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Optimal Control of Distributed Energy Resources and Demand Response under Uncertainty

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Abstract

We take the perspective of a microgrid that has installed distribution energy resources (DER) in the form of distributed generation with combined heat and power applications. Given uncertain electricity and fuel prices, the microgrid minimizes its expected annual energy bill for various capacity sizes. In almost all cases, there is an economic and environmental advantage to using DER in conjunction with demand response (DR): the expected annualized energy bill is reduced by 9% while CO_2 emissions decline by 25%. Furthermore, the microgrid's risk is diminished as DER may be deployed depending on prevailing market conditions and local demand. In order to test a policy measure that would place a weight on CO_2 emissions, we use a multi-criteria objective function that minimizes a weighted average of expected costs and emissions. We find that greater emphasis on CO_2 emissions has a beneficial environmental impact only if DR is available and enough reserve generation capacity exists. Finally, greater uncertainty results in higher expected costs and risk exposure, the effects of which may be mitigated by selecting a larger capacity.

Keywords: Distributed generation, demand response, CO_2 emissions, stochastic optimization

1 Introduction

Distributed energy resources (DER) such as distributed generation (DG), combined heat and power (CHP) equipment, batteries, and other on-site storage devices provide an alternative means for commercial and industrial sites to meet their energy service demands. Indeed, rather than relying on central-station electricity generation and purchase of natural gas for heating, they may generate and store electricity on-site when economical and recover and store waste heat in the process. Consequently, it may be possible to improve system energy efficiency, reduce CO_2 emissions, and lower energy costs via DER. However, unlike passive reliance on a centralized system, the optimal adoption and operation

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of DER requires active control algorithms and technology in order to coordinate the various components of the system along with external energy purchases and deployment of demand response. Previous work ([7] and [3]) has demonstrated how DER comprising heat and electrical storage devices may be adopted and operated optimally over a test year in various building types under deterministic electricity and fuel prices. Recent work has shown how to use a similar formulation for handling demand response and incorporation of risk under uncertain prices ([8] and [2], respectively). In this paper, we aim to bridge the gap between these two approaches by performing operational analysis of DER under uncertain energy prices with demand response and CO_2 minimization as part of the objective function in addition to cost minimization.

The structure of this paper is as follows:

- Section 2 formulates the problem and outlines the simulation algorithm to solve it

- Section 3 presents the data used in the numerical examples

- Section 4 illustrates how cost and CO_2 emissions are affected by demand response and DER under uncertain electricity and natural gas prices

- Section 5 summarizes the findings of this paper and offers directions for future research

2   Problem Formulation

Since the objective is to minimize the expected discounted cost of meeting energy loads over a test year, it is first necessary to determine the DER equipment that will be installed. A deterministic approach has been used in the past by solving a mixed-integer linear program (MILP) that includes amortized capital costs for the on-site equipment (see [4], [5], [7], and [3]). Here, we fix the capacity installed at various levels and run a stochastic dynamic program (SDP) to minimize the expected discounted cost of meeting energy loads. The objective function in the SDP may be modified to consist of a weighted average of costs and CO_2 emissions (see [8]).

In order to find optimal DER adoption under stochastic prices, however, a stochastic programming approach needs to be used, possibly employing Benders' decomposition. Here, we take the approach of [2] and [6] in solving the optimal DER operational problem under uncertainty for various levels of installed capacity. We assume that the microgrid is on a real-time pricing (RTP) tariff, which means that it must buy all energy at the daily average price. No demand charges or hedging opportunities are considered.

2.1   Nomenclature

Denoting each day as $t = 1, \ldots, T$ and each DER generator as $i = 1, \ldots, I$, we define the parameters for this program as follows:

- $EDemand_t$: electricity demand during day t (in MWh$_e$)

- $HDemand_t$: heat demand during day t (in MWh)

- $\Delta t = \frac{1}{T}$: length of each time step (in years)

- r: discount rate per annum

- $\overline{h}$: number of hours per day

- EP_t: wholesale electricity price during day t (in \$/MWh$_e$)

- $ETDCharge_t$: electricity transmission and distribution (T&D) charge during day t (in \$/MWh$_e$)

- FP_t: wholesale natural gas price during day t (in \$/MMBTU)

- $FTDCharge_t$: natural gas T&D charge during day t (in \$/MWh)

- $EEff_i$: energy-conversion efficiency of DER generator i (in MWh_e/MWh)

- $PPEEff$: energy-conversion efficiency of central power plant (in MWh_e/MWh)

- HR_i: amount of useful heat produced by DER generator i per MWh_e (in MWh/MWh_e)

- $HEff$: energy-conversion efficiency for natural gas to heat (in MWh/MWh)

- $MaxP_i$: rated power capacity of DER generator i (in MW_e)

- $CapCost_i$: capacity cost of DER generator i (in $\text{\$/MW}_e$)

- $OMFix_i$: fixed O&M cost of DER generator i (in $\text{\$/(MW}_e \text{ year))}$)

- $Life_i$: lifetime of DER generator i (in years)

- $Annuity_i$: annuity factor for DER generator i

- $OMVar_i$: variable O&M cost of DER generator i (in $\text{\$/MWh}_e$)

- $NGCRate$: CO_2 emissions rate of natural gas (in tCO_2/MWh)

- $UCRate$: CO_2 emissions rate of utility-provided electricity (in $\text{tCO}_2/\text{MWh}_e$)

- $\Phi = 3.412$: conversion factor from MMBTU to MWh for fuel (in MMBTU/MWh)

- $EDRCost$: variable cost of reducing electricity demand (in $\text{\$/MWh}_e$)

- $HRDCost$: variable cost of reducing heat demand (in \$/MWh)

- $MaxEDR$: maximum fraction of electricity demand to be met by demand response during any day

- $MaxHDR$: maximum fraction of heat demand to be met by demand response during any day

- $DGInv_i \in \mathbb{Z}_+$: number of units of DER generator i adopted

- $DNCost$: annualized deterministic energy bill of a microgrid without DER equipment installed (in \$)

- $DNEmissions$: annual CO_2 emissions of a microgrids without DER equipment installed (t CO_2)

- $\Psi_t \equiv \{EP_t, FP_t\}$: set of stochastic state variables at time t

The corresponding decision variables are as follows:

- $Gen_{i,t} \geq 0$: operating level of DER generator i during day t (in MWh_e)

- $DGHeat_t \geq 0$: heat load met from DER generation during day t (in MWh)

- $NGHeat_t \geq 0$: heat load met from natural gas purchases during day t (in MWh)

- $EPurchase_t \geq 0$: electricity purchased from utility during day t (in MWh_e)

- $EDResponse_t \geq 0$: electricity demand response during day t (in MWh_e)

- $HDResponse_t \geq 0$: heat demand response during day t (in MWh)

- $\Xi_t \equiv \{Gen_{i,t}, DGHeat_t, NGHeat_t, EPurchase_t, EDResponse_t, HDResponse_t\}$: set of all decision variables at time t

2.2 Stochastic Dynamic Program

Using $0 \leq \alpha \leq 1$, the objective function is a weighted average of the annualized costs and CO_2 emissions. If we allow for stochastic electricity and natural gas prices, then $V_t(\Psi_t)$ is the minimized value function to go (consisting of the weighted expected discounted operating cost and CO_2 emissions to go) at the start of day t given current energy prices. In the SDP, we take the DER technologies as fixed and dispatch them optimally under uncertainty. Their amortized investment cost is added to the minimized operating cost at $t = 1$. For convenience, we let $Cost_t(\Xi_t; \Psi_t)$ be the period-t weighted operating cost and $Emissions_t(\Xi_t; \Psi_t)$ be the corresponding CO_2 emissions:

$$Cost_t(\Xi_t; \Psi_t) \equiv \sum_{i=1}^{I} Gen_{i,t} OMVar_i + \sum_{i=1}^{I} Gen_{i,t} \frac{(FP_t\Phi + FTDCharge_t)}{EEff_i} \tag{1}$$

$$+EPurchase_t(EP_t + ETDCharge_t) + NGHeat_t \frac{(FP_t\Phi + FTDCharge_t)}{HEff}$$

$$+EDRCost\ EDResponse_t + HDRCost\ HDResponse_t$$

$$Emissions_t(\Xi_t; \Psi_t) \equiv \sum_{i=1}^{I} Gen_{i,t} \frac{NGCRate}{EEff_i} + NGHeat_t \frac{NGCRate}{HEff} + EPurchase_t UCRate \tag{2}$$

Thus, the SDP to be solved from any day t is:

$$V_t(\Psi_t) = \min_{\Xi_t} \frac{\alpha}{DNCost} Cost_t(\Xi_t; \Psi_t) + \frac{(1-\alpha)e^{r\Delta t}}{DNEmissions} Emissions_t(\Xi_t; \Psi_t) \tag{3}$$

$$+e^{-r\Delta t}\mathbb{E}_{\Psi_t}[V_{t+1}(\Psi_{t+1})]$$

subject to the following terminal condition and constraints:

$$V_T(\Psi_T) = \min_{\Xi_T} \frac{\alpha}{DNCost} Cost_T(\Xi_T; \Psi_T) \tag{4}$$

$$+\frac{(1-\alpha)e^{r\Delta t}}{DNEmissions} Emissions_T(\Xi_T; \Psi_T)$$

$$Gen_{i,t} \leq DGInv_i MaxP_i \overline{h} \; \forall \; i, t \tag{5}$$

$$EDResponse_t + \sum_{i=1}^{I} Gen_{i,t} + EPurchase_t = EDemand_t \; \forall \; t \tag{6}$$

$$DGHeat_t \leq \sum_{i=1}^{I} HR_i Gen_{i,t} \; \forall \; t \tag{7}$$

$$HDResponse_t + DGHeat_t + NGHeat_t = HDemand_t \; \forall \; t \tag{8}$$

$$EDResponse_t \leq MaxEDR\ EDemand_t \; \forall \; t \tag{9}$$

$$HDResponse_t \leq MaxHDR\ HDemand_t \; \forall \; t \tag{10}$$

2.3 Energy Price Models

We assume that the logarithms of the deseasonalized electricity and natural gas prices, X_t and Y_t, respectively, evolve according to correlated mean-reverting Ornstein-Uhlenbeck (OU) processes, i.e.,

$$dX_t = \kappa_X(\theta_X - X_t)dt + \sigma_X dS_t \tag{11}$$

$$dY_t = \kappa_Y(\theta_Y - Y_t)dt + \rho\sigma_Y dS_t + \sqrt{1-\rho^2}\sigma_Y dW_t \tag{12}$$

Here, for process k, θ_k is the long-term mean, κ_k is the rate of mean reversion, σ_k is the annualised volatility, and $\rho = \rho_{XY}\frac{1}{2}\frac{(\kappa_X+\kappa_Y)}{\sqrt{\kappa_X\kappa_Y}}$, where ρ_{XY} is the instantaneous correlation coefficient between $\{X_t, t \geq 0\}$ and $\{Y_t, t \geq 0\}$. Furthermore, $\{S_t, t \geq 0\}$ and $\{W_t, t \geq 0\}$ are independent standard Brownian motion processes. Thus, the natural logarithms of the electricity and natural gas prices are:

$$\ln EP_t = X_t + f_t^X \tag{13}$$

$$\ln FP_t = Y_t + f_t^Y \tag{14}$$

where $f_t^k = \sum_{\ell=1}^{[s/2]}\left(\gamma_{1\ell}^k \cos\lambda_\ell t + \gamma_{1\ell}^{*k}\sin\lambda_\ell t\right) + \sum_{\ell=1}^{[s'/2]}\left(\gamma_{2\ell}^k \cos\lambda_\ell' t + \gamma_{2\ell}^{*k}\sin\lambda_\ell' t\right)$ is the seasonality function that detects weekly and annual trends for $k = X, Y$, $t = 1, \ldots, T$, $s = 7$, $s' = 365$, and

$$[a/2] = \begin{cases} a/2 & \text{if } a \text{ is even} \\ (a-1)/2 & \text{otherwise} \end{cases} \tag{15}$$

We use the procedure described in [1] for estimating the parameters in both the OU processes and the seasonality functions.

The OU processes in Equations 11 and 12 may be simulated as follows using two independent standard normal random variables ϵ_X and ϵ_Y:

$$X_{t+1} = X_t + \kappa_X(\theta_X - X_t)\Delta t + \sigma_X\epsilon_X\sqrt{\Delta t} \tag{16}$$

$$Y_{t+1} = Y_t + \kappa_Y(\theta_Y - Y_t)\Delta t + \sigma_Y\rho\epsilon_X\sqrt{\Delta t} + \sqrt{1-\rho^2}\sigma_Y\epsilon_Y\sqrt{\Delta t} \tag{17}$$

2.4 Solution Procedure

In order to solve the SDP in Equations 3 to 8, we proceed via simulation by first generating N sample paths for the electricity and natural gas prices. Next, we calculate the expected minimized objective function in the terminal time step, T, for each sample path. Finally, we work backwards recursively along each sample path to solve the constrained minimization problem for each t until we reach the first time step. Taking the mean of the value functions at $t = 1$ gives us the expected minimized value function for the microgrid to which is added the amortized DER capital cost. The algorithm is summarized in Figure 1.

For a given level of α, the minimized objective function value for various levels of installed DER capacity may be compared with and without the availability of demand response (DR). Features to note will be the expected values and variances of the objective functions. As α is varied, the objective function puts different weights on minimizing the cost and CO_2 emissions. After running the SDP for different values of α, an efficient frontier is created that indicates the tradeoff between economic and environmental objectives. By allowing for DR, we can show how the efficient frontier is altered.

3 Data

Since we assume decisions are made daily over a test year, $T = 365$ and $\Delta T = \frac{1}{365}$. Furthermore, the discount rate is $r = 0.10$.

```
1    Generate Ψ_t^(n), t = 1, ..., T, n = 1, ..., N
```

$$2 \quad V_T^{(n)}(\Psi_T^{(n)}) = \min_{\Xi_T^{(n)}} \left\{ \frac{\alpha Cost_T^{(n)}(\Xi_T^{(n)};\Psi_T^{(n)})}{DNCost} + \frac{(1-\alpha)Emissions_T^{(n)}(\Xi_T^{(n)};\Psi_T^{(n)})}{e^{-r\Delta t}DNEmissions} \right\} \texttt{ s.t. } \text{Equations 5 to 10,}$$

```
     n = 1, ..., N;

3    For t = T − 1, ..., 1

4      For n = 1, ..., N
```

$$5 \qquad V_t^{(n)}(\Psi_t^{(n)}) = \min_{\Xi_t^{(n)}} \left\{ \frac{\alpha Cost_t^{(n)}(\Xi_t^{(n)};\Psi_t^{(n)})}{DNCost} + \frac{(1-\alpha)Emissions_t^{(n)}(\Xi_t^{(n)};\Psi_t^{(n)})}{e^{-r\Delta t}DNEmissions} \right\} + e^{-r\Delta t}V_{t+1}^{(n)}(\Psi_{t+1}^{(n)})$$

```
       s.t. Equations 5 to 10;

6      End

7    End
```

$$8 \quad \texttt{Min Function} = \frac{\sum_{n=1}^{N} V_1^{(n)}(\Psi_1^{(n)})}{N};$$

Figure 1: Solution Procedure for SDP

3.1   Energy Prices

We use electricity and gas prices for San Francisco during the years 2006 and 2007 in order to estimate the parameters for our seasonal and mean-reverting processes. The electricity and natural gas price data are available from the California Independent System Operator (`http://www.caiso.com`) and the Pacific Gas and Electric Company (`http://www.pge.com`), respectively. In deseasonalizing the data, we first obtain the deterministic seasonality functions during 2006 and 2007 for the natural logarithms of the prices indicated in Figure 2. Next, we estimate parameters for the OU processes via ordinary least-squares (OLS) regression on the lagged residuals (see Table 1). In addition, $X_1 \equiv \ln EP_1 - f_1^X = 3.82$, $Y_1 \equiv \ln FP_1 - f_1^Y = 2.08$, and $\rho = 0.33$.
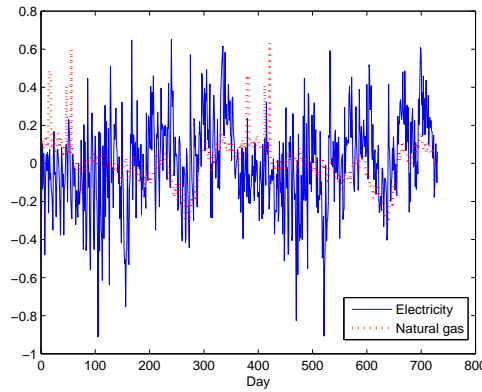

Figure 2: Seasonality Functions for Natural Logarithms of Electricity and Natural Gas Prices during 2006 and 2007

Simulated sample paths for the electricity and natural gas prices during 2006 are shown in Figures 3 and 4. As indicated, it is usually not economical to produce electricity on-site when CHP applications do not exist. We also perform sensitivity analyses to the volatility and correlation coefficient. The constant T&D adders are $ETDCharge_t = 100$ and $FTDCharge_t = 20$. Unless otherwise stated, we will base our subsequent numerical examples on 2006 simulated prices.
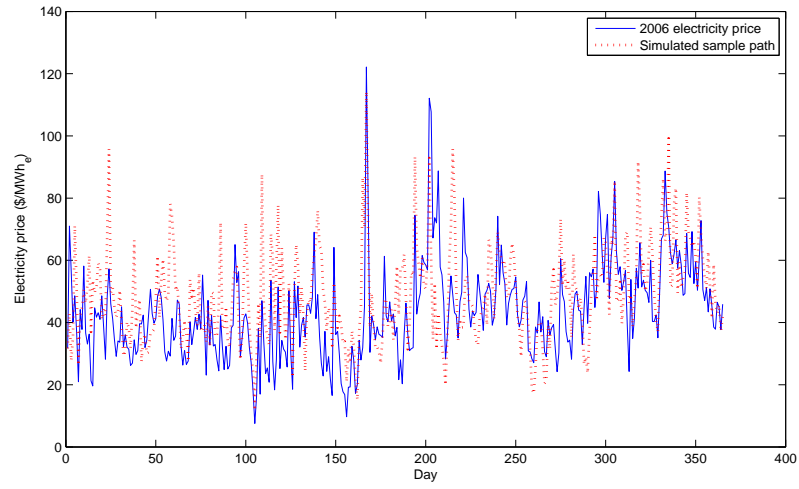
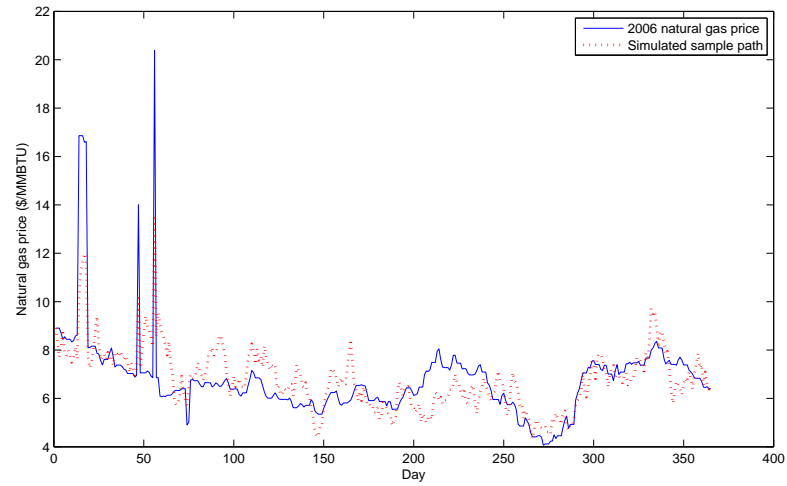Figure 3: Actual and Simulated Daily Electricity Prices for 2006

Figure 4: Actual and Simulated Daily Natural Gas Prices for 2006

Process, k	θ_k	κ_k	σ_k
X	3.84	110.37	3.33
Y	1.92	53.00	1.10

Table 1: Parameter Estimates for OU Processes

3.2 Energy Loads

The load data are taken from a northern California nursing home and are plotted in Figure 5. Mean daily demands are 15.79 MWh$_e$ and 15.62 MWh for electricity and heat, respectively, while the standard deviations are 1.34 MWh$_e$ for electricity and 2.37 MWh for heat. Furthermore, the correlation coefficient between the two is -0.91 because heat demand usually peaks during winter months when the electrical demand for cooling is at its lowest. This negative correlation between electricity and heat demand could limit the potential for CHP applications.
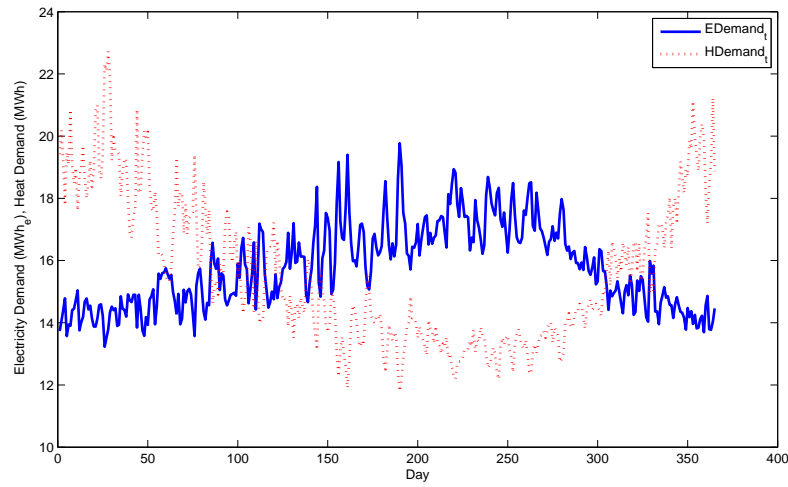

Figure 5: Daily Electricity and Heat Demands at a Northern California Nursing Home

3.3 DER Equipment

We assume that the only available technology is discrete units of 0.10 MW$_e$ DG engines with CHP capability that can operate without failure throughout the year. The associated cost and performance parameters are: $HR = 1.74$, $CapCost = 3000000$, $EEff = 0.26$, $OMFix = 0$, $PPEEff = 0.35$, $OMVar = 20$, $HEff = 0.8$, $NGCRate = 0.1836$, and $UCRate = 0.55$.

3.4 Demand Response

We assume that $EDRCost = 60$, $HDRCost = 30$, $MaxEDR = 0.10$, and $MaxHDR = 0.20$.

4 Numerical Examples

We assume three price scenarios (deterministic, stochastic, and high electricity price volatility), and do three cases for each: do nothing (DN), DER without DR (ND), and DER with DR (DER). For the two

cases in each scenario where DER equipment is installed, we consider three capacity levels: 0.10 MW$_e$, 0.30 MW$_e$, and 0.50 MW$_e$. The annualized capital cost of DG equipment is \$352379 per MW$_e$, which is added to any operational costs to determine the total annual energy bill. All stochastic scenarios use $N = 1000$ sample paths.

4.1 Deterministic Prices

We run the model with $\sigma_X = 0$ and $\sigma_Y = 0$ for $\alpha = 1$, $\frac{2}{3}$, and $\frac{1}{3}$, where decreasing α places greater emphasis on minimizing CO_2 emissions. The results are indicated in the form of an efficient frontier between the two objectives (see Figure 6). In the DN case, the annual energy bill is \$1.1 million with 4480 tons of CO_2 produced. Compared to this benchmark, almost any ND or DER case dominates from both economic and environmental perspectives. In particular, for the ND cases, the 0.50 MW$_e$ unit provides a nearly 18% reduction in the CO_2 emissions (albeit at a slightly higher cost), while the 0.30 MW$_e$ unit delivers a 3% reduction in the annualized energy bill along with CO_2 emissions that are 14% lower than in the DN case. The 0.10 MW$_e$ unit provides modest savings in both criteria due to its limited size, which cannot leverage significant differences in the electricity and heat prices to produce enough energy on-site when warranted. Varying α for the ND cases has almost no impact on the output as without DR, there is hardly any scope to produce more heat on-site.
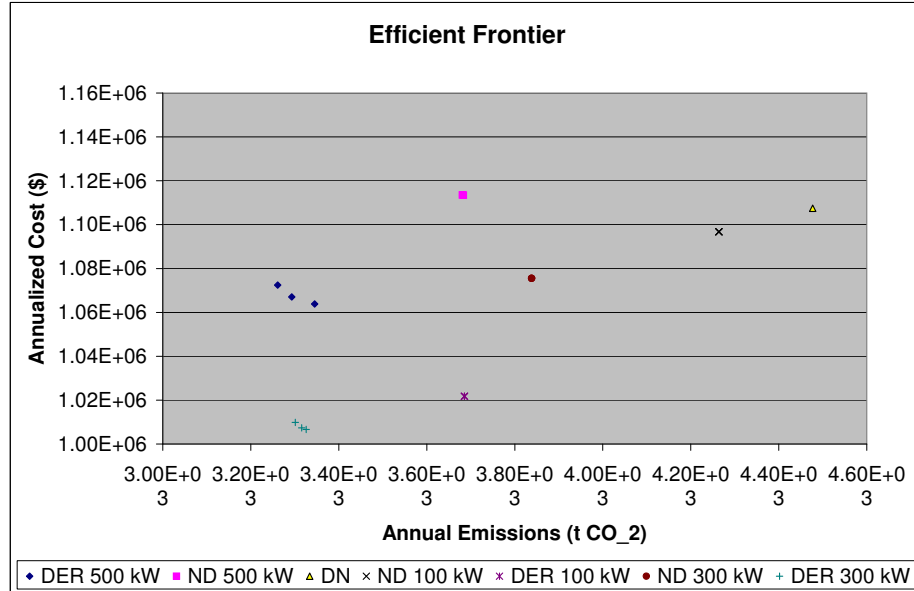

Figure 6: Efficient Frontier Between Cost and CO_2 Emissions Minimization with Deterministic Energy Prices

Once DR is allowed, further savings in both the cost and CO_2 emissions are achieved. Even the 0.10 MW$_e$ unit provides cost savings of 8% along with a CO_2 emissions reduction of nearly 18%. However, it is outperformed by the larger 0.50 MW$_e$ unit in the environmental criterion and by the 0.30 MW$_e$ unit in terms of both cost (9% lower than in the DN case) and emissions (25% lower than in the DN case). Furthermore, as α is varied, we observe a clearer tradeoff between the two objective function criteria: greater emphasis on the environmental criteria duly reduces CO_2 emissions but at a slightly higher cost.

4.2 Stochastic Prices

Here, we use the energy price parameters given in Table 1 for all cases. From the efficient frontier between cost and CO_2 minimization (see Figure 7), we observe a similar pattern as in Figure 6: the large unit is

the most environmentally attractive one, whereas the medium unit minimizes expected cost. Again, DR is crucial in reducing both costs and emissions. The main difference here is that due to uncertain energy prices, expected costs are slightly higher, especially for the small DG unit, which has less leverage to respond to adverse market conditions.
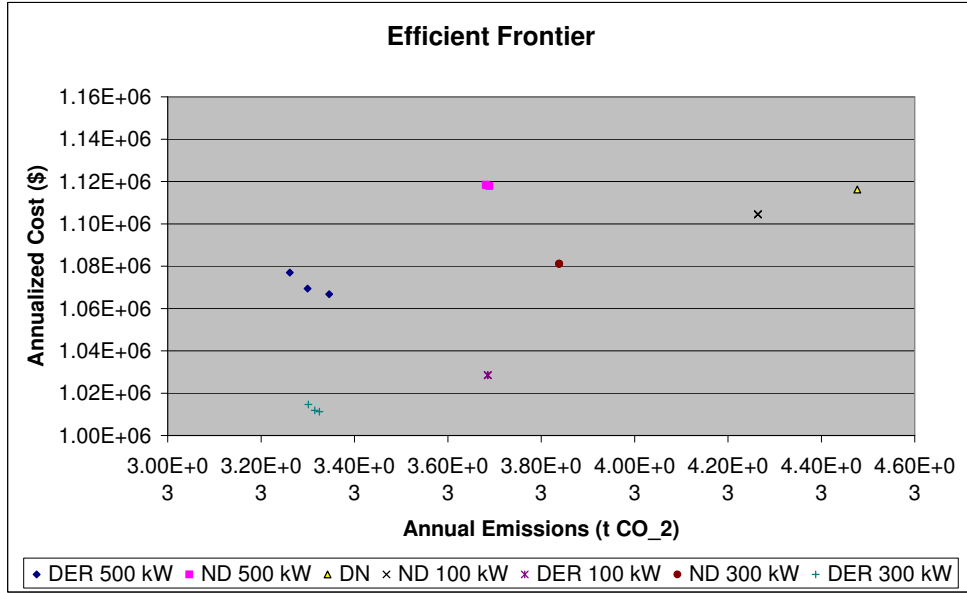

Figure 7: Efficient Frontier Between Cost and CO_2 Emissions Minimization with Stochastic Energy Prices

In order to examine the risk implications of DER equipment under uncertainty, we next compare the expected minimized cost with the 95%-level conditional value-at-risk (cVaR), which is the conditional expected cost given that the cost lies in the upper 5% of the possible outcomes. This is also known as the expected tail loss (ETL). The results in Figure 8 indicate that while the large DG unit in the ND case is not very effective at managing risk, both the small and medium units control costs well. In fact, the cVaR for the medium DG unit is $1.1 million, which is even less than the expected cost in the DN case. Once DR is allowed, the risk management potential of DER improves even further: in the extreme case, the 95% cVaR with the medium DG unit is $1.03 million. Another finding here is that as greater emphasis is placed on minimizing CO_2 emissions, the cVaR increases perceptibly, an effect that was not noticeable in the ND cases.

4.3   Stochastic Prices with High Electricity Price Volatility

In this price scenario, we double the instantaneous volatility of the natural logarithm of the deseasonalized electricity price process, σ_X, *ceteris paribus*. Consequently, higher electricity prices are more likely to occur, which results in higher expected minimized costs (see Figure 9). Besides this change, the pattern observed in Figure 7 is not affected since CO_2 emissions do not change much except for the large DG unit. Indeed, since it has spare capacity to produce more electricity, there is more on-site generation in response to higher electricity prices without a corresponding increase in heat capture. As on-site generation is more carbon-intensive than via the macrogrid, CO_2 emissions increase.

The pattern for risk management in Figure 10 is also similar to the one in Figure 8 except that the cVaR is perceptibly higher here. In the ND cases, the cVaR increases by between 1.3% (for the large unit) and 3.5% (for the small unit) as higher electricity prices occur more frequently. Here, the large DG unit is relatively less affected because it has enough reserve capacity to run during periods of
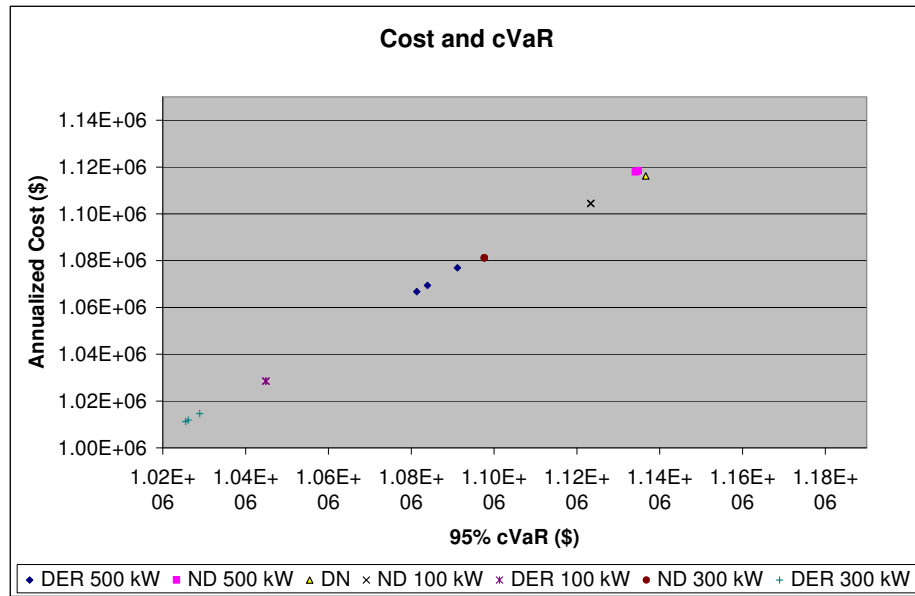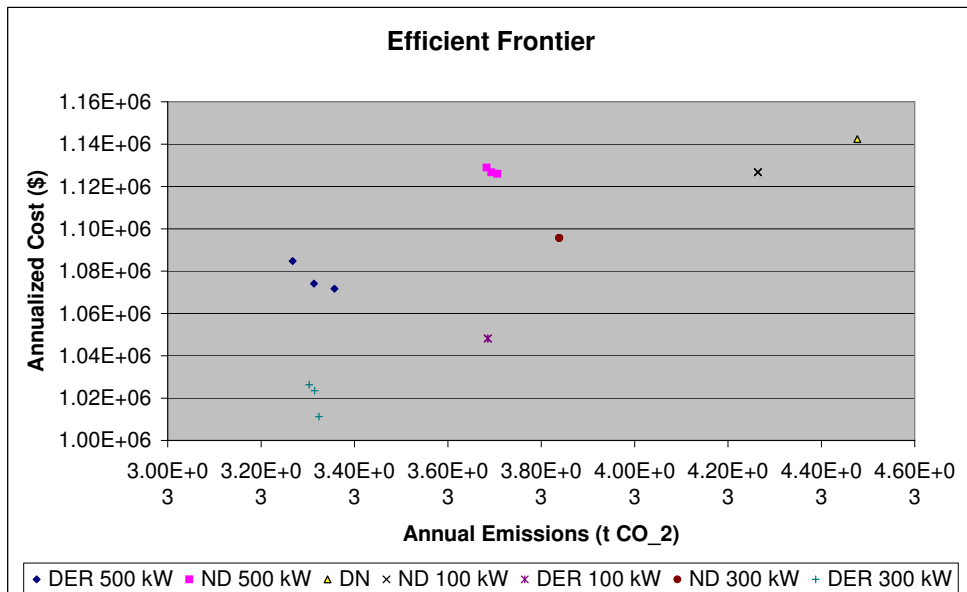
Figure 8: Tradeoff Between Cost and Risk with Stochastic Energy Prices

Figure 9: Efficient Frontier Between Cost and CO_2 Emissions Minimization with High Electricity Price Volatility

high electricity prices. Consequently, the large DG unit now outperforms the small one in terms of risk management. For the cases with DR, the increase in the cVaR is more modest: only 2% for the medium DG unit, 3.3% for the small one, and 1% for the large one. The availability of DR provides greater flexibility to control costs, although emphasis on CO_2 emissions minimization increases risk exposure.
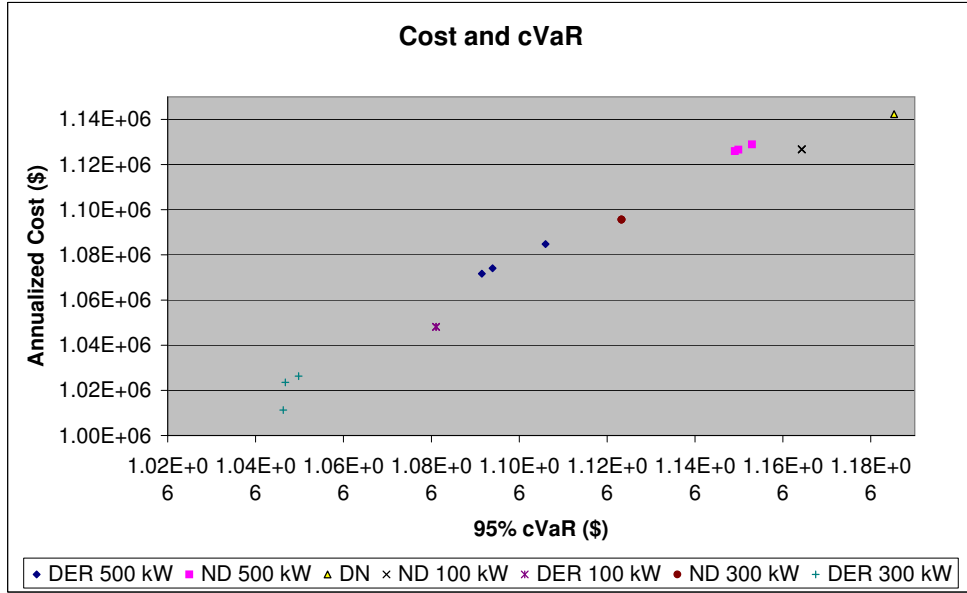

Figure 10: Tradeoff Between Cost and Risk with High Electricity Price Volatility

5   Conclusions

Microgrids that employ DG with CHP applications potentially provide a more sustainable pathway to capacity expansion. Indeed, the waste heat from on-site generation can be captured and partially used to offset heat loads, thereby reducing the need for natural gas purchases. In this paper, we illustrate not only the economic and environmental benefits of microgrids, but also their risk management capability as energy prices are allowed to be stochastic. For almost all cases, DER dominates the baseline case of doing nothing in terms of lower expected energy bills, reduced CO_2 emissions, and lower cVaR. Furthermore, the availability of DR makes DER more attractive, especially when the electricity price becomes more uncertain.

Via a multi-criteria objective function, we also examine the prospect of minimizing CO_2 emissions directly. This leads to an efficient frontier, in which the large DG unit performs the best in terms of the environmental aspect, while the medium DG unit continues to have the lowest expected cost. Again, the availability of DR reduces both cost and emissions, while greater emphasis on minimizing CO_2 emissions increases not only the microgrid's expected cost, but also its risk exposure, especially in a scenario with higher electricity price volatility.

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