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Estimating Returns to Scale and Scale Efficiency for Energy Consuming Appliances

A Data Envelopment Approach

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Abstract

Energy consuming appliances accounted for over 40% of the energy use and $17 billion in sales in the U.S. in 2014. Whether such amounts of money and energy were optimally combined to produce household energy services is not straightforwardly determined. The efficient allocation of capital and energy to provide an energy service has been previously approached, and solved with Data Envelopment Analysis (DEA) under constant returns to scale. That approach, however, lacks the scale dimension of the problem and may restrict the economic efficient models of an appliance available in the market when constant returns to scale does not hold. We expand on that approach to estimate returns to scale for energy using appliances. We further calculate DEA scale efficiency scores for the technically efficient models that comprise the economic efficient frontier of the energy service delivered, under different assumptions of returns to scale. We then apply this approach to evaluate dishwashers available in the market in the U.S. Our results show that (a) for the case of dishwashers scale matters, and (b) the dishwashing energy service is delivered under non-decreasing returns to scale. The results further demonstrate that this method contributes to increase consumers' choice of appliances.

1. Introduction

Energy consuming appliances provide their users with an energy service. Blum (2015) develops on production economics to represent the energy service provided by any energy consuming equipment as the output of a production function that relies on capital and energy to provide that service. The approach uses Data Envelopment Analysis (DEA) to estimate – under the assumption of constant returns to scale (CRS) technology – technical and allocative efficiency scores for a set of peer energy consuming equipment that delivers same amount of an energy service. Whereas the method can be useful to compare models of appliances with same capacity, it may restrict the set of technically efficient models when used to benchmark appliances whose capacity may vary across peer models and CRS does not hold. Take for example dishwashers, a ubiquitous appliance in households in the Unites States (U.S.). The U.S. Department of Energy (US DOE) classifies residential dishwashers into standard-sized and compact dishwashers. The classification is based on the number of place settings and serving pieces that can be accommodated in the appliance.[1] While there is a minimum capacity defined for a model to be classified as a standard-sized dishwasher, there is no upper limit to the number of place settings and serving pieces that can be washed in a dishwasher of that category. As a consequence, one can find in the market standard-sized dishwashers with different capacities. The larger the capacity of a model, the more likely the model will be more expensive and consume more energy to wash the dishes. Whether the increased equipment price and energy consumption comprise a technically efficient combination of capital and energy to provide the amount of energy service expressed by the model's capacity is not a straightforward conclusion, and has economic implications.

A technology that does not exhibit CRS over all of its possible range of outputs implies that the production of some units might be too large or too small for the amount of inputs used. This is particularly relevant in the case of appliances, as households are likely to approach their purchase choice to the optimal size of the appliance that meets their specific demand for an energy service. While several studies have looked at choices among household energy durables (Hausman, 1979; Revelt & Train, 1998; Lee et al, 2005; Lee et al, 2008), very few have looked at how economically efficient the suite of energy durable products available in the market are (Chung & Magrabi, 1994; Fernadez-Castro & Smith, 2002). To our knowledge, there are no existing studies looking at returns to scale and scale efficiency of energy using durables. We develop on Blum (2015) to evaluate returns to scale for appliances and estimate technical efficiency under the assumption of non-CRS. In addition, we estimate DEA scale efficiency scores for the technical efficient models that comprise the efficient frontier of the energy service delivered by the appliance. We further demonstrate this approach by estimating returns to scale and scale efficiency for dishwashers marketed in the U.S.,[2] and show that the approach increases consumers' choice of technically efficient models. In the following we discuss how to estimate returns to scale and scale efficiency for energy consuming appliances (Section 2); calculate technical and scale efficiencies, as well as estimate returns to scale

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[1] A standard-sized dishwasher has capacity equal to or greater than eight place settings plus six serving pieces; a compact dishwasher has capacity less than eight place settings plus six serving pieces (DOE, 2012).

[2] Richter (2010, 2011) has also studied the efficiency of dishwashers, yet with focus on consumer habits related to the use of the appliance.

for residential standard-sized dishwashers (Section 3); and conclude with some findings and limitations of the proposed approach (Section 4).

2. Returns to Scale and Scale Efficiency of Energy Consuming Appliances

The market for a certain type of appliance includes several alternative models. They provide consumers with some amount of the (same) energy service, at different equipment prices and levels of energy consumption. Models that operate with lower energy consumption will save on the energy bill for the life of the unit, but are likely to cost more than the ones that demand more energy to provide the same amount of the service. This results from higher development costs and lack of economies of scale that typically underlie the manufacturing of more energy efficient appliances.[3] In addition, models with larger capacity provide larger amounts of the energy service, and are more likely to be more expensive and to consume more energy than their peers with lower capacity. Therefore, a consumer shopping for an appliance faces a decision where (a) the upfront cost of the purchase is traded against operating costs over the life of the unit to be purchased, and (b) the total life-cycle cost of the unit is traded against the amount of energy service the consumer will enjoy.

Energy consuming equipment combines capital and energy to provide an energy service (Blum, 2015). It can be represented as a production technology that relies on two input factors – capital (k) and energy (g) – to yield an amount of the energy service (s). We develop on such approach and on DEA scale efficiency to evaluate returns to scale for energy consuming appliances. DEA has been broadly used to estimate scale efficiency, from banking (Wheelock & Wilson, 2012) to manufacturing (Kim, 2000) to health services (Brown & Pagán, 2006) to agriculture (Watkins et al, 2014). In our approach, we start with estimating DEA[4] technical efficiency scores for the set of $m = 1..M$ peer models of the appliance available in the market, under the alternative assumptions of constant returns to scale (CRS) and variable returns to scale (VRS).[5]

In our DEA linear program, each model m is a decision-making unit (DMU) that converts k and g into s. The technical efficiency θ_m of a model m under CRS is calculated from the following two-input (k_m,g_m), one-output (s_m), input-minimizing envelopment program:[6]

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[3] Higher prices may also result from the fact that some manufacturers consider energy efficiency as a value-added feature, and pack the more energy efficient designs with other high value-added, more costly design options.

[4] For more on DEA see Coopers et al (2011).

[5] DEA has also been used to compare durables by Doyle & Green (1991), for computer printers; Doyle and Green (1994) for microcomputers; Khouja (1995) and Baker and Talluri (1997) for industrial robots; Odeck and Hjalmarsson (1996) for trucks used in road construction, Fernandez-Castro and Smith (2002) for diesel cars, and Staat et al (2002) for compact cars.

[6] In the DEA program proposed by Blum (2015), it is assumed that all models provide the same amount of energy service. That allows for the third constraint in the DEA program in [1] to be reduced to $\sum_{i=1,M} \lambda_i \geq 1$. In this case, however, because we are assuming that different models can provide different amounts of the energy service, we do not use the reduced form of that constraint.

$$Min\ \theta_m \qquad [1]$$
s.t.

$$\sum_{i=1,M} k_i\lambda_i \leq \theta_m k_m$$

$$\sum_{i=1,M} g_i\lambda_i \leq \theta_m g_m$$

$$\sum_{i=1,M} s_i\lambda_i \geq s_m$$

$$\lambda_i \geq 0, i = 1..M$$

where:

θ_m technical efficiency of model m under CRS,
k_x capital required by model x,
g_x energy consumed by model x, and
λ_i weight of model i in the estimate of θ_m.

The technical efficiency θ'_m of a model m under VRS is calculated from a similar, two-input (k_m,g_m), one-output (s_m), input-minimizing envelopment program:

$$Min\ \theta'_m \qquad [2]$$
s.t.

$$\sum_{i=1,M} k_i\lambda_i \leq \theta'_m k_m$$

$$\sum_{i=1,M} g_i\lambda_i \leq \theta'_m g_m$$

$$\sum_{i=1,M} s_i\lambda_i \geq s_m$$

$$\sum_{i=1,M} \lambda_i = 1$$

$$\lambda_i \geq 0, i = 1..M$$

where θ'_m is the technical efficiency of model m under VRS, and the rest of the variables are as defined above.

The technical efficiency coefficients θ_m and θ'_m calculated in the DEA programs in [1] and [2] indicate the extent to what each model provides the energy service at minimum levels of capital and energy, under CRS and VRS respectively. The highest technical efficiency coefficient a model can score is 1, and that is the efficiency score of the technically efficient models that comprise the efficient frontier at each assumption of returns to scale. Models with efficiency scores lower than 1 are considered non-efficient, since the existing data suggest that the amount of energy service provided could be delivered with lower amounts of capital and/or energy.

We proceed with the evaluation of returns to scale for the appliance by comparing the technical efficiency coefficients θ_m and θ'_m. A model that is technically efficient under VRS, but not at CRS, belongs to a subset of the production possibilities set where models operate either at non-decreasing (NDRS) or non-increasing (NIRS) returns to scale. Whether such a model operates under NDRS or NIRS can be identified from the DEA technical efficiency coefficients θ^{nd} (NDRS) and θ^{ni} (NIRS), calculated from two slightly modified versions of the DEA program in [2] where the fourth constraint is replaced respectively by:

$$\sum_{i=1,M} \lambda_i \geq 1 \quad \text{(for NDRS)} \qquad [3a]$$

$$\sum_{i=1,M} \lambda_i \leq 1 \quad \text{(for NIRS)} \qquad [3b]$$

A model that is technically efficient under both VRS and NDRS [3a] operates below the optimal scale size. Similarly, a model that is technically efficient under both VRS and NIRS [3b] operates above the optimal scale size. A DEA scale efficiency coefficient σ_m can be estimated for a model m from the technical coefficients θ_m and θ'_m calculated above. The scale efficiency indicates how close the model is to its optimal scale size, given the amount of energy service it can deliver and the capital and energy it requires for that. The scale efficiency is calculated as (Bogetoft & Otto, 2011):[7]

$$\sigma_m = \frac{\theta_m}{\theta'_m} \qquad [4]$$

The highest scale efficiency coefficient a model can score is 1, and that is the efficiency score of the scale efficient models. Technical efficiency under CRS implies technical efficiency under VRS. Models that are technically efficient under CRS (θ_m =1) are therefore technically efficient under VRS (θ'_m =1) and, consequently, scale efficient (σ_m =1). They are also said to be in the *most productive scale size* (MPSS) (Banker, 1984) region of the production function. As for the non-efficient models under CRS (θ_m <1), accounting for the scale they operate in a VRS approach may eventually turn them technically efficient (θ'_m =1). In that case – and in the cases where a model is non-efficient under both CRS ($\theta_m < 1$) and VRS ($\theta'_m < 1$) – the ratio between the CRS and VRS technical efficiency coefficients expresses the model's loss from not operating at optimal scale size, namely its scale efficiency.[8]

3. Estimating Returns to Scale for Dishwashers

We rely on government and manufacturers' data to build a dataset with price (k), energy consumption per cycle (g) and capacity (s) of 372 representative standard-sized dishwashers (out of 636 models).[9] The notion of representative refers here to a dishwashing production unit,

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[7] For more on scale efficiency in DEA see Banker et al (1984).

[8] Notice that for any model m: $\theta_m \leq \theta'_m$ and $\sigma_m \leq 1$.

[9] Energy consumption and capacity are from the US DOE's Compliance Certification Database (DOE, 2015). The database lists all dishwashers approved by the US DOE. Price refers to manufacturer suggested retail price (MSRP), and is from manufacturer's catalogs.

characterized by a unique combination of the two input factors and the output. Table 1 provides descriptive statistics of the models in the dataset.

Table 1: Descriptive statistics of the dishwasher models

	Min	*Q25*	*Median*	*Q75*	*Max*	*Mean*
Price (*2015$*)	279.99	594.00	799.00	1199.00	3999.00	943.26
Energy (*kWh/cycle*)	1.00	1.21	1.26	1.30	1.47	1.26
Capacity (*place settings*)	8.0	12.0	14.0	15.0	16.0	13.4

We evaluate the DEA programs in [1] and [2] to estimate technical efficiency scores under CRS and VRS. Figure 1 presents the distribution of the technical efficiency scores. Technical efficiency under VRS is greater than under CRS, and a higher percentage of the models are technically efficient – or close to technical efficiency – under VRS than under CRS. We select the efficient models from the DEA program in [2] to determine the efficient frontier under VRS. Table 2 lists the efficient combinations of capital and energy for each dishwasher capacity under that assumption. The VRS efficient frontier is comprised of 23 models, but some of them are *weakly-efficient* models.[10] Take, for example, models number 87, 104, 88, 93, 91, 110, 109, 94 and 97 (the nine models listed in the very bottom of Table 2). They all require the same amount of energy to wash the same amount of dishes (same number of place settings), but have different prices. The difference in prices may result from additional features included in the more expensive models. Those features, however, while adding value to the model, do not contribute to the amount of energy service they deliver.[11] Consequently, from that subset of nine peer models, we account only for model 87 – the one with the lowest price – as the efficient one. The same situation occurs across models 59 and 60, and models 98 and 134. The efficient models under VRS, excluding the weakly-efficient ones, are highlighted (bold) in Table 2.

Table 2 also presents some other interesting cases. Models 674 and 687, for example, have same energy consumption yet different prices, which grow with capacity. On the other hand, models 61 and 88 have same price yet different energy requirements, which also grow with capacity. One can also observe the capital-energy substitution among models 250, 674, 61, 16 and 25, which have all same capacity and across which the energy requirements decrease as the price increases.

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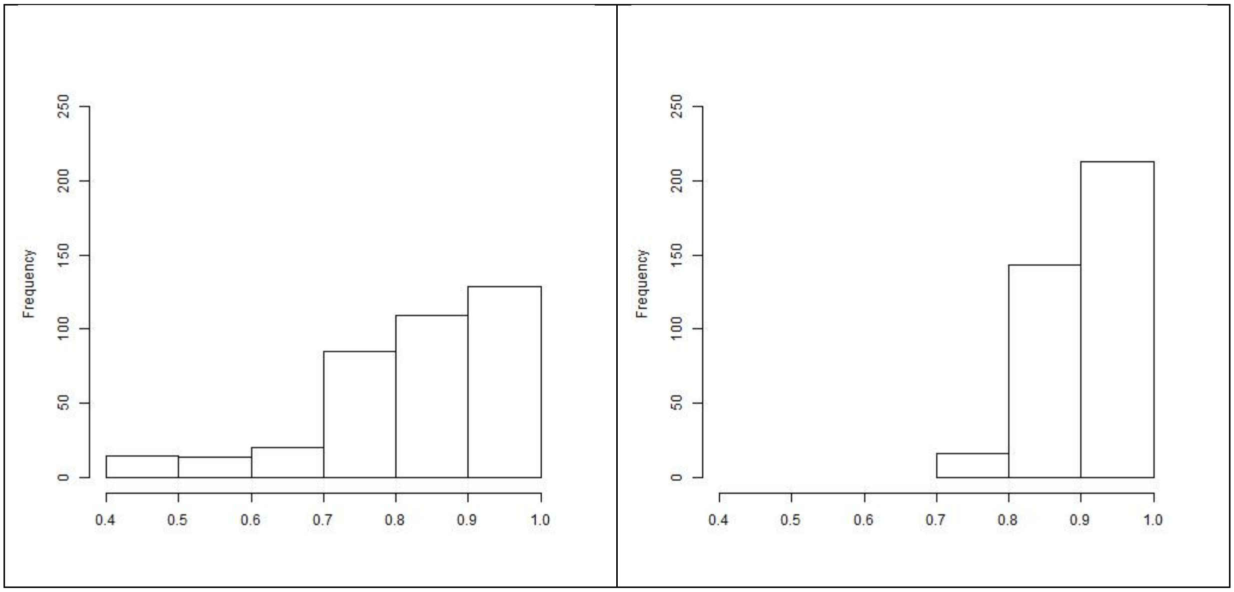
[10] The notion of *weakly-efficient* model refers to the case where a model's DEA technical efficiency coefficient is equal to 1, even though there is data evidence that the energy service can be delivered by another model that requires the same amount of one of the inputs and lower amount of the other input. For more on weakly efficiency in DEA please refer to Cooper et al (2011).

[11] Notice the only utility of a dishwasher we account for in this approach is its ability to wash a certain amount of dishes, regardless, for example, of the time it takes to wash the dishes, the noise the unit makes when washing the dishes or any other functionalities and aesthetic characteristics that may add value to the model.

Figure 1: Distributions of the DEA technical efficiency scores (Left to right: CRS, VRS).



We evaluate the DEA program variants in [3a] and [3b] to estimate technical efficiency scores under NDRS and NIRS. Table 3 presents the DEA coefficients calculated for four assumptions regarding returns to scale: CRS, VRS, NDRS and NIRS. While the VRS frontier is comprised of 13 models (excluding the weakly-efficient ones), the CRS frontier is restricted to only six of those models. The latter are the scale efficient models, and they all operate in the most productive scale size. The scale efficiency of the seven remaining models, because they are all technically efficient under VRS (θ'_m =1), corresponds to their CRS technical efficiency coefficient. Notice that the seven scale inefficient models, despite not operating at the most productive scale size, still represent efficient combinations of capital and energy given the amount of dishes they can wash. A consumer shopping, for example, for a 13-place setting dishwasher would be misled by the CRS frontier that does not include any model at that capacity.

Results in Table 3 also show that the scale inefficient models are all efficient under NDRS. That indicates they are all part of the dishwashing production function where the energy service is delivered with NDRS. This is relevant information that shows that – in the case of dishwashers – scale matters, and that the dishwashing energy service is provided under NDRS.[12]

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[12] Notice, for example, that none of the models with capacity equal to 8 place settings are in either of the frontiers.

Table 2: Efficient combinations of capital and energy under VRS

Model	Capacity (s)	Price (k)	Energy (g)
734	12	280	1.288
59	12	1465	0.995
60	12	1532	0.995
670	13	379	1.209
250	14	349	1.251
674	14	399	1.200
61	14	899	1.088
16	14	1050	1.074
25	14	1420	1.070
687	15	444	1.200
703	15	594	1.140
98	15	1899	1.112
134	15	1999	1.112
271	16	449	1.279
87	16	849	1.205
104	16	894	1.205
88	16	899	1.205
93	16	944	1.205
91	16	949	1.205
110	16	994	1.205
109	16	999	1.205
94	16	1299	1.205
97	16	1499	1.205

Table 3: Efficient combinations of capital and energy with their corresponding DEA coefficients*

Model	Capacity (s)	Price (k)	Energy (g)	DEA CRS (θ)	DEA VRS (θ')	DEA NDRS (θ^{nd})	DEA NIRS (θ^{ni})
734	12	280	1.288	1	1	1	1
59	12	1465	0.995	0.89677	1	1	0.89677
670	13	379	1.209	0.90945	1	1	0.90945
250	14	349	1.251	1	1	1	1
674	14	399	1.200	0.95865	1	1	0.95865
61	14	899	1.088	0.96662	1	1	0.96662
16	14	1050	1.074	0.97683	1	1	0.97683
25	14	1420	1.070	0.97567	1	1	0.97567
687	15	444	1.200	0.99349	1	1	0.99349
703	15	594	1.140	1	1	1	1
98	15	1899	1.112	1	1	1	1
271	16	449	1.279	1	1	1	1
87	16	849	1.205	1	1	1	1

* Excluding the weakly-efficient models.

4. Conclusion

Energy consuming equipment combines capital and energy to provide an energy service. The models of a given type of energy consuming equipment comprise the production possibilities set of the energy service they provide. DEA can be used to estimate technical efficiency coefficients for those models and to estimate the efficient frontier of the service provided. Whereas this has been previously proposed, the approach does not account for possible non-CRS of the technology when models of the equipment have different capacities. We elaborate on that approach to estimate returns to scale for energy consuming appliances, and to estimate scale efficiency for the technical efficient models available in the market. We further demonstrate our approach using market data on price, energy consumption and capacity of dishwashers.

Looking at scale efficiency of household appliances is particularly important, as several microeconomic and econometric models that evaluate the use and choice of this equipment often make simplifying assumptions about the returns to scale of the technology set. Further, evaluating the efficient frontier of a non-CRS technology under the assumption of CRS can be misleading, as it may restrict the number of efficient production possibilities. For the case of dishwashers our results show that scale matters, and that the dishwashing energy service is delivered under non-decreasing returns to scale.

The approach introduced here is suitable to assess any energy durable. It is to be verified, however, whether for washing equipment – including dishwashers – water is a relevant input factor, in which case the DEA programs proposed should be redefined to account for water consumption.

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