universal approach and their suitability for computer programming. Nevertheless, we have retained, for the purposes of this paper, the equivalent three-level formalism of our earlier work to better demonstrate the physical phenomena and to facilitate the description of standard procedures.

Throughout this section various radiometric properties such as reflectance and transmittance are considered to be implicit functions of wavelength, angle of incidence and polarization. Calculations are carried out with all properties at given values of those three parameters. Average values over a range of wavelengths, angles and/or polarizations can be taken following the procedures of Section 4.4.

4.1 External Radiometric Model

The spectral properties of the glazing system are determined using equations that take into account the multiple internal reflections within the glazing system. There are at least three equivalent approaches to this problem: the ray-tracing method,[5] the "embedding" or energy-balance method,[6] and the transformation-matrix method.[7] Consider a set of L glazing elements making up a window system as in Figure 1.

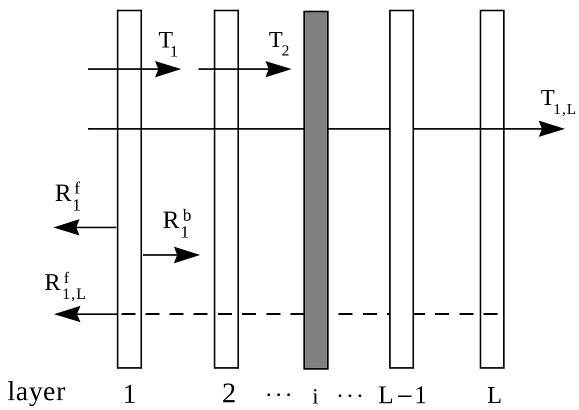

Figure 1. Window system consisting of L plane parallel layers separated by gas-filled gaps.

For two elements *i* and *j* the net transmittance $T_{i,j}$, the front reflectance $R^f_{i,j}$ and back reflectance $R^b_{i,j}$ for the subunit are given by:

$$T_{i,j} = \frac{T_{i,j\text{-}1}\ T_j}{1-\ R^b_{j-1\,,\,i}\ R^f_j} \qquad R^f_{i,j} = R^f_{i,j-1} + \frac{T^2_{i,j-1}\ R^f_j}{1-R^b_{j-1,i}R^f_j} \qquad R^b_{j,i} = R^b_j + \frac{T^2_j\ R^b_{j-1,i}}{1-R^b_{j-1,i}R^f_j}$$ **Equation 4-1**

In the above equations, a single subscript refers to the property of a single glazing element whose properties have to be measured. By convention, element 1 is outermost towards the sun. For a system of L elements, the solution is built up by adding one element at a time as described in Section 5.

The absorption $\tilde{A}^f_j$ of each element as part of the stack can be calculated from values for the transmittance and reflectance of the surrounding substacks obtained from Equation 4-1 and the from the *external* absorptance of the isolated elements A_j as follows:

$$\tilde{A}^f_j = \frac{T_{1,j-1}A^f_j}{1-R^b_{j-1,1}R^f_{j,L}} + \frac{T_{1,j}R^f_{j+1,L}A^b_j}{1-R^b_{j,1}R^f_{j+1,L}} \quad \text{where} \quad \begin{aligned} A^f_j &= 1-T_j - R^f_j \\ A^b_j &= 1-T_j - R^b_j \end{aligned}$$ **Equation 4-2**

4.2 Internal Radiometric Model

Now let us consider "internal properties such as the reflection and transmission through an interface or absorption within a medium. The previous section referred to manipulations on *external* properties which are measured on a specimen with light incident through air on one side and emerging into air on the other side. To go beyond this point, we must dig deeper into the structure of the materials. Methods for obtaining the needed information will be discussed in later sections. For now let us set up the formalism for calculating the properties of the multilayer assuming that we can obtain the necessary parameters to perform the calculation. In the most general case, we can imagine a structure consisting of *L* layers and *L*+1 interfaces as in Figure 2.

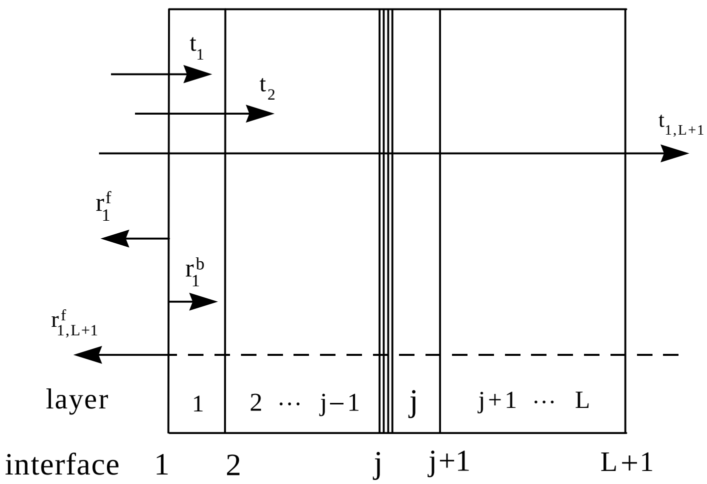

Figure 2. Stack of L layers separated by L+1 asymmetric interfaces.

The interfaces may be "simple" , i.e., an infinitesimally thin boundary between homogeneous materials for which $r_f = r_b = r = 1-t$. The interfaces may also be considered "complex" in the sense that they have a finite thickness, possibly consisting of many thin-film layers. The *complex* interface could even be a sequence of thick and thin layers if that part of the internal structure is irrelevant to the specific problem. For *complex* interfaces, $r^f \neq r^b \neq 1-t$. By analogy to the *external* model above, with the *complex* interfaces replacing the elements, expressions can be written directly for the system properties. An additional factor of τ_i is introduced to account for the possibility of absorption within a layer *i*. For a subsystem bounded by interfaces i and j:

$$\mathrm{T}_{i,j} = \frac{t_{i,j-1}\ t_j\ \tau_{j-1}}{1 - r^b_{j-1,i}\ r^f_j \tau^2_{j-1}}$$ **Equation 4-3**

$$R^f_{i,j} = r^f_{i,j-1} + \frac{t^2_{i,j-1}\ r^f_j \tau^2_{j-1}}{1 - r^b_{j-1,i} r^f_j \tau^2_{j-1}}$$ **Equation 4-4**

$$R^b_{j,i} = r^b_j + \frac{t^2_j\ r^b_{j-1,i} \tau^2_{j-1}}{1 - r^b_{j-1,i} r^f_j\ \tau^2_{j-1}}$$ **Equation 4-5**

Simple interfaces have no absorption, but complex interfaces may contain one or more absorbing layers with total absorption considered as part of the stack:

$$\tilde{A}_j^f = \frac{t_{1,j-1}\tau_{j-1}a_j^f}{1-r_{j-1,1}^b r_{j,L}^f \tau_{j-1}^2} + \frac{t_{1,j}r_{j+1,L}^f \tau_{j-1}a_j^b}{1-r_{j,1}^b r_{j+1,L}^f \tau_{j-1}^2} \quad \text{where} \quad \begin{aligned} a_j^f &= 1-t_j - r_j^f \\ a_j^b &= 1-t_j - r_j^b \end{aligned}$$

Absorption within a substrate or other monolithic layer is given by a similar but separate equation.

$$A_j = \frac{t_{1,j}(1-\tau_j)}{1-r_{j,1}^b r_{j+1,L+1}^f} + \frac{t_{1,j+1}r_{j+1,L+1}^f(1-\tau_j)}{1-r_{j,1}^b r_{j+1,L+1}^f}$$

4.3 Optical-Index Model

In some situations, it may be necessary to calculate the system properties from the most fundamental optical properties, i.e., the optical indices and thickness of each layer. If these properties are known, then any radiometric property can be calculated as a function of angle or polarization. Our physical model remains the same as in Section 4.2 for *internal* properties. For simple interfaces between two monolithic materials, the reflectivity is calculated according to Fresnels equations. Introducing polarization explicitly for the first time, we have for transverse electric TE and transverse magnetic TM polarizations:

$$r = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2 \qquad \text{substituting } n_i \equiv \begin{cases} n_i \cos\theta_i & \text{for TE polarization} \\ n_i / \cos\theta_i & \text{for TM polarization} \end{cases}$$

and $n_i \equiv n_i - ik_i$ for a strongly absorbing material

where k is the extinction coefficient and the angles between the direction of propagation and the normal to the boundary in the two media are related by Snell's law of Refraction:

$$\cos\theta_2 = \left[1-\left(n_1 / n_2\right)^2 \sin^2\theta_1\right]^{1/2}$$

If the material is weakly absorbing, then the interface reflectivity above depends only on the real part of n and the internal transmittance depends only on k through Beer's law:

$$\tau = \exp\left[-\frac{4\pi k}{\lambda}\frac{d}{\cos\theta}\right] \qquad \Rightarrow \qquad k = -\frac{\lambda\cos\theta}{4\pi d}\ln\tau$$

The thin-film stacks must be treated separately, because of interference effects within the stacks. From the equations of Heavens[8] we can write three equations for transmittance and reflectance from each side of a stack in incident medium n_1 (usually air n=1) and exit or substrate medium n_s:

$$t = \frac{n_s}{n_1}\left|\frac{2}{an_1 + bn_1n_s - c - dn_s}\right|^2 \quad r^f = \left|\frac{an_1 + bn_1n_s - c - dn_s}{an_1 + bn_1n_s + c + dn_s}\right|^2 \quad r^b = \left|\frac{-an_1 + bn_1n_s - c + dn_s}{an_1 + bn_1n_s + c + dn_s}\right|^2$$

The coefficients in the equations above are given by the elements *a,b,c,d* of a product matrix *M* whose factors are individual matrices M_j , for each thin layer:

$$M_j = \begin{bmatrix} \cos\delta_j & \dfrac{i}{n_j}\sin\delta_{j+1} \\ in_j \sin\delta_j & \cos\delta_j \end{bmatrix} \qquad \text{where the phase retardation } \delta_j = \frac{2\pi\,\mathrm{n_j h_j}}{\lambda}\cos\theta_j$$

where imaginary *i* is √-1. Within the stack, we need not worry about the spatial distribution of absorbed energy because layers are so thin. The stack may now be considered as a *complex* interface whose calculated properties may be used in the *internal* radiometric model.

4.4 Averaged Properties

The *external* and *internal* radiometric properties discussed above are all implicit functions of wavelength, incident angle and polarization. The optical indices are not dependent on either angle of incidence or polarization, although they may be strongly dependent on wavelength. Permanently oriented thin films of certain noncubic crystallites or liquid crystals may be optically anisotropic or birefringent so that the index and optical properties would be dependent on orientation with respect to the crystal axes. These cases are beyond the scope of this paper. Various useful quantities can be derived by averaging over a suitable range of one or more of these parameters

4.4.1 Spectral Averages

Having calculated the properties at all desired wavelengths, various types of average properties commonly used as figures of merit for the glazing system can then be calculated using the general equation:

Equation 4-6

$$P_x = \frac{\int_a^b P(\lambda)\,\Phi_x(\lambda)\,\Gamma_x(\lambda)d\lambda}{\int_a^b \Phi_x(\lambda)\,\Gamma_x(\lambda)d\lambda}$$

In the above equation, *P* is the measured spectral radiometric property such as transmittance or reflectance; *Φ* is the weighting function for the strength of source (sunlight or thermal radiation) under appropriate conditions at each wavelength; and Γ is a weighting function for the response of the "detector." The subscript *x* indicates the type of average to be taken. The practical wavelength limits are specified by *a* and *b*, although the finite extent of the source or response functions effectively bound the integral.

Table 4-1 summarizes some of the quantities commonly defined in standard test methods. Many other types could be defined as needed for color rendering, night vision, plant growth, retinal damage, etc.

Table 4-1 Values used in Equation 4-6.

Property Type x	Property P_x	Lower Wavelength a (μm)	Upper Wavelength b(μm)	Source Function Φ_x	Detector Function Γ_x
Solar	T, R, A	0.30	2.5	AM 1.5 Global Irradiance [ISO 9845/ASTM 892] [9]	1.0
Visible	T	0.38	0.78	CIE D65 Illuminant [ISO/CIE 10526] [10]	CIE 1931 Observer [ISO/CIE 10527] [11]
Thermal	E, T, R,	5.0	≥ 25 (50)	Blackbody	1.0
Color	T, R	0.38	0.78	CIE D65 Illuminant [ISO/CIE 10526][10]	CIE 1968 Observer [ISO/CIE 10527] [11]
Skin Protection Factor	T	0.28	0.40	Sayre et al.[12]	McKinlay and Diffey[13]
Damage*	T	0.3	0.5	AM 1.5 Global Irradiance [ISO 9845/ASTM 892][9]	Krochmann[14]

* The calculation of "ultraviolet" damage to fabrics is a controversial issue. For information on common practice only.

4.4.2 Directional Averages

The general formula for a conical average on an azimuthally symmetric material (properties depend only on θ) is:

$$P = \frac{\iint_{\phi\theta} P(\theta)\cos\theta\sin\theta d\theta d\phi}{\iint_{\phi\theta} \cos\theta\sin\theta d\theta d\phi}$$

Ideally directional averages would be taken over the specific angular field of view. Such averages could be in building energy simulation programs to account for limited view of ground or sky. Nevertheless, lack of information about the surroundings of the glazing usually confines the calculated averages to the hemispherical quantity. Integrating over the hemisphere from $\theta = 0 \rightarrow \pi/2$ and $\phi = 0 \rightarrow 2\pi$:

Equation 4-7

$$P_{\cap} = 2\int_{0}^{\pi/2} P(\theta)\cos(\theta)\sin(\theta)d\theta$$

4.4.3 Polarization Average

For oblique incidence, polarization effects must be considered. Natural sunlight is "unpolarized" so that the angle of polarization with respect to the plane of incidence fluctuates randomly. Only the time average is measurable for any radiometric property *P* which reduces to a simple average of the properties for the two polarization states:

Equation 4-8

$$P = \frac{1}{2}(P_{TE} + P_{TM})$$

5. Synthesis of Window Systems

The simplest case is that of a complete window system consisting of multiple elements separated by air gaps. In this case, the equations of Section 4.1 can be used directly with commonly measured reflectance and transmittance at normal incidence on each element in air. Neither wavelength, angle of incidence nor polarization appear explicitly in these equations. The most common usage of these equations is to calculate window system properties wavelength-by-wavelength at normal incidence, but they are equally valid if angle-dependent or polarization-dependent properties are available.

A recursive procedure is carried out until the transmittance and reflectance of the entire glazing system has been determined. Simply measure the properties T_i, R_i^f, and R_i^b of each element in the system as a function of wavelength according to standard practice.[1,2] Then for double glazing, the combination of layers 1 and 2 has the following properties:

Equation 5-1

$$T_{12} = \frac{T_1\ T_2}{1 - R_1^b\ R_2^f} \qquad R_{12}^f = R_1^f + \frac{T_1^2\ R_2^f}{1 - R_1^b R_2^f} \qquad R_{21}^b = R_2^b + \frac{T_2^2\ R_1^b}{1 - R_1^b R_2^f}$$

Similarly, for triple glazing, the system properties can be calculated starting with the properties calculated for double-glazing above and the measured properties of the added third layer:

Equation 5-2

$$\mathrm{T}_{13} = \frac{\mathrm{T}_{12}\ \mathrm{T}_3}{1 - \mathrm{R}_{12}^{\mathrm{b}}\ \mathrm{R}_3^{\mathrm{f}}} \qquad R_{13}^f = R_{12}^f + \frac{T_{12}^2\ R_3^f}{1 - R_{12}^b R_3^f} \qquad R_{31}^b = R_3^b + \frac{T_3^2\ R_{21}^b}{1 - R_{12}^b R_3^f}$$

For higher orders continue the progression above. For example, to progress to quadruple glazing, mechanistically exchange index $3 \rightarrow 4$ and $2 \rightarrow 3$ in the expressions for triple glazing above. In all cases calculate the absorption in each layer using the T and R values calculated in the previous steps and Equation 4-2. The final step should always be to calculate the relevant spectral and or directional average properties as in Section 4.4

6. Two Useful Component Solutions

It can not be mentioned too frequently that sufficient data will rarely be available to resolve the full internal optical properties of a window material or system. Furthermore, even when such data is available, closed-form solutions are possible only for the most simple cases. In this section, we present closed-form solutions for two simple but extremely useful component models. These solutions will serve as building blocks for many of the solutions to more complex problems developed in later sections.

6.1 Monolithic-Substrate (MS) Solution

An uncoated substrate is symmetric and thus completely characterized in principle by only 2 independent measured quantities, the *external* transmittance T_s, and the reflectance, which we assume to be exactly the same from both sides, so that $R_s{}^f = R_s{}^b$. From Equation 4-3 and Equation 4-4 (or Equation 4-5), we can write these measured radiometric quantities in terms of the interfacial reflectance r_s or transmittance t_s=1-r_s of the substrate in air, and the internal transmission τ_s:

Equation 6-1

$$T_s = \frac{(1-r_s)^2\tau_s}{1-r_s{}^2\tau_s^2} \quad \text{and} \qquad R_s = r_s + \frac{(1-r_s)^2 r_s \tau_s^2}{1-r_s{}^2\tau_s^2}$$

After substituting and solving the quadratic we have the *internal* radiometric properties r_s and τ_s in closed form[15]:

Equation 6-2

$$r_s = \frac{\beta - \sqrt{\beta^2 - 4(2-R_s)R_s}}{2(2-R_s)} \quad \text{where} \quad \beta = T_s^2 - R_s^2 + 2R_s + 1$$

Equation 6-3

$$\text{and} \quad \tau_s = \frac{R_s - r_s}{r_s T_s}$$

In this simplest situation, the optical indices can be determined directly. Inverting Fresnel's equation for the special case of normal incidence on a weakly absorbing material in air and inverting Beer's law at normal incidence gives:

Equation 6-4

$$n = \frac{1+\sqrt{r_s}}{1-\sqrt{r_s}} \quad \text{and} \quad k = -\frac{\lambda}{4\pi d}\ln\tau_s$$

The above method works for any unknown dielectric material. For a material of well known optical index such as soda-lime silica glass,[16] a preferable method is to solve for τ (or k) directly from T. This avoids the measurement of R which carries a higher uncertainty than T.[17] So from Equation 6-1:

Equation 6-5

$$\tau_s = \frac{\left[(1-r_s)^4 + 4r_s{}^2T_s^2\right]^{1/2} - (1-r_s)^2}{2r_s{}^2T_s}$$

6.2 Coating-on-Known-Substrate (CKS) Internal Solution

The case of an unknown film on a known substrate, like the above case of an unknown monolithic substrate, has an exact solution. For an asymmetric material, there are three independent measured quantities: the transmittance, T_c, the reflectance from the front (uncoated or substrate side *s*) $R_c{}^f$ and the reflectance from the back (coating side *c*) $R_c{}^b$. Again following Equation 4-3 through Equation 4-5:

$$T_c = \frac{(1-r_s)t_c\tau_s}{1-r_s r_c{}^f \tau_s^2}$$ **Equation 6-6**

$$R_c^f = r_s + \frac{(1-r_s)^2 r_c{}^f \tau_s^2}{1-r_s r_c{}^f \tau_s^2}$$ **Equation 6-7**

$$R_c{}^b = r_c{}^b + \frac{t_c{}^2 r_s \tau_s^2}{1-r_s r_c{}^f \tau_s^2}$$ **Equation 6-8**

From the previous section we know how to find r_s and τ_s for the substrate. Then, there are only three unknowns in these three equations: the transmittance and reflectance from each side of the coating, t_c, $r_c{}^f$ and $r_c{}^b$, respectively. Only one of these unknowns $r_c{}^f$ appears linearly in Equation 6-7 so that:

$$r_c{}^f = \frac{R_c{}^f - r_s}{[1+r_s(R_c{}^f - 2)]\tau_s^2}$$ **Equation 6-9**

Once $r_c{}^f$ has been calculated t_c and in turn $r_c{}^b$ can be written in terms of known quantities:

$$t_c = \frac{T_c(1-r_s r_c{}^f \tau_s^2)}{(1-r_s)\tau_s}$$ **Equation 6-10**

$$r_c{}^b = R_c{}^b - \frac{t_c{}^2 r_s \tau_s^2}{1-r_s r_c{}^f \tau_s^2}$$ **Equation 6-11**

Unfortunately, we cannot proceed to calculate the optical indices as we could in the monolithic-substrate case. This will limit our ability to solve several problems involving coatings in later Sections.

7. Extrapolation of Substrate Thickness or Substitution of Coatings

The widely used multilayer calculation in Section 5 exhausts the possibilities for the use of *external* properties only. The process could be repeated for every permutation of glazing materials but some analysis will greatly reduce the burden of measurement. In this section we present the first simple examples of the use of *internal* properties. From a single careful measurement, it is possible to simulate the properties of families of similar products. Not only would such procedures reduce the measurements on existing products but they would also allow the creation of "virtual" products without the trouble of manufacturing prototypes. For example, A product can be measured with a standard substrate thickness and then the properties could be calculated for all other available substrate thickness. Similarly, a coating could be transferred to an entirely different type of substrate.

First we must disassemble the layers. Start by calculating the *internal* radiometric properties of the substrate r_s and τ_s. For a glass substrate, use known values of the index of refraction[16] and Equation 6-5. For other substrate materials use Equation 6-2 and Equation 6-3. If the substrate is coated, calculate the *internal* properties of the coating t_c, $r_c{}^f$ and $r_c{}^b$ via Equation 6-9, Equation 6-10, and Equation 6-11.

If only a different thickness is desired, then choose new thickness d_2 different from the original thickness d_1, and recalculate the internal absorption of the substrate. Introducing the thickness explicitly via $\tau = e^{-\alpha d}$, gives $\tau_2 = \tau_1^{d2/d1}$. The absorption coefficient α is a property of the glazing and we assume that is constant for all thickness. Then it is simply a matter of finding the transmission and reflection of the new glazing, using τ_2 instead of τ_1 (all of the other internal properties are unchanged) from Equation 6-1 for monolithic glazing or from Equation 6-6, Equation 6-7, and Equation 6-8 for coated glazing. To put the coating on a different type of substrate, disassemble the new substrate as above and then combine the coating with the new substrate properties.

8. Synthesis of Laminates and Applied Films

In Section 5, we treated the problem of synthesis of window systems from available component glazings separated by air gaps. Synthesis of other layer structures like applied films and laminates is considerably more involved. The properties of the adhesive subcomponents are not well known. Also, they are difficult to fabricate in free standing form for direct measurement. The rigid subcomponents can be measured, but they are not in the same environment, i.e., immersed in air, when part of the solid structure. Methods for circumventing these difficulties are discussed.

8.1 Properties of Adhesive Materials

Concerning unknown adhesive materials, we took the direct approach of asking major manufacturers for information about the index of refraction. Unfortunately, there were no records of such measurements, although it was obvious that by some process materials had been developed that were at least approximately index matched to glass. We cast free-standing films of PVB materials supplied by DuPont used to construct laminated glazing. Using both variable-angle ellipsometry as well as transmittance and reflectance measurements over the full solar spectrum, we confirmed that the index of refraction was indeed very closely matched to glass (see Figure 3). At 550 nm the reflection at a PVB/glass interface would be less that 0.01 %. A similar degree of index matching was found for two common application adhesives from 3M and Courtaulds (see Figure 3). There are some variations in the formulation of such adhesives, but we can safely neglect any reflections over the range of likely variations. Consider that even a much larger difference in index between glass and adhesive of 1.5 versus say 1.6 would only cause an interface reflectivity of 0.1%.

The extinction coefficient or imaginary part of the index k is usually negligible, but can be significant in the ultraviolet and varies between the two products tested (Figure 4). A much greater variation is found in PVB which is offered by the manufacturers in a range of colors for design purposes. The ultraviolet absorption can be accounted for, but it was found to produce only minor variance in the visible and solar transmittance for a range of test glazing. No such assumption can be made about the PVB and the spectral absorption coefficients must be known for each specific case. A simpler method than casting films is given in Section 8.2 for routine determination of *k*.

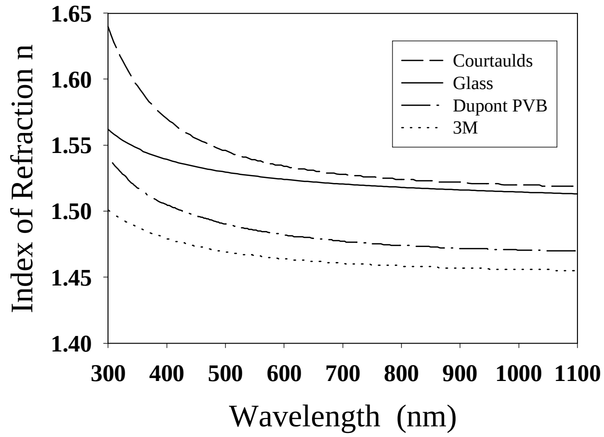

Figure 3. Index of refraction of adhesive components for applied films and laminates.

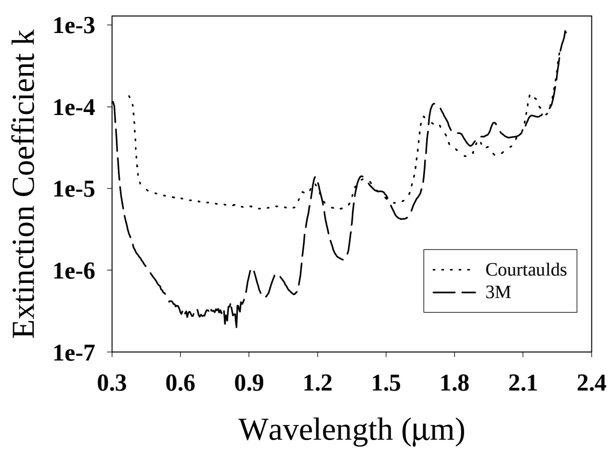

Figure 4. Extinction coefficient of adhesives for applied films.

8.2 Synthesis of Laminates without Coatings

Laminated glazing will generally be constructed of glass or rigid plastic sheets stuck together with layers of PVB. Any rigid component could have a coating, but the most likely scenario is to have a coating on only one of the glazing layers. In this section we will consider the case of a laminate of any number of layers but without any coatings as in the model of Figure 5.

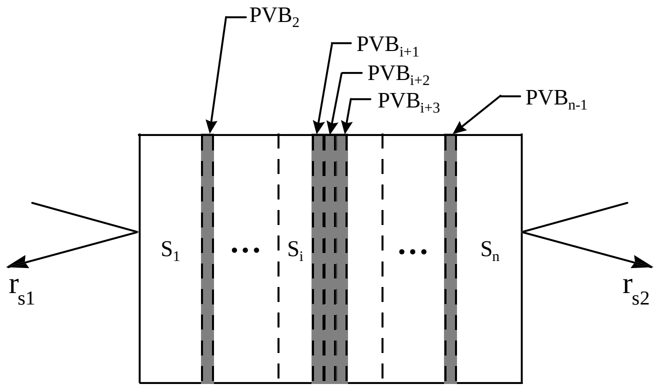

Figure 5. Structure of a laminate with alternating glass and PVB layers.

The properties of the rigid layers can be found by again using the monolithic-substrate Solution in Section 6.1. We then assume that reflection at interfaces between glass and PVB is zero. The index-matching between glass (or acrylic or polycarbonate) and PVB is so close that practically zero reflection takes place within the structure. There may be changes in absorption coefficient among the layers, but that can be handled as a sequence of internal transmittance factors, so that:

$$T_{lam} = \frac{(1-r_{s1})\tau_{s1}\ldots\tau_{\mathrm{PVB}}\ldots\tau_{\mathrm{sL}}(1-r_{sL})}{1-r_{s1}[\tau_{s1}\ldots\tau_{\mathrm{PVB}}\ldots\tau_{\mathrm{sL}}]^2 r_{\mathrm{sL}}}$$ **Equation 8-1**

$$R_{lam}^{f} = r_{s1} + \frac{(1-r_{s1})^2 r_{sL}[\tau_{s1}\ldots\tau_{\mathrm{PVB}}\ldots\tau_{\mathrm{sL}}]^2}{1-r_{s1}[\tau_{s1}\ldots\tau_{\mathrm{PVB}}\ldots\tau_{\mathrm{sL}}]^2 r_{\mathrm{sL}}}$$ **Equation 8-2**

$$R_{L}^{b} = r_{s2} + \frac{(1-r_{sL})^2 r_{s1}[\tau_{\mathrm{sL}}\ldots\tau_{\mathrm{PVB}}\ldots\tau_{s1}]^2}{1-r_{\mathrm{sL}}[\tau_{\mathrm{sL}}\ldots\tau_{\mathrm{PVB}}\ldots\tau_{s1}]^2 r_{s1}}$$ **Equation 8-3**

The ellipsis indicates that any number of additional internal transmittance factors may be added for 5-layers, 7-layers or higher numbers of periods. The above equations are perhaps more general than needed for current purposes. Usually, if the rigid layers are glass or plastic sheet, the reflection from each air/ substrate interface is identical. Then the 2 expressions for reflectance become one. If asymmetry is suspected, however, in a nominally monolithic substrate, due perhaps to the inherent asymmetry of the float process, then the more general forms could prove useful.

The properties of the rigid layers are obtained as usual by the monolithic-substrate solution. Having confirmed the index-matching condition for cast PVB layers as described above, a more practical approach can be taken to build a library of absorption factors τ_{PVB} for various colors of PVB. All of the standard colors are available as single layers of PVB between two uncoated substrates of known properties. Assuming first, that both surrounding layers are glass with known and equal surface reflectivities, $r_s=r_{s1}=r_{s2}$, then Equation 8-1 becomes:

Equation 8-4

$$T_{\text{lam}} = \frac{(1-r_s)^2 \tau_{s1}\tau_{s2}\tau_{PVB}}{1-r_s^{\,2}(\tau_{s1}\tau_{s2}\tau_{PVB})^2}$$

By comparison to Equation 6-1 we can write a solution analagous to Equation 6-7:

Equation 8-5

$$\tau_{PVB} = \frac{1}{\tau_{s1}\tau_{s2}} \frac{\left[\left(1-r_s\right)^4 + 4r_s^{\,2}T_{lam}^2\right]^{1/2} - \left(1-r_s\right)^2}{2r_s^{\,2}T_{lam}}$$

This solution is sufficient for most cases, but it should be noted that the more general problem of two unknown rigid layers can also be solved exactly. A PVB layer between two clear glasses is a common configuration, caution is advised in choosing the sample, because it is also common to mix several PVB layers of different primary colors between two pieces of glass to obtain a wider range of color. In this case the absorption coefficient of the different PVB layers cannot be obtained simultaneously. The problem of laminates with coatings is much more difficult and so will be deferred until later in this paper.

8.3 Synthesis of Applied Films with or without Coatings

An flexible film applied as a retrofit to existing glazing may have a very complex structure. Figure 6 shows the breakdown of a typical substrate with film applied. Two polyester (PET) substrates are glued together with a "laminating" adhesive. The inner PET layer may have a vacuum-deposited solar-control coating and the outer layer may have an abrasion-resistant coating. Sometimes the solar-control coating and outer PET layer are reversed to induce a lowered emittance. Our only concern is to isolate the effect of the interface between air and adhesive, so that we can combine the two halves of the structure. Accordingly, we adopt a model in which we view the adhesive as the substrate and all the rest of the applied film structure, including the polyester layers, as a complex interface. It is irrelevant that in this "interface" the vacuum deposited portion alone may consist of many unknown sublayers or that the polyester is relatively thick.

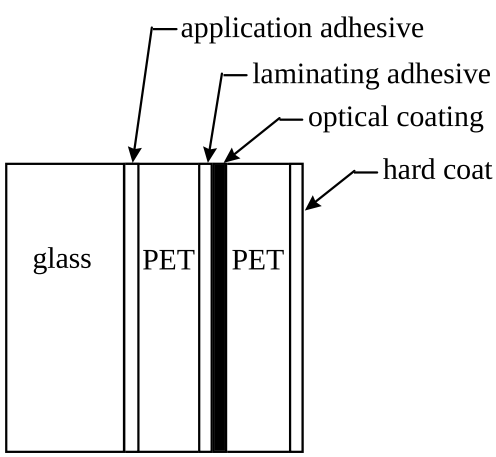

Figure 6. Structure of a typical applied film on glass substrate.

First we determine the properties of the applied film using the coated-substrate solution and specifically Equation 6-9, Equation 6-10, and Equation 6-11 to find t_{af}, r_{af}^{f}, and r_{af}^{b}, respectively. We know that the adhesive "substrate" has roughly the index as glass so that $r_s \approx r_a$. Also, the absorption coefficient of the adhesive can often be taken as zero, but we include it in the formulation for cases where more accurate spectral values are needed. It is now possible to write expressions for the properties of the glass substrate glued to the applied film by analogy to the laminate equations:

$$T_{af} = \frac{(1-r_s)\tau_s\ \tau_a\ t_{af}}{1-r_s\tau_s{}^2\tau_a{}^2 r_{af}^f}$$

$$R_{af}^f = r_s + \frac{(1-r_s)^2\tau_s{}^2\tau_a{}^2 r_{af}^f}{1-r_s\tau_s{}^2\tau_a{}^2 r_{af}^f}$$

$$R_{af}^b = r_{af}^b + \frac{t_{af}^2\tau_a{}^2\tau_s{}^2 r_s}{1-r_{af}^f\tau_a^2\tau_s{}^2 r_s}$$

8.4 Laminates with coatings

In the case of applied films above, the two materials that are joined together have known indices (and they are the same values), so that we can easily account for the change in interface reflectivity. In a coated laminate, however, the PVB may be put against an unknown thin-film material. We have not been able to find a general correlation that will account for the change in reflectance and transmittance at any such interface when the adjacent medium changes from air to PVB. It seems that the only acceptable solution involves a determination of the optical indices of the coating. A discussion of this problem is given in Section 9.3 on prediction of angular properties.

Knowing these optical indices we can make an estimate of the change in *internal* optical properties at each complex coated interface in the laminate using the most basic optical-index model in Section 4.3. Unlike the uncoated laminate, reflection will now occur at some internal interfaces. *External* properties for the complete laminate can then be calculated from the general functions given in terms of the *internal* optical properties of a stack in Section 4.2.

9. Extrapolation to Oblique Angles

It is possible, in principle, to calculate the reflection and transmission of a glazing material at any angle of incidence from the measured values at normal incidence. The solution is not exact or even single valued, in general, so the accuracy will depend on the choice of model. Choices are limited by the lack of experimental data, as well as by ignorance of the layer structure and thickness. Even the type of material may not be known, preventing a good choice for a dispersion model. The physical parameters needed to set up such a model are unlikely to come from the manufacturers for every case. It would be possible to set up an "expert system" that runs through a series of likely models and initial guesses for the parameters. For current purposes of standardization, only the simplest acceptable models can be used reliably. So, let us turn to the question of what is acceptable. The answer will of course depend on the requirements of the specific problem: In order of difficulty, the need is for average solar properties, average visible properties, color shift with angle, change in coating environments as in the case of laminates, and finally new coating design.

9.1 Angular Properties of Uncoated Glazing

The monolithic model of Section 6.1 should be and is sufficient for the solution to the optical indices of an uncoated substrate material. As suggested in that section, if the material is a clear or tinted glass, then begin with known values of the real part of the index. Extrapolation of the properties of uncoated glazing is then straightforward by using Fresnel's equation and Beer's law to obtain the internal properties and from there the external properties from Equation 6-1. This validity of this procedure was confirmed experimentally by Petit,[18] although he solved for the optical indices numerically rather than applying the exact solution.

9.2 Hemispherical Infrared Transmittance

This topic may seem out of place, but it is a specialized example of the extrapolation to oblique properties for a monolithic material. Windows are sometimes constructed with suspended or stretched layers of thin plastic film between glass panes to make triple or quadruple glazing. When these layers are covered with a low-emittance coating, then they are opaque in the infrared and an approximate correspondence can be found between normal and hemispherical emittance.[19] Recently, there has been renewed interest in windows which incorporate uncoated plastic film which is partially transparent in the thermal infrared (5-50 microns). The monolithic-substrate solution of Section 6.1 will predict the angle-dependent properties with sufficient accuracy and then the hemispherical transmittance and reflectance can be calculated via Equation 4-7 and Equation 4-8.

In practice, the only plastic film that has ever been used for this application is polyethelyne terepthalate (PET or polyester). There are some variations in composition among PET products, but they do not strongly affect the infrared properties. Another concern is that the absorption may become large enough in regime of molecular resonance so that reflectivity would depend on k. This is indeed the case over very narrow portions of the complex infrared spectrum of PET, but the effect is negligible when averaged over the full infrared spectral range. Optical constants have already been determined for this material and used to calculate the infrared hemispherical transmittance and emittance (or reflectance) as a function of PET thickness.[20] Exponential decay of transmittance is the primary effect and so this data can be fit with simple exponential forms valid from d=0.02-0.3 mm (approximately 0.001-0.01 inches):

$$T(d) = 0.720\exp(-14.2d + 11.1d^2)$$

$$E(d) = 0.846 - 0.811\exp(-15.0d + 18.0d^2)$$

9.3 Angular Properties of Coated Glazing

It is quite possible to unravel the internal structure and optical indices of very complex coating structures, thereby enabling the prediction of oblique properties. The most versatile measurement instruments, however, such as spectroscopic ellipsometers or variable angle radiometers, are neither widely available nor standardized. It might take many measurements and hours of analyzing structural and spectral models to arrive at an acceptable solution for the optical constants of a single glazing sample. In the usual context of coating analysis, “acceptable” refers to the ability to carry out further coating design, accounting for sensitive factors such as color shift with angle. For routine analysis of glazing products, fortunately, the requirements are not as great, at least not for prediction of total solar or visible properties.

One approach has been to apply the angular dependence of clear glass to all glazing materials. This would not work particularly well even for other uncoated glasses of different tint.[15] Another somewhat more accurate approach has been to apply the angle dependence of a tinted glass whose normal transmittance is matched to that of the coated glass. The most sophisticated model that we can apply to obtain effective optical indices without resorting to numerical solution is the monolithic model whose solution was given earlier in this paper; Simply apply the monolithic solution described above to the coated glazing as if it were uncoated. As far as we know, the monolithic model has not been used with such generality, for the simple reason that it would seem to be a very poor description of the true structure and index distribution in a coated glazing. Nevertheless, if only the angular dependence of average solar and visible properties is needed, then the monolithic model gives surprisingly good results, as we will demonstrate. There are actually several variants of the monolithic model. In this case we use the average of the two normal reflectance values to calculate angular transmittance, and the asymmetric values of normal reflectance to calculate the asymmetric angular variation of reflectance from each side. A complete analysis and comparison of the monolithic models is given in a paper devoted to the subject of angle dependence.[21]

For low-emittance coatings of the pyrolytic tin oxide types, the monolithic model fits to better than 2% over the entire angular range in both the visible and solar spectra. Silver multilayers also behave well in the visible with respect to the monolithic model, but tend tend to have much higher deviations over the solar because of the dominance of their metallic character in the solar infrared. Coatings containing transition metals tended to fit in transmittance about the same as noble metals typical of Figure 7. The maximum deviation of the monolithic model for this case is about 4% in transmission at 75 degrees. The glass model is significantly less accurate. The worst case attempted so far is a very high-index Si-based coating as shown in Figure 8. Here the maximum deviation of the monolithic model approaches 10% in reflection at 80 degrees. Errors in energy calculations caused by this large deviation are mitigated by the reduced irradiance at such high angles. In this case, the glass model is as poor in reflection as the monolithic model but far worse in transmission.

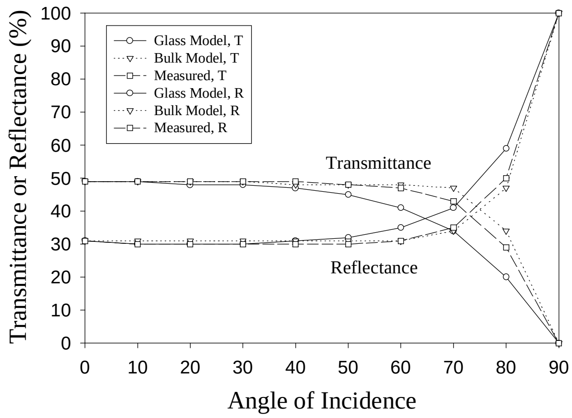

Figure 7. Models of angle-dependent optical properties for a transition metal coating.

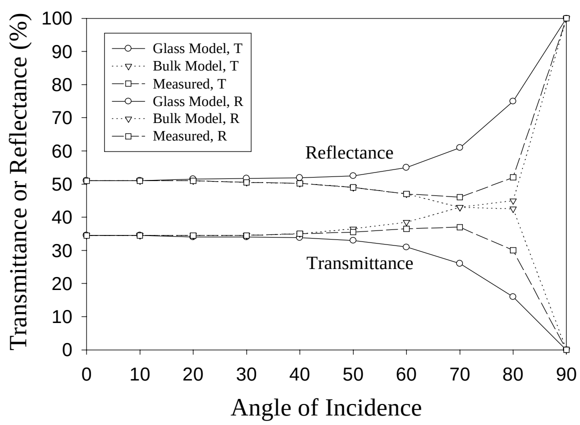

Figure 8. Models of angle-dependent optical properties for a high-index Si coating.

The monolithic model tends to break down when the index of the coating has a large deviation from the index of glass, although it is usually better than the glass model. For prediction of angle-dependent total properties, the monolithic model may then be a good choice, but a true thin-film model will be required for predicting the spectral interference effects for coated glazings and for simulation of coated laminate structures . Prudently assuming nothing about the coatings, Molina and Maestre[22] applied a single-film model to low-emittance glazings. Fitting wavelength-by-wavelength resulted in a variation in thickness at each wavelength which of course does not make physical sense. Thus they were forced to assume a constant thickness which caused the optical constants to vary with thickness. Nevertheless, the predictions for average solar properties were very close to measured values at 60 degrees, except in one case. Others[23,24,25,26,27] have performed global fits including thickness with extremely close agreement, but they began with some physical knowledge of the coating structure and independently determined initial values of the optical indices for the specific materials. These works illustrate the power of an analytical approach to predict accurate and detailed properties given some physical insight to the coating, but they do not fulfill the need for a blind analytical tool.

Montecchi and Polato[28] took an important first step in creating a test set of glazings with a range of optical characteristics. We looked for close analogues to that set in our Window 4.1 spectral data library which provides normal transmittance and reflectance of all U.S. and some European products.[29] A silver-based low-emittance coating was included in the set of Montecchi and Polato and we added a double-silver spectrally selective coating. Although the materials involved are the same, it is possible that the more complex structure would put the double-silver in a different category. We added two other glazings, PK_K_0.PLK and IP_i_0.INT, based on pyrolytic tin oxide and sputtered silver, respectively. Although duplicative of other glazing types in the set, these coatings are desirable for having been thoroughly characterized under IEA Task 10. Ten coatings were chosen as shown in Table 2.

Wavelength-by-wavelength fits with five single-film models converged for every coating (including those known to have multiple layers) with a wide range of guesses for the initial values. It should be mentioned that this type of fit often had to proceed from the shortest wavelength to prevent failure. Even these relatively poor fits would still predict solar properties better than the monolithic model in most cases. A successful global fit including thickness for all cases required us to try a set of five starting solutions approximating the range of coating types used. All of the models were of the single-film type, although some of the coatings were known to be multilayer types. As an example, Figure 9 shows that the global fit to the data for the Si-based coating is quite good over the entire solar spectrum. On the other hand, the deviation in the visible transmittance might be enough to cause problems with color predictions. Even a minor refinement in the model to two homogeneous layers improves the fit (Figure 10) to a satisfactory level for any purpose.

Table 2. Convergence of five models to a standard set of glazing materials.

	Ag	**SnO2:F**	**TiN**	**Stainless Steel**	**a-Si**
Material	**10 nm**	**100 nm**	**10 nm**	**10 nm**	**10 nm**
E78.CIG	**x**		**x**		
EE72.CIG			**x**		
Sn45.CIG	**x**			**x**	
IP_i_0.INT				**x**	
LowE.LOF		**x**			
PK_K_0.PLK		**x**			
S120.AFG	**x**			**x**	
B130.AFG				**x**	
P120.AFG		**x**		**x**	
EclR.LOF					**x**

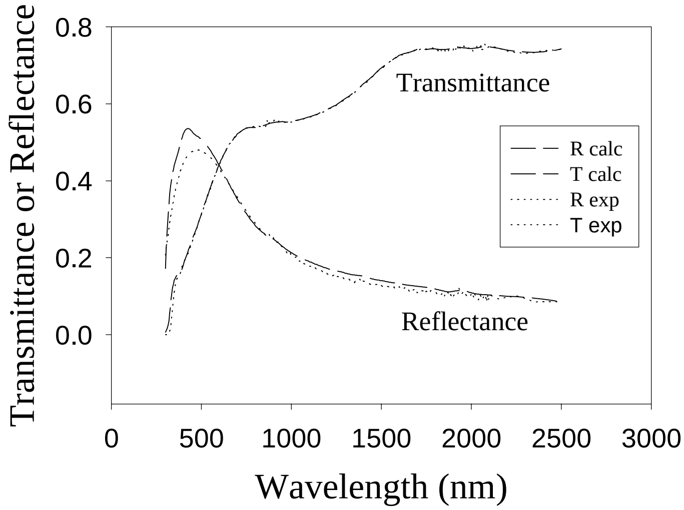

Figure 9. High-index coating fit with a single layer model and a blind starting guess

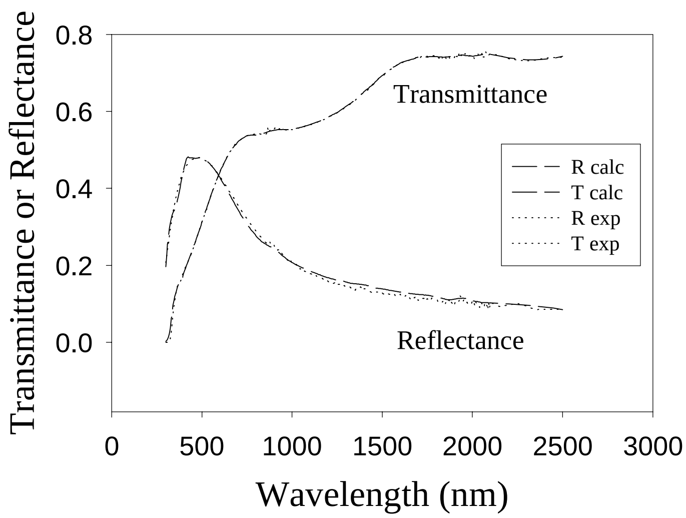

Figure 10. High-index coating fit with a realistic 2-layer model and close starting guess.

10. Conclusions

A comprehensive set of procedures is presented for calculating various properties of specular glazings. Some procedures enable the determination of properties needed to synthesize various structures from component properties or to predict the effects of variation in coating or substrate. These accurate and reliable procedures can greatly reduce the number of measurements that would be required for every combination of components. The monolithic model is shown to be an acceptable representation for angle-dependent properties of coated glazings when only visible or solar averaged properties are required, as in the case of annual energy calculations. Detailed spectral and directional properties can be determined with the use of more realistic thin-film models, but more work is required to build intelligence into these numerical solutions and make them more reliable.

11. Acknowledgment

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