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How to proceed with competing alternative energy technologies: A real options analysis☆

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article info abstract

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Concerns about $CO₂$ emissions create incentives for the development and deployment of energy technologies that do not use fossil fuels. Indeed, such technologies would provide tangible benefits in terms of avoided fossil-fuel costs, which are likely to increase as restrictions on $CO₂$ emissions are imposed. However, a number of challenges need to be overcome prior to market deployment, and the commercialisation of alternative energy technologies may require a staged approach given price and technical risk. We analyse how a firm may proceed with staged commercialisation and deployment of competing alternative energy technologies. An unconventional new alternative technology is one possibility, where one could undertake cost-reducing production enhancement measures as an intermediate step prior to deployment. By contrast, the firm could choose to deploy a smaller-scale existing renewable energy technology, and, using the real options framework, we compare the two projects to provide managerial implications on how one might proceed.

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1. Introduction

Climate change due to greenhouse gas emissions from human activities poses a threat to the planet, and the adoption of alternative energy technologies that do not rely on fossil fuels has been proposed as part of a mitigation strategy in [IPCC \(2007\)](#page-13-0). Indeed, curbing greenhouse gas emissions from electricity generation would facilitate the stabilisation of $CO₂$ concentration in the atmosphere. Consequently, international agreements are being proposed to limit emissions levels via mechanisms such as the EU Emission Trading System. By accounting for the marginal social cost of pollution, it is anticipated that such programmes will encourage investment in cleaner energy technologies.

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The development of new energy technologies to meet growing demand for electricity will require considerable effort and expense, however. Typical phases in new technology development prior to deployment include research and development (R&D), demonstration, and commercialisation [\(Rothwell, 2007\)](#page-13-0). In the latter phase, a successful prototype is prepared for deployment via incremental improvement of its performance and cost effectiveness based on learning effects. Thus, the first-of-a-kind unit may be enhanced when going to an nth-of-a-kind one. In this setting, the learning curve relates the cost of a technology to the accumulation of experience during its commercialisation stage. [Rothwell \(2007\)](#page-13-0) takes an R&D manager's perspective to consider the technology development process by using advanced energy systems as an example for allocating funds between R&D and demonstration phases in technology development. Optimal decisions are found in a model where funding impacts stage durations, target costs, and stage transition probabilities. Empirically, [Kobos et al. \(2006\)](#page-13-0) estimate learning curves of wind and photovoltaic technologies, where learning is achieved both by R&D and by actual installation and operation.

Since the benefit from deploying a new energy technology depends on uncertainty in energy prices and technological factors, proceeding with new technology development in phases may maximise overall project value. Such an approach is amenable to real options analysis, which considers the interaction between uncertainty in cash flows and managerial flexibility, e.g., discretion over timing of decisions [\(Dixit](#page-13-0) [and Pindyck, 1994](#page-13-0)). In contrast to the traditional now-or-never

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discounted cash flow (DCF) approach, real options trade off in continuous time the marginal benefits and costs of making decisions under uncertainty. Furthermore, it is able to cope with investment opportunities that have embedded options, such as discretion to abandon, expand, or modify existing projects, which is also not possible via the now-or-never DCF approach. Indeed, several papers have applied real options analysis to R&D management. Early work such as [Roberts and Weitzman \(1981\)](#page-13-0) (actually predating the term "real options") analyses an investment project where R&D effort reduces the variability of the cash flows of the project. [Newton and](#page-13-0) [Pearson \(1994\)](#page-13-0) use the Black–Scholes formula to value R&D, [Grenadier](#page-13-0) [and Weiss \(1997\)](#page-13-0) use real options to analyse the decision to replace an existing technology with a randomly arriving new technology, and [Jensen and Warren \(2001\)](#page-13-0) consider two distinct phases in project development. Recent work such as [Malchow-Møller and Thorsen](#page-13-0) [\(2005\)](#page-13-0) has extended real options analysis to repeated investment. Within the context of renewable energy (RE) R&D projects, both [Davis](#page-13-0) [and Owens \(2003\)](#page-13-0) and [Siddiqui et al. \(2007\)](#page-13-0) have used real options to value government programmes under uncertain fossil-fuel prices.

In this paper, we take the perspective of a private firm that would like to commercialise a new alternative energy technology to meet given electricity demand. This unconventional energy technology (UET), which has been through the R&D and demonstration phases, requires funding to move down the learning curve via an intermediate enhancement step in order to make it viable for the market. The enhancement step reduces operating costs, which are uncertain. Due to economies of scale, there is a minimum capacity level necessary to develop this new technology. By contrast, the firm may choose to deploy more of an existing RE technology, which has a lower capacity since it is based on an intermittent or a space-constrained resource such as wind or hydropower, whereas the UET would provide an opportunity to capture more market share. Given uncertainty in the electricity price and the UET operating cost, we use the real options approach to determine both technology-deployment and timing decisions. We assume that due to limited funds, the firm faces such a choice between two mutually exclusive projects. Consequently, we use the approach of [Décamps et al. \(2006\)](#page-13-0) in order to find optimal investment and waiting regions. In energy economics, this technique has been used to evaluate investment timing and sizing decisions in wind and transmission projects (see [Fleten et al. \(2007\)](#page-13-0) and [Siddiqui](#page-13-0) [and Gupta \(2007\)](#page-13-0), respectively). We find that the interaction of the two mutually exclusive projects increases the value of the firm's entire alternative energy portfolio while making the selection of any given technology less likely.

The remainder of this paper is organised as follows:

- Section 2 states the assumptions, illustrates the decision-making problem via a simple now-or-never DCF example, and formulates the problem using the real options approach for various technology cases.
- [Section 3](#page-4-0) presents the results of the numerical examples.
- [Section 4](#page-10-0) summarises the contribution of this work, discusses its limitations, and offers directions for future research.

2. Model and assumptions

In formulating the firm's decision-making problem under uncertainty, we assume that the long-term electricity price is exogenous to the model and, thus, unaffected by any technology-deployment decisions. This is justified by the fact that although the scale of the potential investment, i.e., 1250 MW $_{e}$,¹ may be a sizeable fraction of a small state's installed capacity, it is, nevertheless, minor compared to the worldwide consumption of energy. Furthermore, we analyse a one-time investment opportunity, the effects of which are unlikely to influence the long-term electricity price as it will have already anticipated the consequences of such technology adoption.

We assume that the long-term, time-t electricity price, P_t (in \$/ MWh), depends chiefly on fossil fuels and evolves according to a geometric Brownian motion (GBM) process, i.e., $dP_t = \alpha P_t dt + \sigma P_t dz_t$, where α is the annualised growth rate of P_t , σ is the annualised percentage volatility of P_t , and dz_t is an increment to the Wiener process.2 The firm may capture additional electricity demand via either an existing RE technology at constant operating cost C^E (in \$/ MWh) or the UET at operating cost C_t (in \$/MWh), which evolves stochastically according to a GBM process once the firm starts commercialisation, i.e., $dC_t = -\lambda C_t dt + \sigma_c C_t dz_t^C$. Here, λ is the annualised rate of decrease in the UET's operating cost, while σ_c denotes the level of technical risk associated with the commercialisation programme. All cash flows are discounted using the real risk-adjusted rate of return, ρ , which is assumed to be greater than α . We assume that the UET's operating cost is uncorrelated with fluctuations in the long-term electricity price.³ If the UET is deployed early, then there will be increased costs associated with lack of movement down the learning curve. For this reason, we assume that $C_0 > C^E$, but that $C_{t^*} < C^E$ for some t^* >0 once the commercialisation programme has lowered the cost of UET generation sufficiently.

If the firm commercialises the UET, then it must pay a lump sum of I (in \$), which covers the start-up cost of the programme. After the UET commercialisation programme has been under way to lower the operating cost, the firm may decide to deploy the technology to meet the electricity demand, X (in MWh). In this case, X MWh of electricity are provided by the UET each year, and the learning effects accrue indefinitely, thereby reducing the cost of electricity production forever. Instead of undertaking the staged commercialisation and deployment of UET, the firm may choose to proceed with an existing RE technology by paying a lump-sum cost $I^E < I$ (also in \$), which allows it to meet a more modest electricity demand, X^E (in MWh), per year forever at operating cost C^E plus the right to switch to the UET commercialisation programme at any point by paying I. Due to the intermittency and lack of additional suitable sites for RE technologies, we assume that $X^E < X$. Furthermore, we assume that all investment and deployment options are perpetual, which not only eases the analysis, but also reflects the flexibility a firm may have over timing of its projects.

The limitations of our approach include the assumption of an exogenous long-term electricity price, a lower capacity for additional existing RE technology installment, and the treatment of the existing RE technology and UET as mutually exclusive alternative projects due to limited project funding.⁴ We have provided justifications for these assumptions, but for future work, it would be instructive to relax them. In particular, optimising the level of funding for a portfolio of energy technology programmes would be closer to a typical firm's decision-making problem. In [Sections 2.1, 2.2, and 2.3](#page-2-0), we formulate the firm's problem and find analytical solutions where possible. However, for illustration, we first present a now-or-never DCF analysis of UET commercialisation.

A firm with a now-or-never decision to deploy the UET (with ongoing operating cost reduction due to learning effects) would first

¹ Assuming a 90% capacity factor, this corresponds to approximately 10 TWh of annual energy output.

 $^{\rm 2}$ The difference between the long-term electricity price and the spot price is related to the fact that electricity prices are affected by fluctuations in short-term supply and demand and in expectations regarding long-term supply and demand. One can think of the long-term electricity price as the electricity price where short-term deviations have been removed from the spot price, so that the only source of uncertainty in the long-term electricity price is long-term uncertainty, related to changing expectations regarding future supply and demand. See [Schwartz \(1998\)](#page-13-0) for an example of how this can be estimated and operationalised.

³ Incorporating instantaneous correlation between dz_t and dz_t^C poses no analytical difficulty in our model.

⁴ The mutually exclusive investment strategy may also arise out of necessity because proceeding with both technologies may stretch the firm's capabilities.

calculate the expected net present value (NPV) of such an investment and then determine the electricity price threshold, $P^{NN}(C_0)$, at which to deploy it. Since the expected NPV of the UET project, $V^{NN}(P, C_0) - I$, is the expected PV of the operating cash flows minus the investment cost, upon setting it equal to zero we have:

$$
V^{NN}(P, C_0) - I = 0
$$

\n
$$
\Rightarrow X(\int_0^\infty \mathcal{E}[P_t | P] e^{-\rho t} dt - \int_0^\infty \mathcal{E}[C_t | C_0] e^{-\rho t} dt) - I = 0
$$

\n
$$
\Rightarrow X\left(\frac{P}{\rho - \alpha} - \frac{C_0}{\lambda + \rho}\right) = I
$$

\n
$$
\Rightarrow P^{NN}(C_0) = \frac{(\rho - \alpha)}{X} \left(I + \frac{C_0 X}{\lambda + \rho}\right).
$$
 (1)

For a simple numerical example, we use the following parameters: $P_0 = $60/MWh$, $\alpha = 0.04$, $\rho = 0.10$, $I = 1 billion, $\lambda = 0.04$, $C_0 =$ \$100/MWh, and $X = 1 \times 10^7$ MWh (10 TWh). Inserting these values into Eq. (1), we find $P^{NN}(C_0)$ = \$48.86/MWh, i.e., the UET commercialisation project should proceed for a long-term electricity price above this threshold. With an initial electricity price of \$60/MWh, this project would be favourable according to the now-or-never DCF approach and would be worth \$1.86 billion even though $P_0 < C_0$. While this may seem like a large return for an investment of \$1 billion, it should be noted that we are analysing only the commercialisation stage of the project, i.e., the R&D and demonstration stages have been completed, and the expected electricity price (UET operating cost) will increase (decrease) indefinitely upon deployment. In the next section, we consider how the investment decision is affected when there is discretion to defer UET commercialisation under uncertainty.

2.1. Case 1: no existing renewable energy technology

For now, we ignore the opportunity to use the existing RE technology and focus on the staged commercialisation of the UET project. The state transition diagram for this simplified problem may be seen in [Fig. 1](#page-3-0). There are, thus, three states of the world:

- State 0, in which no commercialisation programme exists.
- State 1, in which the commercialisation programme exists, thereby decreasing the UET operating cost, C_t , but no revenues from electricity sales accrue since the UET has not been deployed.
- State 2, in which UET has been deployed with ongoing learning that lowers its operating cost and is accruing savings relative to fossilfuel generation.

In order to solve the firm's UET commercialisation problem, we start in state 2 and work backwards. Given that UET has been deployed and will operate forever, its expected PV is simply the difference between the PVs of two perpetuities:

$$
V_2(P,C) = X \left(\frac{P}{\rho - \alpha} - \frac{C}{\lambda + \rho} \right). \tag{2}
$$

In state 1, while the commercialisation programme is ongoing, the firm holds a perpetual option to deploy UET. The value of this option to the firm is $V_1(P, C)$, which we find by applying Itô's Lemma to expand dV_1 and then use the Bellman Equation to equate the expected appreciation of V_1 to the instantaneous rate of return on V_1 . As we show in [Appendix A](#page-11-0), the value of the deployment option from state 1 is:

$$
V_1(P,C) = a_1 C^{1-\gamma_1} P^{\gamma_1}
$$
\n(3)

where $\gamma_1 = \frac{-(\alpha + \lambda - \frac{1}{2}(\sigma^2 + \sigma_c^2)) + \sqrt{(\alpha + \lambda - \frac{1}{2}(\sigma^2 + \sigma_c^2))}^2 + 2(\sigma^2 + \sigma_c^2)(\lambda + \rho)}{\sigma^2 + \sigma_c^2}$ $\frac{2}{\sigma^2 + \sigma_c^2}$ > 1 is an exogenous constant and $a_1 > 0$ is an endogenous constant. Intuitively, the value of the option to deploy is worth more (less) when the electricity price (UET operating cost) is higher (lower). The UET price–cost threshold ratio for deployment is:

$$
p^* = \left(\frac{\gamma_1}{\gamma_1 - 1}\right) \frac{\rho - \alpha}{\lambda + \rho}.\tag{4}
$$

[Dixit and Pindyck \(1994\)](#page-13-0) note that p^* is increasing in both σ and σ_c , i.e., greater uncertainty postpones deployment even as the value of the option to deploy is worth more.

Finally, in state 0, the value of the option to start UET commercialisation is:

$$
V_0(P; C_0) = A_1 P^{\beta_1}
$$
\n(5)

where $\beta_1 = \frac{-\left(\alpha-\frac{1}{2}\sigma^2\right) + \sqrt{\left(\alpha-\frac{1}{2}\sigma^2\right)^2 + 2\sigma^2\rho^2}}{\sigma^2}$ $\frac{1}{\sigma^2}$ > 1 is an exogenous constant and $A_1 > 0$ is an endogenous constant that depends on C_0 , which is constant in state 0 since learning does not occur until state 1 (see Eqs. (A-14) to (A-19)). Note that the value of the option to start UET commercialisation is an increasing convex function of the long-term electricity price and decreases with the initial UET operating cost, C_0 . Here, greater uncertainty in the electricity price increases the value of waiting, thereby delaying investment. This is also observed from the optimal investment threshold price:

$$
P_I = \left[\left(\frac{I\beta_1}{\beta_1 - \gamma_1} \right) \frac{\left(C_0 \right)^{\gamma_1 - 1}}{a_1} \right] \frac{1}{\gamma_1}.
$$
\n
$$
(6)
$$

Standard comparative statics from [Dixit and Pindyck \(1994\)](#page-13-0) imply that greater uncertainty in the output price causes β_1 to decrease, which results in a higher threshold price, P_I .

Although we solve the problem backwards, in terms of implementation, the firm would first wait until the electricity price reaches P_I before paying I to enter state 1. Once the UET learning process begins, the operating cost decreases stochastically. Crucially, the firm does not care about the absolute level of the cost of UET generation; instead, it deploys the UET once the ratio of the electricity price to the cost of UET operation reaches p^* . What makes this possible is the homogeneity in the value of the option to deploy UET and the conglomeration of any deployment costs into the investment cost, $I⁵$ We will illustrate the intuition with a numerical example in [Section 3.1.](#page-4-0) Before that, we formulate the firm's problem with a mutually exclusive investment opportunity in an existing RE technology.

2.2. Case 2: existing renewable energy technology without switching option to the unconventional energy technology

We now include the flexibility of using the existing RE technology but without the possibility of reverting to the staged commercialisation of the UET project. Here, there are four states of the world (see [Fig. 2\)](#page-3-0):

- State 0, in which neither the UET commercialisation programme exists nor the existing RE technology is deployed.
- State E, in which the existing RE technology has been deployed to meet the available electricity demand.
- State 1, in which the UET commercialisation programme exists, thereby decreasing the UET operating cost, C_t , but no electricity revenues accrue since the UET has not been deployed.

Even if C_t were correlated with P_t , then the result would hold as the expected NPV of the deployed UET depends only on the ratio of the long-term electricity price to the cost of UET operation. Nevertheless, it should be noted that the real options approach becomes analytically intractable with more than two risk factors.

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Fig. 1. State transition diagram for a UET commercialisation project with an intermediate learning step in state 1 that reduces the operating cost prior to deployment.

• State 2, in which UET has been deployed with ongoing learning that lowers its operating cost and is accruing revenues.

Since the switching option is not available, we assume that in state 0, the firm can choose either the existing RE technology or the UET commercialisation programme; however, once state E is entered, it is no longer possible to switch to the UET programme. Following [Décamps et al. \(2006\)](#page-13-0), we note that the value of the option to meet electricity demand via alternative energy sources, $V^{\rm ex}_{0}(P; \, C_0, \, C^E)$, may be dichotomous for small enough σ , with immediate investment occurring in the existing RE technology (UET) for $P_E^{ex} \le P \le P_F^{ex}$ $(P \geq P_G^{\text{ex}})$; specifically, we may have:

$$
V_0^{\text{ex}}(P; C_0, C^E) = \begin{cases} A_1^{\text{ex}} P^{\beta_1} & \text{if } 0 \le P < P_E^{\text{ex}} \\ F^{\text{ex}} P^{\beta_1} + G^{\text{ex}} P^{\beta_2} & \text{if } P_F^{\text{ex}} < P < P_C^{\text{ex}} \end{cases}
$$
(7)

Here, β_1 is defined as in [Section 2.1](#page-2-0), while $\beta_2 = \frac{-(\alpha - \frac{1}{2}\sigma^2) - \cdots}{\sigma^2}$ $\sqrt{\left(\alpha-\frac{1}{2}\sigma^2\right)^2+2\sigma^2\rho^2}$ $rac{2}{\sigma^2}$ \rightarrow 0. The first branch of $V_0^{\text{ex}}(P; C_0, C^E)$ represents the option to deploy the existing RE technology from state 0, while the second branch is the option either to deploy the existing RE technology (if the electricity price decreases) or to initiate the UET commercialisation programme (if the price increases). As $V_E^{\text{ex}}(P; C^E) = X^E\left(\frac{P}{\rho - \alpha} - \frac{C^E}{\rho}\right)$ is the expected PV of profits from deploying the existing RE technology, we use valuematching and smooth-pasting conditions in [Appendix B](#page-12-0) to determine its investment threshold and the option value coefficient:

$$
P_E^{\text{ex}} = \left(\frac{\beta_1(\rho - \alpha)}{X^E(\beta_1 - 1)}\right) \left[\frac{C^E X^E}{\rho} + I^E\right]
$$
\n(8)

$$
A_1^{\text{ex}} = \frac{(P_E^{\text{ex}})^{1-\beta_1} X^E}{\beta_1(\rho - \alpha)}.
$$
\n(9)

Although the value functions in states 1 and 2 are the same as those as in Eqs. (3) and (2), respectively, the endogenous constants, F^{ex} and G^{ex} , and the thresholds, P_F^{ex} and P_G^{ex} , for the second branch of $V_0^{ex}(P;$ C_0 , C^E) have no analytical solution and must be determined numerically for specific parameter values via appropriate value-matching and smooth-pasting conditions (see Eqs. (B-3) through (B-6)). We also know that $P_F^{\text{ex}} < \tilde{P}_{\text{c}}^{\text{ex}}$, where $\tilde{P}_{\text{c}}^{\text{ex}}$ is the price at which $V_E^{\text{ex}}(P; C^E) - I^E$ and $V_1(P, C=C_0)-I$ intersect. Since the latter function is nonlinear, \tilde{P}^{ex} itself must be found numerically. Of course, for large values of σ , it may be preferable to skip considering the state E option, in which case the problem reduces to one of [Section 2.1:](#page-2-0) the key is to check whether $A_1 > A_1^{\text{ex}}$. If so, then the firm can proceed as in [Section 2.1](#page-2-0) ([Dixit, 1993\)](#page-13-0).

From state 0, if the threshold P_E^{ex} is reached, then the existing RE technology is deployed to produce X^E TWh of electricity each year forever at an operating cost of C^E . By contrast, no action will be taken if the electricity price is between P_F^{ex} and P_G^{ex} , while immediate initiation of the UET commercialisation programme (state 1) will occur if the latter threshold price is exceeded. We next consider the case with a switching option, i.e., in which it is possible to proceed from state 1 to E.

2.3. Case 3: existing renewable energy technology with switching option to the unconventional energy technology

Here, the setup is the same as in [Section 2.2](#page-2-0) except that once state E is entered, it is possible for a subsequent transition to state 1 by paying the full UET commercialisation investment cost of I (see [Fig. 3](#page-4-0)). Therefore, while the value functions in states 1 and 2 are still defined by Eqs. (3) and (2), respectively, those in states 0 and E are as follows (see [Décamps et al. \(2006\)](#page-13-0)):

$$
V_0^{\text{sw}}(P; C_0, C^E) = \begin{cases} A_1^{\text{sw}} P^{\beta_1} & \text{if } 0 \le P < P_E^{\text{sw}} \\ F^{\text{sw}} P^{\beta_1} + G^{\text{sw}} P^{\beta_2} & \text{if } P_F^{\text{sw}} < P < P_G^{\text{sw}} \end{cases}
$$
(10)

Fig. 2. State transition diagram with a mutually exclusive existing RE technology option. The firm may choose either to deploy an existing RE technology or to start a UET commercialisation project. If the latter avenue is selected, then the firm may subsequently deploy UET.

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Fig. 3. State transition diagram with a mutually exclusive existing RE technology option and a possibility to switch to the UET. The firm may choose either to deploy an existing RE technology or to commercialise the UET. If the former avenue is selected, then the firm may subsequently switch to the UET commercialisation phase, from where it is then possible to deploy the UET.

$$
V_E^{sw}(P; C_0, C^E) = X^E \left(\frac{P}{\rho - \alpha} - \frac{C^E}{\rho} \right) + B^{sw} P^{\beta_1} \text{ for } 0 \le P < P_{E1}^{sw}.\tag{11}
$$

Again, if $A_1 > A_1^{\text{sw}}$, then the approach of [Section 2.1](#page-2-0) may be used, i.e., there is no need to consider the existing RE technology. However, for small values of σ , it may be relevant, in which case the last term in Eq. (11) is the value of the option to switch to state 1 by paying the full investment cost of the UET commercialisation programme. The endogenous constant, B^{sw} , and the switching threshold price, P_{E1}^{sw} , are found numerically via the value-matching and smooth-pasting conditions in Eqs. (C-1) and (C-2).

The endogenous constant, A_1^{sw} , and the existing RE technologydeployment threshold price, P_{E}^{sw} , for the first branch of $V_0^{\text{sw}}(P; C_0, C^E)$ are found by value-matching and smooth-pasting conditions involving $V_0^{\text{sw}}(P; C_0, C^E)$ and $V_E^{\text{sw}}(P; C_0, C^E)$ as indicated in Eqs. (C-3) and $(C-4)^6$:

$$
P_E^{\text{sw}} = \left(\frac{\beta_1(\rho - \alpha)}{X^E(\beta_1 - 1)}\right) \left[\frac{C^E X^E}{\rho} + I^E\right]
$$
\n(12)

$$
A_1^{\text{sw}} = B^{\text{sw}} + \frac{(P_E^{\text{sw}})^{1-\beta_1} X^E}{\beta_1 (\rho - \alpha)}.
$$
\n(13)

In other words, $P_E^{\text{sw}} = P_E^{\text{ex}}$ and $A_1^{\text{sw}} = B^{\text{sw}} + A_1^{\text{ex}}$, i.e., the existing RE technology with the switching option is deployed at the same price threshold as the one without it. However, the embedded option to switch to the UET commercialisation programme increases its option value.

Finally, the two endogenous constants, F^{sw} and G^{sw} , and threshold prices, P_F^{sw} and P_G^{sw} , for the second branch of $V_0^{\text{sw}}(P; C_0, C^E)$ are determined numerically by solving the value-matching and smoothpasting conditions between $V_0^{\rm sw}(P; C_0, C^E)$ and $V_E^{\rm sw}(P; C_0, C^E)$ as well as between $V_0^{\text{sw}}(P; C_0, C^E)$ and $V_1(P, C_0)$ indicated in Eqs. (C-5) through (C-8). In terms of implementation, the strategy is similar to that outlined in [Section 2.2](#page-2-0) except that now if threshold price $P_{E1}^{\rm sw}$ is reached in state E, then it is optimal to switch to state 1 with the UET commercialisation programme. Next, we illustrate the intuition and policy insights of the models we have developed via numerical examples.

3. Numerical examples

3.1. Numerical example 1: no existing renewable energy technology

In addition to the parameter values used in [Section 2](#page-1-0) for the nowor-never DCF example, we allow σ to vary between 0.15 and 0.40 as parameter estimates. Initially, in Section 3.1.1, we set $\sigma_c = 0$ to abstract from technical uncertainty in the UET's intermediate com-mercialisation stage. Then, in [Section 3.1.2,](#page-6-0) we set $\sigma_c = 0.10$ to examine how the results are affected by technical uncertainty.

3.1.1. No technical uncertainty in UET commercialisation

For $\sigma = 0.20$ and $\lambda = 0.04$, we obtain $\beta_1 = 1.79$, $\gamma_1 = 1.54$, $A_1 = 2.29$, $P_1 = 82.13$, and $p^* = 1.22$. According to Figs. 4 and 5, the firm's strategy is to wait until the long-term electricity price reaches \$82.13/MWh before initiating the UET commercialisation programme and then to wait again until the long-term electricity price is 1.22 times the nominal UET operating cost before deployment. With σ = 0.20, once state 1 is entered, the commercialisation programme will continue since the ratio of the long-term electricity price to the

Fig. 4. Value of option to invest in UET commercialisation without an existing RE technology (σ = 0.20). The UET commercialisation programme value is nonlinear because it is an option to deploy the UET.

⁶ We assume here that investment is sequential, i.e., $P_E^{\text{sw}} < P_{E1}^{\text{sw}}$. Otherwise, it is optimal to invest directly in the UET commercialisation programme at a cost of $I^E + I$.

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Fig. 5. Value of option to deploy the UET without an existing RE technology from a learning stage (σ = 0.20).

UET operating cost is $\frac{82.13}{100} = 0.8213 < p^*$. In other words, there will not
be an instantaneous transition from state 0 to state 2. From [Fig. 4](#page-4-0), the value of the option to invest in UET commercialisation at the threshold P_I is worth approximately $V_0(P_I; C_0) = V_1(P_I, C_0) - I$ $$6.17 \times 10^9$, i.e., around \$6.17 billion, at deployment, which is equal to the initial value in Fig. 5 minus the investment cost: $C_0v_1(p=P_1/C_0)-I$. Finally, the value of the investment opportunity in state 0 for $P_0=60$ is $V_0(P_0; C_0) = 3.52×10^9 , i.e., around \$3.52 billion. Recall that the nowor-never DCF approach in [Section 2](#page-1-0) values the benefit of the UET at \$1.86 billion, i.e., $V_2(P_0, C_0) - I = X \left(\frac{P_0}{\rho - \alpha} - \frac{C_0}{\lambda + \alpha} \right)$ $\left(\frac{P_0}{\rho-\alpha}-\frac{C_0}{\lambda+\rho}\right)-I$, which is almost 50% lower than the value from the real options approach.

By contrast, if state 1 were avoided, i.e., if the firm had only the option to deploy the UET at initial generating cost C_0 without waiting to improve its performance via the intermediate learning stage, then the value of the entire programme in state 0 would be:

$$
V_0^D(P; C_0) = A_1^D P^{\beta_1}.
$$
\n(14)

Solving simultaneously for the deployment threshold, P_I^D , and endogenous constant, A_1^D , via the value-matching and smooth-pasting conditions between $V_0^D(P; C_0)$ and $V_2(P, C)$, i.e., $V_0^D(P_I^D; C_0) = V_2(P_I^D, C_0) - I$ and $\frac{dV_0^D}{dP}|_{P=P_i^D} = \frac{\partial V_2}{\partial P}|_{P=P_i^D, C=C_0}$, we obtain the following deployment threshold price and option value coefficient:

$$
P_I^D = \left(\frac{\beta_1(\rho - \alpha)}{X(\beta_1 - 1)}\right) \left[\frac{C_0 X}{\rho + \lambda} + I\right].
$$
\n(15)

$$
A_1^D = \frac{(P_1^D)^{1-\beta_1} X}{\beta_1 (\rho - \alpha)}\tag{16}
$$

Upon solving for the base–case parameter values, i.e., with σ = 0.20 and λ = 0.04, we find A_I^D = 2.25 and P_I^D = 110.60 as opposed to A_1 = 2.29 and P_1 = 82.13 when state 1 was available (see Fig. 6). In effect, there is option value to improving the performance of the UET before deploying it. Quantitatively, it is worth:

$$
\mathcal{F}(P_0) = \begin{cases}\nV_0(P_0; C_0) - V_0^D(P_0; C_0) & \text{if } P_0 < P_I \text{ and } P_0 < P_I^D \\
V_1(P_0, C_0) - I - V_0^D(P_0; C_0) & \text{if } P_0 \ge P_I \text{ and } P_0 < P_I^D \\
V_0(P_0; C_0) - V_2(P_0, C_0) + I & \text{if } P_0 < P_I \text{ and } P_0 \ge P_I^D \\
V_1(P_0, C_0) - V_2(P_0, C_0) & \text{if } P_0 \ge P_I \text{ and } P_0 \ge P_I^D\n\end{cases} \tag{17}
$$

Fig. 6. Value of option to deploy the UET without intermediate learning (σ =0.20).

For σ = 0.20 and λ = 0.04, this option value to perform intermediate learning is worth \$73 million, which is 2.1% of the entire programme in state 0. Notably, with increasing uncertainty, the value of intermediate learning decreases as the greater probability of higher electricity prices makes the existing UET more attractive even without the performance enhancement provided by learning in state 1 (see Fig. 7). Indeed, only in a scenario with low electricity price volatility does intermediate UET learning add value by making the technology more cost effective. Furthermore, as λ increases ceteris paribus, i.e., as the UET commercialisation programme becomes more effective, the option value of the intermediate learning stage becomes more valuable. For example, for λ = 0.08, it is worth 8.33% of the entire programme.

Varying estimates of the volatility of the long-term electricity price, σ , reveals that the commercialisation programme investment price threshold increases with uncertainty as the value of waiting also increases (see [Fig. 8](#page-6-0)). As indicated earlier, since greater volatility diminishes the value of intermediate learning from state 1, the investment threshold price, P_I , and direct deployment threshold price, P_I, converge. Similarly, the UET deployment price-cost ratio increases as the volatility increases (see [Fig. 9\)](#page-6-0). Moreover, although the ratio $\frac{P_l}{C_C}$

Fig. 7. Option value of intermediate learning stage without an existing RE technology. A higher learning rate to reduce the UET's operating cost makes intermediate learning more valuable. The learning option value decreases with uncertainty since the probability of high long-term electricity prices makes even the non-learning-enhanced UET increase in option value.

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Fig. 8. Investment thresholds as functions of long-term electricity price uncertainty. The P_I^D curve indicates when to build a 1250 MW_e UET plant when there is no intermediate learning step. This occurs at high electricity prices, and the trigger level increases with uncertainty. If such a learning step is available, then the trigger is lower due to the improved possibility of managing deployment timing and is shown as P_I .

is quite close to p^* for $\sigma = 0.40$, instantaneous deployment of the UET still does not occur. Hence, for reasonable values of σ , it is always optimal to perform intermediate learning before deployment.

3.1.2. Technical uncertainty in UET commercialisation

Here, we allow for uncertainty in the commercialisation of the UET, i.e., the decrease in its operating cost is not deterministic after state 1 is entered. We use a representative value of $\sigma_c = 0.10$ to capture this technical risk. Referring to our base–case parameter values of σ = 0.20 and λ = 0.04, we find that the inclusion of technical uncertainty increases the option value of the entire UET programme to \$3.58 billion at P_0 = 60 from \$3.52 billion and decreases the longterm electricity price threshold, P_I , at which to initiate commercialisa-tion (see [Fig. 10](#page-7-0)). Indeed, we find that $P_I = 75.21$ as opposed to \$82.13/MWh as in the case with $\sigma_c = 0$. This is because the value of the option to deploy the learning-enhanced UET from state 1 increases with technical uncertainty as discretion over timing implies that it is possible to take advantage of rapid decreases in the operating

Fig. 9. Deployment threshold ratio indicates when to leave the UET learning programme and deploy a 1250 MW_e UET plant. This decision depends on the ratio $p^* \equiv P/C$, where P follows a GBM process and C is reduced gradually in the learning stage. The graph shows the familiar result that the value of waiting increases with uncertainty.

cost without being adversely affected by unexpected increases. In effect, the firm has a greater option value in state 0 without having to worry about technical risk until state 1. Thus, it is easier for it to initiate the UET programme. On the other hand, in [Fig. 11](#page-7-0), it is optimal to wait longer than in the case without technical uncertainty, i.e., until p^* = 1.27, before deploying the learning-enhanced UET as greater uncertainty also increases the value of waiting and, therefore, the opportunity cost of killing the waiting option in state 1.

Examining the value of the intermediate learning stage for the UET under technical uncertainty, we find that it is greater than in the case with $\sigma_c = 0$ (see [Fig. 12](#page-7-0)). Intuitively, this result arises for two reasons: first, the investment threshold for initiating the commercialisation programme is lower, thereby implying that state 1 is entered sooner than in the example considered in [Section 3.1.1](#page-4-0); second, deployment occurs later in order to mitigate the effects of technical uncertainty. At the same time, technical uncertainty does not change the option value of direct deployment, $V_0^D(P; C_0)$, because the expected NPV from direct deployment, $V_2(P, C) - I$, depends only on the average rate of decrease (and not the uncertainty) in the UET's operating cost. Hence, the option value of the intermediate learning stage as captured by $\mathcal{F}(P_0)$ in Eq. (17) increases.

The other qualitative results of [Section 3.1.1](#page-4-0) also hold, viz., the investment thresholds all increase as parameter estimates of the longterm electricity price's volatility are increased. Again, P_I is lower here as the option value of the UET commercialisation programme in state 0 is higher due to a higher expected value in moving to state 1 without facing any technical risk until the learning programme starts (see [Fig. 13](#page-8-0)). Conversely, p^* is higher because technical uncertainty implies that more time must be spent in the intermediate learning state to offset the effects of any adverse movements in the UET's operating cost (see [Fig. 14\)](#page-8-0).

3.2. Numerical example 2: existing renewable energy technology without switching option to the unconventional energy technology

Assuming the same parameter values as in [Section 2](#page-1-0) for the UET and using I^E = \$200 million, X^E = 5 TWh, and C^E = \$25/MWh, we illustrate the intuition for UET commercialisation when there is also an existing RE technology.⁷ We keep $\sigma_c = 0$ here because numerical examples with technical uncertainty do not reveal any insights additional to those discussed in Section 3.1.2. However, we will comment on how the numerical results are affected if $\sigma_c = 0.10$ were used. For σ = 0.20, we find that $A_1^{ex} > A_1$, which implies that the waiting region is dichotomous around the indifference point, $\tilde{P}^{ex} = 64.10$, i.e., the firm's optimal policy is to deploy the existing RE when the longterm electricity price is in the range $[P_{E}^{\text{ex}}, P_{F}^{\text{ex}}] = [39.39, 52.05]$ and to start the UET commercialisation programme if the long-term electricity price is greater than $P_G^{\text{ex}} = 87.80$ (see [Fig. 15\)](#page-8-0). 8 For all other prices, it is optimal to wait. Note that the threshold for initiating the UET commercialisation programme is greater than what it was without the availability of the existing RE technology, $P_I= 82.13$, as the presence of an alternative project reduces the attraction of the UET.

Even though the commercialisation initiation threshold increases, immediate deployment does not take place once learning commences because the threshold ratio is still less than p^* , i.e., $\frac{p_c^{\text{ex}}}{C_0} = 0.88 \le p^*$. The value of the entire investment opportunity at the initial longterm electricity price, $P_0 = 60$, is worth $V_0^{\text{ex}}(P_0; C_0, C^E) = F^{\text{ex}} P_0^{\beta_1} +$ $G^{\text{ex}}P_0^{\beta_2} =$ \$3.61 billion, which is \$90 million higher than the value in [Section 3.1.1](#page-4-0) (an increase of 2.56%).⁹ If the waiting region is ignored

With technical uncertainty, this option value increases to \$3.64 billion.

We assume that $X^E < X$ because the capacity of the existing RE technology is limited either by the number of additional desirable sites (e.g., for solar panels or windmills) or the intermittency issues.

⁸ By comparison, for $\sigma_c = 0.10$, we have $\tilde{P}^{ex} = 60.93$, $[P_E^{ex}, P_F^{ex}] = [39.39, 50.24]$, and $P_G^{\text{ex}} = 80.46$. Intuitively, greater technical uncertainty facilitates initiation of UET commercialisation and reduces the immediate deployment region for the existing RE technology due to the greater potential upside of the UET project.

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Value of Option to Invest in UET Commercialisation with Technical Uncertainty (σ = 0.20, σ_c = 0.10)

Fig. 10. Value of option to invest in UET commercialisation without an existing RE technology under technical uncertainty (σ = 0.20, σ_c = 0.10).

Fig. 11. Value of option to deploy the UET without an existing RE technology from a learning state under technical uncertainty (σ = 0.20, σ _C = 0.10).

and the existing RE technology is deployed immediately as recommended by [Dixit \(1993\),](#page-13-0) then the firm would lose \$62 million from acting too quickly, which is 1.75% of the expected NPV. In effect, by using the approach of [Décamps et al. \(2006\),](#page-13-0) we show how the firm is able to optimise investment in the two mutually exclusive projects for all long-term electricity prices.

Fig. 12. Option value of intermediate learning stage without an existing RE technology under technical uncertainty. Relative to the case with $\sigma_c = 0$, the case here with $\sigma_c = 0.10$ implies that greater value is placed on intermediate learning. The other attributes of the option value are similar to those in the case without technical uncertainty.

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Fig. 13. Investment thresholds as functions of long-term electricity price uncertainty under technical uncertainty. The trends are the same as in the case without technical uncertainty except that the investment threshold for the UET commercialisation programme is lower.

At higher levels of estimated volatility, the viability of the existing RE technology as an alternative to the UET commercialisation programme gradually diminishes (see [Fig. 16\)](#page-9-0). Here, as σ increases, the region for immediate deployment of the RE technology shrinks as there exists greater probability of high electricity prices in the future. Furthermore, the indifference point between the two projects, \tilde{P}^{ex} , decreases as the UET commercialisation programme starts to look more promising. Indeed, for high enough levels of volatility, the option to deploy the existing RE technology may be disregarded, which then reduces the problem to a simple real options one with the same investment threshold as in [Section 3.1.1](#page-4-0).

As in [Section 3.1,](#page-4-0) instead of managing the UET project in a staged manner, the firm may choose to pursue a more direct strategy in which state 1 is skipped. In terms of [Fig. 2](#page-3-0), the firm may transition from state 0 either to state E or directly to state 2, i.e., deploying the existing UET at its initial operating cost with ongoing learning. In this case, the value of the entire programme in state 0 is similar to that in Eq. (7):

$$
V_0^{D,ex}(P; C_0, C^E) = \begin{cases} A_1^{ex} P^{\beta_1} & \text{if } 0 \le P < P_E^{ex} \\ F^{D,ex} P^{\beta_1} + G^{D,ex} P^{\beta_2} & \text{if } P_F^{D,ex} < P < P_G^{D,ex} \end{cases} \tag{18}
$$

Fig. 15. Value of option to proceed with UET commercialisation when an existing RE technology is available (σ =0.20). $V_E^{ex}(P; C^E) - I^E$ is the expected NPV of the existing RE technology, while $V_1(P, C_0) - I$ is the option value to deploy the UET from an intermediate learning stage. The $V_0^{\text{ex}}(P; C_0, C^E)$ curves are the option values for state 0 when neither project has yet been selected.

Now, the coefficients, $F^{D,ex}$ and $G^{D,ex}$, together with the threshold prices for the indifference zone, $P_F^{D,ex}$ and $P_G^{D,ex}$, must be found numerically via value-matching and smooth-pasting conditions analogous to those in Eqs. (B-3) through (B-6). The only difference is that the second set of value-matching and smooth-pasting conditions are defined with respect to a contact point on the expected NPV curve in state 2, $V_2(P, C) - I$. In [Fig. 17](#page-9-0), we plot the option value and expected NPV curves for the direct investment strategy. We note that due to the lack of intermediate learning opportunities with the UET, the second waiting region widens. Indeed, since the UET project's timing cannot be managed as precisely now, deployment of it is less likely to be precipitated, a fact that is also captured by the effect of varying the volatility parameter on the investment thresholds (see [Fig. 18](#page-9-0)). However, it is still the case that it dominates the existing RE technology option for σ > 0.24.

As we did in [Section 3.1.1,](#page-4-0) we now also illustrate the option value of the intermediate learning stage for various levels of σ and λ by

Fig. 14. The deployment threshold ratio under technical uncertainty where the value of waiting has increased. As before, the value of waiting also increases with uncertainty.

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Fig. 16. Investment thresholds when an existing RE technology is available. For low levels of uncertainty, the existing RE technology should be selected at moderate electricity price levels, whereas the UET commercialisation project should be launched at higher electricity price levels. When uncertainty increases, the waiting region increases until the existing RE technology disappears as a candidate solution. Note that there is a lower floor to the existing RE technology-deployment option only when uncertainty is low.

using an analogue of Eq. (17). First, fixing $\lambda = 0.04$ and $\sigma = 0.20$, we find that the option value of this state is less than 1% of the overall project's value. However, it is zero in the range $0.15 \le \sigma \le 0.16$, increasing in the range 0.16< σ <0.25, and decreasing for σ ≥0.25 (see [Fig. 19\)](#page-10-0). The first component can be explained by the fact that both strategies recommend immediate deployment of the existing RE for low levels of volatility as there is not much value to waiting for the UET to become attractive. We use the same intuition from [Section 3.1.1](#page-4-0) to explain why the option value decreases for high levels of σ : the prospect of sustained price increases makes it attractive to disregard the existing RE technology and focus on the UET. However, the value of intermediate learning decreases with σ in this region as greater electricity price uncertainty makes even the existing UET generation capability competitive. By contrast, over a moderate range of σ , there is not enough information to make a decision between existing RE

Value of Option to Deploy the UET without Intermediate Learning and with Existing RE Technology (σ = 0.20)

Fig. 17. Value of option to deploy the UET without intermediate learning when an existing RE technology is available (σ = 0.20). $V_E^{\text{ex}}(P; C^E) - I^E$ is the expected NPV of the existing RE technology, while $V_2(P, C_0) - I$ is the expected NPV of the deployable UET. The $V_0^{D,\text{ex}}(P;\mathcal{C}_0,\mathcal{C}^E)$ curves are the option values for state 0 when neither project has yet been selected.

Fig. 18. Investment thresholds with direct deployment of the UET when an existing RE technology is available. The existing RE project is chosen only for low electricity price uncertainty levels and low levels of long-term electricity prices. The waiting region increases with uncertainty, and the UET is deployed at high long-term electricity prices.

deployment and pursuing UET generation at the initial price, P_0 . Consequently, the resulting indifference zone also widens with more uncertainty starting from a low level of σ (until the existing RE technology is no longer considered). The intermediate option value increases in this range because the learning programme provides a way to time the deployment of the new technology. Finally, note that the option value of intermediate learning is much higher (over 8% of the total project value for σ = 0.20) when λ is increased to 0.08. Due to the greater effectiveness of the UET learning process, there is more value to the intermediate state. And, precisely due to its attraction, the UET commercialisation programme is started more quickly, which then causes the option value to decrease with σ again as there is little competition with the existing RE technology.10

3.3. Numerical example 3: existing renewable energy technology with switching option to the unconventional energy technology

For completeness, we perform a numerical example involving switching using the same data as in [Sections 2 and 3.2](#page-1-0) without technical uncertainty. We consider the case in which either deployment of the existing RE technology (with a subsequent option to deploy the UET directly with ongoing learning) or direct deployment of the UET with ongoing learning is possible. In terms of [Fig. 3,](#page-4-0) we suppose that the arrow from state E leads to state 2, i.e., there is no intermediate learning stage for the UET.¹¹ At the initial long-term electricity price of \$60/MWh, we obtain that it is optimal to deploy the existing RE technology and wait for the opportunity to switch to deployment of the UET when the long-term electricity price reaches \$187.25/MWh. The expected value of the entire alternative energy programme with the switching option is \$4.68 billion, which is more than a \$1 billion increase relative to the example in [Section 3.2](#page-6-0) with direct deployment of the UET.

Intuitively, the subsequent option to switch to the UET (even without the intermediate learning stage) facilitates the deployment of the existing RE technology as this decision is now reversible. Indeed, until the electricity price reaches suitably high levels for deployment of the UET to become viable, the firm is able to profit from deploying the existing RE technology. The option value and expected NPV curves

¹⁰ These results also hold for the case in which C_t is stochastic. ¹¹ Similarly, the arrows out of state 0 lead either to state E or to state 2.

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Fig. 19. Option value of intermediate learning stage with an existing RE technology. Faster learning to reduce the UET's operating cost makes the intermediate learning stage more valuable. For high learning rates, the intermediate learning option value decreases with uncertainty since the probability of higher long-term electricity prices means that even the existing UET dominates the smaller RE project. By contrast, for low learning rates and moderate level of uncertainty, the intermediate learning option value increases with uncertainty since learning provides a way to optimise the timing of the deployment of the UET.

in Fig. 20 indicate how the situation changes from that illustrated in [Fig. 17](#page-9-0) without the switching option: the region for immediate investment in the existing RE technology widens, the indifference zone between the two alternative energy projects occurs at a much higher electricity price and is narrower, and the threshold for deploying the UET is much higher. In particular, $P_{E}^{\rm sw}$ = 39.39 as before, but $[P_F^{D,sw}, P_G^{D,sw}] = [149.37, 157.79]$ and $P_{E1}^{D,sw} = 187.25$. [Fig. 21](#page-11-0) illustrates how these thresholds behave with varying estimates of the long-term electricity price volatility.

4. Conclusions

Given the concern about climate change, the development of alternative energy technologies with lower rates of $CO₂$ emissions is gaining prominence. Within the domain of existing RE technologies, biofuels, hydroelectric power, solar-based technologies, wave generation, and windmills have all demonstrated various levels of effectiveness and gained some measure of public support in contributing to the world's energy supply. As cap-and-trade systems for $CO₂$ emissions gain popularity, such technologies will become only more competitive with fossil-fuelled technologies. On the other hand, the limited scale of such existing RE technologies means that investors will have the incentive to branch out in order to commercialise UETs that could capture more market share.

In this paper, we examine how a staged commercialisation programme for a UET could proceed under uncertainty. By taking the real options approach, we find that the option to commercialise and deploy such a technology would have considerable value. In particular, the value of the intermediate learning stage is worth more if its effectiveness increases, while it decreases with the volatility of the long-term electricity price. The latter, seemingly counterintuitive, result holds because it is only in a scenario with low price volatility that the intermediate learning stage of the programme makes the UET competitive. Otherwise, a high level of volatility makes even the rudimentary UET attractive since there is a high probability of sustained electricity price increases. With the addition of an existing RE technology, we have the problem of mutually exclusive investment in alternative staged projects under uncertainty. We find that the addition of an existing RE technology increases the value of the overall programme from the perspective of the firm. However, it delays the potential initiation of the UET commercialisation programme as the existing RE technology is more beneficial for a moderate range of electricity prices. Furthermore, the value of the intermediate learning stage increases for an intermediate range of price volatility as such activity provides additional information about the relative benefit of the UET versus the existing RE technology. For high volatility levels, the existing RE technology is not considered at all, which causes the value of the intermediate learning stage to decrease as before. Hence,

Value of the Option to Deploy the UET without Intermediate Learning and with a Switching Option from the Existing RE Technology (σ = 0.20)

Fig. 20. Value of option to deploy the UET without intermediate learning when an existing RE technology is available with a switching option (σ = 0.20). V $_{\rm e}^{\rm ex}(P;\,C^{\rm E})-I^{\rm E}$ is the expected NPV of the existing RE technology, while V₂(P, C₀) − I is the expected NPV of the deployable UET. Representing the value of the option to switch to the UET after the existing RE technology has been selected is $V_{E}^{Dsw}(P; C_0, C^E) - I^E$, while $V_2(P, C_0) - I - I^E$ is the expected NPV of the deployed UET after the existing RE technology was already used. Finally, the lower branch of V $_0^{D, \text{sw}}$ (P; C₀, C^E) is the value of the option to invest in the existing RE technology with a subsequent option to switch to the UET, and the upper branch of V $_0^{D, \text{sw}}$ (P; C₀, C₀) C^E) is the value of the option to invest in either the existing RE or the UET.

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Fig. 21. Investment thresholds with direct deployment of the UET when an existing RE technology is available with a switching option. Unlike the case without the switching option, the existing RE project is always available at moderate levels of the long-term electricity price. The waiting region increases with uncertainty, and the UET is deployed at high longterm electricity prices as before, but these regions are relatively narrower than before. Finally, if the existing RE technology is deployed, then the switch to the UET is made at even higher electricity price levels than for those at which the UET would have been deployed.

firms planning to initiate similar commercialisation programmes would be prudent not to neglect the effects of their interactions with existing RE technologies. Indeed, in future work, it would be beneficial to explore pursuing both projects jointly by allocating budget shares to each rather than proceeding in a mutually exclusive sense as in this paper.

Appendix A. Analytical solution to case 1

 $V(f)$ \mathbb{R}^k C^* $V(f)$ \mathbb{R}^k C^*

First, we find the expected appreciation of the value of the option to deploy:

$$
dV_1 = \frac{1}{2} \frac{\partial^2 V_1}{\partial P^2} (dP)^2 + \frac{1}{2} \frac{\partial^2 V_1}{\partial C^2} (dC)^2 + \frac{\partial V_1}{\partial P} dP + \frac{\partial V_1}{\partial C} dC
$$

\n
$$
\Rightarrow \mathcal{E}[dV_1] = \frac{1}{2} \frac{\partial^2 V_1}{\partial P^2} \sigma^2 P^2 dt + \frac{1}{2} \frac{\partial^2 V_1}{\partial C^2} \sigma^2_C C^2 dt + \frac{\partial V_1}{\partial P} \alpha P dt - \frac{\partial V_1}{\partial C} \lambda C dt.
$$
\n(A-1)

Next, we equate the expected appreciation of V_1 to the instantaneous rate of return on V_1 via the Bellman Equation:

$$
\mathcal{E}[dV_1] = \rho V_1 dt
$$

\n
$$
\Rightarrow \frac{1}{2} \frac{\partial^2 V_1}{\partial P^2} \sigma^2 P^2 + \frac{1}{2} \frac{\partial^2 V_1}{\partial C^2} \sigma^2 C^2 + \frac{\partial V_1}{\partial P} \alpha P - \frac{\partial V_1}{\partial C} \lambda C - \rho V_1 = 0.
$$
 (A - 2)

Eq. (A-2) is solved subject to the following value-matching and smooth-pasting conditions:

$$
v_1(r^*, c^*) = v_2(r^*, c^*)
$$

\n
$$
\Rightarrow V_1(P^*, C^*) = X\left(\frac{P^*}{\rho - \alpha} - \frac{C^*}{\lambda + \rho}\right)
$$
 (A - 3)

$$
\frac{\partial V_1}{\partial P}\Big|_{P=P^*,C=C^*} = \frac{\partial V_2}{\partial P}\Big|_{P=P^*,C=C^*}
$$

$$
\Rightarrow \frac{\partial V_1}{\partial P}\Big|_{P=P^*,C=C^*} = \frac{X}{P-\alpha}
$$
 (A-4)

$$
\frac{\partial V_1}{\partial C}\Big|_{P=P^*,C=C^*} = \frac{\partial V_2}{\partial C}\Big|_{P=P^*,C=C^*}
$$

$$
\Rightarrow \frac{\partial V_1}{\partial C}\Big|_{P=P^*,C=C^*} = -\frac{X}{\lambda + \rho}.
$$
 (A-5)

Eq. (A-3) states that at deployment, the value of the option to use UET generation equals the expected NPV of an active investment. Meanwhile, Eqs. (A-4) and (A-5) are first-order necessary conditions that equate the marginal benefit of delaying deployment with its marginal cost. Since the solution to system of Eqs. (A-2) to (A-5) involves a free boundary, i.e., P^* depends on C, we convert the partial differential equation (PDE) to an ordinary differential equation (ODE)

as discussed in [Dixit and Pindyck \(1994\).](#page-13-0)
We start by defining $p=\frac{P}{C}$ and assuming that $V_1(P, C)$ is homogenous of degree one in (P, C). Then, we note that $V_1(P, C) = Cv_1(P/C) = Cv_1(p)$. Using the definition of p and $v_1(p)$, we re-write Eqs. (A-2) through (A-5) as follows:

$$
\frac{1}{2}v_1^{''}(p)(\sigma^2 + \sigma_C^2)p^2 + v_1^{'}(p)(\alpha + \lambda)p - v_1(p)(\lambda + \rho) = 0 \qquad (A - 6)
$$

$$
\nu_1(p^*) = X\left(\frac{p^*}{\rho - \alpha} - \frac{1}{\lambda + \rho}\right) \tag{A-7}
$$

$$
v_1'(p^*) = \frac{X}{\rho - \alpha} \tag{A-8}
$$

$$
v_1(p^*) - p^* v_1'(p^*) = -\frac{X}{\lambda + \rho}.
$$
\n(A - 9)

Since Eq. (A-9) follows from Eqs. (A-7) and (A-8), we may ignore it. The solution to the ODE in Eq. (A-6) is:

$$
v_1(p) = a_1 p^{\gamma_1}.
$$
 (A-10)

This is the normalised value of the option to deploy the UET, where γ_1 is a positive exogenous constant that is the solution to the characteristic quadratic equation, i.e.,

γ1= [−] ^α ⁺ ^λ−¹ 2 ^ðσ² ⁺ ^σ² C Þ + ffi ^α ⁺ ^λ−¹ 2 ^ðσ² ⁺ ^σ² C Þ ² + 2ðσ² ⁺ ^σ² ^C Þðλ + ρÞ r σ² + σ² C : ðA 11Þ

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Using Eqs. (A-7) and (A-8), we can solve simultaneously for the deployment price–cost threshold ratio, p^* , and the positive endogenous constant, a_1 :

$$
p^* = \left(\frac{\gamma_1}{\gamma_1 - 1}\right) \frac{\rho - \alpha}{\lambda + \rho} \tag{A-12}
$$

$$
a_1 = \frac{X(p^*)^{1-\gamma_1}}{\gamma_1(p-\alpha)}.
$$
\n(A-13)

From Eqs. (A-10) and (A-12), the value of the commercialisation programme and the deployment threshold price–cost ratio, respectively, may be determined.

Moving once step back, we would like to obtain the value of the perpetual option to invest in the commercialisation programme, $V_0(P;$ C_0), along with the investment threshold price, $P₁$. By following reasoning similar to that in Eqs. (A-1) and (A-2), we obtain the option value to start the commercialisation programme:

$$
V_0(P; C_0) = A_1 P^{\beta_1}.
$$
\n(A - 14)

In order to find the investment threshold price, P_I , and the endogenous constant, A_1 , we use the following value-matching and smooth-pasting conditions:

$$
V_0(P_I; C_0) = V_1(P_I, C_0) - I
$$

\n
$$
\Rightarrow A_1 P_I^{\beta_1} = a_1 (C_0)^{1 - \gamma_1} P_I^{\gamma_1} - I
$$
 (A - 15)

$$
\frac{dV_0}{dP}\Big|_{P=P_1} = \frac{\partial V_1}{\partial P}\Big|_{P=P_1, C=C_0}
$$

\n
$$
\Rightarrow \beta_1 A_1 P_1^{\beta_1 - 1} = \gamma_1 a_1 (C_0)^{1 - \gamma_1} P_1^{\gamma_1 - 1}.
$$
 (A - 16)

Here, β_1 is a positive exogenous constant:

$$
\beta_1 = \frac{-\left(\alpha - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\alpha - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 \rho}}{\sigma^2}.
$$
 (A - 17)

Note that in Eqs. (A-15) and (A-16) we use the fact that $v_1(p) \equiv \frac{V_1(P,C)}{C}$, which implies that $V_1(P,C) = Cv_1(p) = Ca_1(\frac{P}{C})$ $\left(\frac{P}{C}\right)^{\gamma_1}$. Solving Eqs. (A-15) and (A-16) simultaneously, we obtain the following:

$$
P_I = \left[\left(\frac{I\beta_1}{\beta_1 - \gamma_1} \right) \frac{\left(C_0 \right)^{\gamma_1 - 1}}{a_1} \right]^{1/2} \tag{A-18}
$$

$$
A_1 = \frac{\gamma_1 a_1 (C_0)^{1 - \gamma_1} P_I^{\gamma_1 - \beta_1}}{\beta_1}.
$$
\n(A - 19)

Appendix B. Analytical solution to case 2

In order to find A_1^{ex} and P_E^{ex} analytically for the first branch of $V_0^{\text{ex}}(P; C_0, C^E)$, we use the following value-matching and smoothpasting conditions between $V_0^{\text{ex}}(P; C_0, C^E)$ and $V_E^{\text{ex}}(P; C^E)$:

$$
V_0^{\text{ex}}(P_E^{\text{ex}}; C_0, C^E) = V_E^{\text{ex}}(P_E^{\text{ex}}; C^E) - I^E
$$

$$
\Rightarrow A_1^{\text{ex}}(P_E^{\text{ex}})^{\beta_1} = X^E \left(\frac{P_E^{\text{ex}}}{\rho - \alpha} - \frac{C^E}{\rho} \right) - I^E
$$
 (B - 1)

$$
\left. \frac{dV_0^{\text{ex}}}{dP} \right|_{P=P_E^{\text{ex}}} = \left. \frac{dV_E^{\text{ex}}}{dP} \right|_{P=P_E^{\text{ex}}}
$$

$$
\Rightarrow \beta_1 A_1^{ex} (P_E^{ex})^{\beta_1 - 1} = \frac{X^E}{\rho - \alpha}.
$$
 (B - 2)

Solving Eqs. (B-1) and (B-2) simultaneously, we obtain the investment threshold price and endogenous constant for the existing RE technology in Eqs. (8) and (9), respectively.

For the second branch of $V_0^{\text{ex}}(P; C_0, C^E)$, we determine the two endogenous constants, F^{ex} and G^{ex} , and threshold prices, P_F^{ex} and P_G^{ex} , by using the following four value-matching and smooth-pasting conditions:

$$
V_0^{\text{ex}}(P_F^{\text{ex}}; C_0, C^E) = V_E^{\text{ex}}(P_F^{\text{ex}}; C^E) - I^E
$$

\n
$$
\Rightarrow F^{\text{ex}}(P_F^{\text{ex}})^{\beta_1} + G^{\text{ex}}(P_F^{\text{ex}})^{\beta_2} = X^E \left(\frac{P_F^{\text{ex}}}{\rho - \alpha} - \frac{C^E}{\rho}\right) - I^E
$$
(B-3)

$$
\frac{dV_0^{\text{ex}}}{dP}\Big|_{P=P_F^{\text{ex}}} = \frac{dV_E^{\text{ex}}}{dP}\Big|_{P=P_F^{\text{ex}}}
$$

$$
\Rightarrow \beta_1 F^{\text{ex}} (P_F^{\text{ex}})^{\beta_1 - 1} + \beta_2 G^{\text{ex}} (P_F^{\text{ex}})^{\beta_2 - 1} = \frac{X^E}{\rho - \alpha}
$$
 (B - 4)

$$
V_0^{\text{ex}}(P_G^{\text{ex}}; C_0, C^E) = V_1(P_G^{\text{ex}}, C_0) - I
$$

\n
$$
\Rightarrow F^{\text{ex}}(P_G^{\text{ex}})^{\beta_1} + G^{\text{ex}}(P_G^{\text{ex}})^{\beta_2} = a_1(C_0)^{1-\gamma_1}(P_G^{\text{ex}})^{\gamma_1} - I
$$

\n
$$
\frac{dV_0^{\text{ex}}}{dP}\Big|_{P=P_G^{\text{ex}}} = \frac{\partial V_1}{\partial P}\Big|_{P=P_G^{\text{ex}}, C=C_0}
$$

\n(B - 5)

$$
\Rightarrow \beta_1 F^{\text{ex}} (P_G^{\text{ex}})^{\beta_1 - 1} + \beta_2 G^{\text{ex}} (P_G^{\text{ex}})^{\beta_2 - 1} = \gamma_1 a_1 (C_0)^{1 - \gamma_1} (P_G^{\text{ex}})^{\gamma_1 - 1}. \quad (B - 6)
$$

Appendix C. Analytical solution to case 3

The endogenous constant, B^{sw} , and the switching threshold price, P_{E1}^{sw} , are found numerically by solving the following system:

$$
V_{E}^{sw}(P_{E1}^{sw}; C_0, C^E) - I^E = V_1(P_{E1}^{sw}, C_0) - I - I^E
$$

\n
$$
\Rightarrow X^E \left(\frac{P_{E1}^{sw}}{\rho - \alpha} - \frac{C^E}{\rho} \right) + B^{sw}(P_{E1}^{sw})^{\beta_1} - I^E = a_1(C_0)^{1 - \gamma_1} (P_{E1}^{sw})^{\gamma_1} - I - I^E
$$

\n(C - 1)

$$
\frac{dV_{E}^{sw}}{dP}\Big|_{P=P_{E1}^{sw}} = \frac{\partial V_{1}}{\partial P}\Big|_{P=P_{E1}^{sw}, C=C_{0}}
$$
\n
$$
\Rightarrow \frac{X^{E}}{\rho-\alpha} + \beta_{1}B^{sw}(P_{E1}^{sw})^{\beta_{1}-1} = \gamma_{1}a_{1}(C_{0})^{1-\gamma_{1}}(P_{E1}^{sw})^{\gamma_{1}-1}.
$$
\n(C-2)

Next, the endogenous constant, A_1^{sw} , and existing RE technology (with a switching option) deployment threshold price, P_E^{sw} , are found by solving the following value-matching and smooth-pasting conditions:

$$
V_0^{sw}(P_E^{sw}; C_0, C^E) = V_E^{sw}(P_E^{sw}; C_0, C^E) - I^E
$$

$$
\Rightarrow A_1^{sw}(P_E^{sw})^{\beta_1} = X^E \left(\frac{P_E^{sw}}{\rho - \alpha} - \frac{C^E}{\rho}\right) + B^{sw}(P_E^{sw})^{\beta_1} - I^E
$$
 (C - 3)

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$$
\frac{dV_0^{\text{sw}}}{dP}\Big|_{P=P_E^{\text{sw}}} = \frac{dV_E^{\text{sw}}}{dP}\Big|_{P=P_E^{\text{sw}}}
$$

$$
\Rightarrow \beta_1 A_1^{\text{sw}} (P_E^{\text{sw}})^{\beta_1 - 1} = \frac{X^E}{\rho - \alpha} + \beta_1 B^{\text{sw}} (P_E^{\text{sw}})^{\beta_1 - 1}.
$$
 (C - 4)

The endogenous constants and threshold prices for the second branch of $V_0^{\rm sw}(P; C_0, C^E)$ are solved via the following value-matching and smooth-pasting conditions:

$$
V_0^{\text{sw}}(P_F^{\text{sw}}; C_0, C^E) = V_E^{\text{sw}}(P_F^{\text{sw}}; C_0, C^E) - I^E
$$

\n
$$
\Rightarrow F^{\text{sw}}(P_F^{\text{sw}})^{\beta_1} + G^{\text{sw}}(P_F^{\text{sw}})^{\beta_2} = X^E \left(\frac{P_F^{\text{sw}}}{\rho - \alpha} - \frac{C^E}{\rho}\right) + B^{\text{sw}}(P_F^{\text{sw}})^{\beta_1} - I^E
$$

\n(C-5)

$$
\frac{dV_0^{sw}}{dP}\Big|_{P=P_F^{sw}} = \frac{dV_E^{sw}}{dP}\Big|_{P=P_F^{sw}}
$$
\n
$$
\Rightarrow \beta_1 F^{sw} (P_F^{sw})^{\beta_1 - 1} + \beta_2 G^{sw} (P_F^{sw})^{\beta_2 - 1} = \frac{X^E}{\rho - \alpha} + \beta_1 B^{sw} (P_F^{sw})^{\beta_1 - 1}
$$
\n(C - 6)\n
$$
V_0^{sw} (P_G^{sw}; C_0, C^E) = V_1 (P_G^{sw}, C_0) - I
$$

$$
\Rightarrow F^{sw}(P_G^{sw})^{\beta_1} + G^{sw}(P_G^{sw})^{\beta_2} = a_1(C_0)^{1-\gamma_1}(P_G^{sw})^{\gamma_1} - I
$$
 (C-7)

$$
\left. \frac{dV_0^{\rm sw}}{dP} \right|_{P=P_C^{\rm sw}} = \left. \frac{\partial V_1}{\partial P} \right|_{P=P_C^{\rm sw}, C=C_0}
$$

$$
\Rightarrow \beta_1 F^{sw} (P_G^{sw})^{\beta_1 - 1} + \beta_2 G^{sw} (P_G^{sw})^{\beta_2 - 1} = \gamma_1 a_1 (C_0)^{1 - \gamma_1} (P_G^{sw})^{\gamma_1 - 1} . (C - 8)
$$

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