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Comparison of Fast Daylighting Climate-Based Simulation Methods for Parametric Design: Two-Phase, Three-Phase Method and Path Tracing

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Comparison of fast daylighting climate-based simulation methods for parametric design: two-phase, three-phase method and path tracing

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ABSTRACT

Computer modeling and simulation are essential for predicting architectural solutions performance. When integrated into the design process; they are known as performance-based design. During the design process, fast analytical cycles are required. However, daylighting simulations are challenging due to computational processing and time requirements. This paper presents a comparative study of fast daylight simulation methods, two-phase, three-phase and path tracing, in a hypothetical commercial building located in Miami, Florida. Five types of static glass and one electrochromic glass were tested with six parametric models of a translucent horizontal shading device. Daysim was used as reference. The two-phase method produced results similar to the reference results with faster processing times for static facades. For dynamic facades, the path tracing method with the tested interface enabled faster simulations and parametric variations. However, the three-phase method yielded a lower average error.

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Parametric facades; daylighting simulation; performance-based design; facade design; electrochromic windows

Introduction

This paper compares fast daylight simulation methods with a validated tool from a performance-based design (PBD) perspective. Assessing efficiency and accuracy, it evaluates methods to accelerate daylighting simulation in the design process. The study highlights the importance of rapid and integrated analyzes to optimize building performance, focusing on the initial stages of architectural design and optimization in PBD methods. A theoretical framework is established, divided into three parts: Simulation in architecture, Performance-based design, and Validation in analysis tools for design process.

Simulation in architecture

Computer modeling and simulation are crucial for predicting architectural performance in areas like comfort, energy efficiency, cost reduction, and material management (Hensen, 2004). Since 1960s, mathematical models and simulation tools have advanced for building analysis and evaluation. Methods have evolved for increased complexity and accuracy, demanding specialized technical knowledge and computational processing (Kusuda, 1999).

Simulation tools are evolving to integrate with architectural models and the design process. They require simplifications and improvements in visual outputs, methods, computing power, algorithmic

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3D models, and input data availability. These advancements enable simulations with greater detail, integration, and speed that were previously unfeasible (Jakubiec & Reinhart, 2011; Jones & Kennedy, 2017; Roudsari, Pak, & Smith, 2013; Wetter, 2012).

Architects ideate high-performance buildings through transdisciplinary considerations with qualitative and quantitative measures (Demers, 2006; Dili, Naseer, & Varghese, 2010). Decision-making needs to satisfy several performance requirements at the same time (Wetter, 2012). Ensuring efficiency in eco-friendly building design is challenging due to the complex and interconnected nature of the process, making it difficult to describe and classify in models (Aish, 2011; Shi & Yang, 2013; Wetter, 2012). Often performance parameters are not completely known (Maciel, Ford, & Lamberts, 2007). Despite recognizing the importance of performance considerations, few architects apply analysis steps in sketch phases, focusing on esthetic issues (S. Li, Liu, & Peng, 2020; Shi & Yang, 2013).

Authors suggest analyzing multiple performance criteria simultaneously in early design stages with fast cycles (Negendahl & Nielsen, 2015; Østergård, Jensen, & Maagaard, 2016; Wetter, 2012). For this reason, tools are created to improve interoperability and visual outputs in architectural design. Fast tools and multi-criteria analysis aid decision-making, guide the design process, and facilitate iterative solutions comparison (Negendahl & Nielsen, 2015).

Using simulation in design has reported barriers. As processing power has improved, simulation tools have become resource intensive (Kusuda, 1999). Their use requires specialized knowledge and skilled specialists for simulation, data entry, and manipulation. Tools often lack interoperability with architectural and simulation models, leading to rework and wasted time (Li et al., 2020).

Performance-based design

Performance-based design (PBD) integrates analytical processes into architectural models to guide solution generation based on performance indicators, aiming to provide quick feedback on the performance of design alternatives(Kalay, 1999). An optimization process can be incorporated to evaluate the effectiveness of alternatives in the modeling system (Al-Masrani, Al-Obaidi, Zalin, Isma, & I, 2018; Eve Lin & Jason Gerber, 2014; Oxman, 2008). This approach is gaining popularity thanks to advances in computer processing, simulation interfaces integrated with modeling tools, processing in the cloud, visual programming languages (VPL) (Bentley & Corne, 2002; Rutten & McNeel, 2007) and development of optimization tools focused on the design process (Kheiri, 2018; Luca, 2019; Rutten, 2013). PBD introduces measurable steps to the solution generation process, reducing subjectivity (Aish, 2011; Nguyen, Reiter, & Rigo, 2014; Østergård et al., 2016).

Four techniques commonly found in PBD models are rule-based design, data-driven design, simulation-based design, and optimization. These techniques can be used individually or in combination to enhance performance-driven models. Rule-based design involves establishing guidelines or rules to guide the design process, considering factors such as esthetics, functionality, materials, and structure. Data-driven design utilizes data and simplified analysis to inform design decisions, including the incorporation of site and climate data to improve efficiency. Simulation-based design employs complex computer simulations to analyze various aspects of building performance, such as structure, energy, and daylighting. Optimization involves adjusting design parameters to achieve desired outcomes, such as energy efficiency, daylighting, stability, cost-effectiveness, and esthetics. In parametric design (PD), optimization is often achieved by iteratively varying the design parameters using computational techniques, such as heuristic or metaheuristic methods, such as genetic algorithms (Aish, 2011; Østergård et al., 2016; Zhao & de Angelis, 2019).

The PBD success depends on how the geometric model is parametrically generated. Logical steps, 3D model flexibility, variable parameter selection, and system constraint design are crucial for effective planning (Aish, 2011; Papalambros & Wilde, 2000). Programming initially requires more time and effort, but it enables the cognitive construction and regeneration of the model within the design logic, resulting in a radical increase in possibilities compared to traditional or non-algorithmic parametric modeling (Aish, 2011; Zhao & de Angelis, 2019).

Simulation in daylighting for architecture

Daylighting simulations have historically posed challenges in integrating them with the design process (Ochoa, Aries, & Hensen, 2012). Validated and reliable methods like ray tracing and radiosity require significant computational time. Recent research has focused on developing faster methods to integrate daylighting simulations with architectural design processes (Subramaniam & Mistrick, 2017), processing techniques (Jones & Reinhart, 2017; Zuo, McNeil, Wetter, & Lee, 2014), and configuration and modeling explorations (Subramaniam, 2018).

Radiance is commonly used for high-impact studies. It's a suite of over 50 free tools for 3D modeling, lighting analysis, and visualization developed at Berkeley Lab for research purposes. Radiance employs a backward ray tracing method to generate realistic images and precise photometric data from 3D models and the optical properties of surfaces and light sources (Compagnon, 1997; Ward & Shakespeare, 1998). Ray tracing simulates light rays in a scene to determine light distribution from surface interaction. Rays stop after a set number of reflections or if the weight ratio falls below a threshold (Ward & Shakespeare, 1998).

Radiance is extensively validated for daylight simulations, and recent improvements have sped up simulations for integration with architectural design tools. Subramanian tested the two-phase method to speed up the simulation by modifying sky discretization (Subramaniam & Mistrick, 2017). Jones and Reinhart (2017) and Zuo et al. (2014), on the other hand, configure Radiance to enable simulation processing on the computer's GPU. Jones considers two-phase and three-phase methods and implemented experimentally a path tracing mode on Radiance for simulation (Jones & Kennedy, 2017). The main idea is the possibility to interrupt simulation to control runtime.

Some of these studies include integration with PD tools for architecture, which indicates a willingness to integrate simulation with the design process. However, there is a gap in the literature regarding the comparative accuracy of the latest method strategies, especially path tracing. Studies on accuracy often do not consider the context of the architectural design process, where faster methods and simplifications are often necessary. This is because there is a lower level of detail in the early design stages, and the demands for responses are more qualitative, such as understanding which type of solution performs better than another (Brown & Mueller, 2017; Ellis & Mathews, 2001; D. H. W. Li, Cheung, & Lau, 2006; Picco, Lollini, & Marengo, 2014). Brembilla and Mardaljevic compared the state of art of climate-based simulation methods to test accuracy using high accuracy configurations settings and considers 15% as baseline for errors (Brembilla, Chi, Hopfe, & Mardaljevic, 2019). However, the study does not consider the design process or settings to produce faster simulations and did not include the path tracing method.

This paper explores three fast simulation methods and compares them to a widely used simulation package for climate-based annual analysis, Daysim. The evaluated methods are the two-phase method (Subramaniam & Mistrick, 2017), three-phase method, and path tracing method. The next paragraphs summarize each explored method. Daysim is a Radiance-based tool commonly used in annual climate-based simulation models that have been extensively validated. The tool was developed by the National Research Council Canada (NRCC) and the Fraunhofer Institute for Solar Energy Systems in Germany. It uses Radiance daylighting coefficients (two-phase dynamic daylighting simulations-DDS method) and simulates annual daylighting metrics using the ray tracing and climate-based sky models developed by Perez, Ineichen, Seals, Michalsky, and Stewart (1990), Reinhart and Breton (2009), Reinhart and Walkenhorst (2001).

Two-phase simulation (daylight coefficient)

The traditional two-phase method uses Radiance's daylight coefficient calculations for non-complex fenestration characterization. In this method, the contributions of the sky and sun are established in a single stochastic sampling and because of that, this method allows getting results quickly (Subramaniam & Mistrick, 2017). Two-phase ray tracing simulation is based on luminance characterization of the sky and optical properties of surfaces. The ray tracing is computed in two stages: outdoor route and indoor route. The two-phase method can be considered when the 3D simulation model and the material settings are static, and the fenestration is composed of coplanar surfaces. The traditional version has less accuracy than the DDS (dynamic daylighting simulation) version embedded in Daysim. The DDS version, in turn, is suitable for simulations with complex geometries and dynamic materials (Subramaniam, 2017). Subramanian implemented a modified version of two-phase method to improve accuracy for use in architectural design. It considers simulations performed in 'rcontrib' instead of simulations in 'rtrace' used in the DDS version. This version is disponible in the Honeybee + plugin for Grasshopper 3D (Subramaniam & Mistrick, 2017).

Three-phase simulation

The three-phase method is the first Radiance-based technique that supports dynamic fenestration and the use of complex fenestration materials (BSDF). It computes ray tracing in three-phases, represented by separate matrices: light propagation from sky to the outside of the fenestration, transmission of light through the fenestration using a Bidirectional Scattering Distribution Function (BSDF) matrix, and light propagation in the interior environment (Brembilla et al., 2019; Subramaniam, 2017; Ward, Mistrick, Lee, McNeil, & Jonsson, 2011). Each matrix contains normalized coefficients. The first stage establishes the sky model and the sun position (sky matrix). The coefficients that characterize the transmission of light through the fenestration are organized in a transmission matrix. The contribution of light from the sky toward the window is represented by the daylight matrix. The transport of light between the window and the sensor is represented by a view matrix. The result is achieved by multiplying the coefficients of these matrices with the contribution of the sky light, allowing the determination of the contribution of daylight to the light distribution in the interior space (McNeil, 2013).

Path tracing simulation

The path tracing method was initially described by Kajiya, and initially implemented by Dutré for rendering (Ayoub, 2020; Dutré, Lafortune, & Willems, 1993; Kajiya, 1986). It addresses a progressive model based on Monte-Carlo methods. Path tracing traces a ray at a time, iteratively, thus constructs a sampling of the scene, and each iteration reduces sampling errors. The main benefit of the method is that it enables the interruption of computational rendering/simulation before it is completed if the margin of errors is acceptable.

In the work described by Jones and Reinhart (2016), and Jones and Kennedy (2017) a path tracing mode was implemented on Radiance for daylight simulation. The aim was to achieve real-time results for static glare simulations. This approach combines GPU processing with path tracing to shorten computation time. The implemented path tracing method in the GPU is integrated into the ClimateStudio software (Solemma, 2020). This method was employed to perform static and dynamic light simulations with reduced speed, integrating with PD. The number of iterations becomes a significant parameter for simulation accuracy.

Validation in analysis tools for design process

Integrated simulation with architectural models simplifies design analysis and synthesis, allowing for greater error tolerance compared to end-stage simulations. In essence, simplified simulation models or rules of thumb are commonly used during the design process (Reinhart, 2019). Despite this, errors should be understood to avoid misunderstandings in interpretation or classification of design solutions. Architects prefer a qualitative approach in the initial design stages to classify and determine the most suitable solution sets compared to other relevant solutions (Kanters, Horvat, & Dubois, 2014).

Two software validation procedures are found in the literature: experimental and comparative (Reinhart, 2018). In the first type, the validation uses experimental data for the comparison and considers the uncertainties of the measurement process for the validation. The second type comparatively evaluates the simulation data of two pieces of software: one validated and the other under review. For daylighting simulation, the *Commission Internationale de l'Eclairage* has developed a set of test procedures presented in 2006 CIE 171 that address accuracy tests of lighting programs (CIE STANDARD, 2006). These are procedures for testing accuracy in an end-use context, and not to aid the design process.

In the search for a validation process for simplified tools for architects, Poehls (2017) proposed an energy analysis tool and a method to evaluate simplified tools for early design stages. Two methods were proposed: classic and quanti-qualitative. The classic method compares differences between validated and reviewed tools within a margin of error. The quanti-qualitative method orders simulation results from validated and reviewed tools, focusing on result ordering rather than numerical accuracy. This method aids in selecting design solutions for further qualitative analysis (Poehls, 2017). Hviid, Nielsen, and Svendsen (2008) emphasize the importance of integrated and fast analysis tools for use in the early stages of a project. The study proposes a simplified mathematical model to predict the performance of lighting, thermal, and energy aspects of buildings. In this context, previous studies considered a 20% margin of error as acceptable for tools to assist in the design process (Hviid et al., 2008; Nielsen, 2005; Picco et al., 2014; Poehls, 2017)

Objective

The goal is to compare fast daylighting simulation methods and simplifications aiming to accelerate analysis cycles with a widely validated method, from the perspective of PBD. The tested methods are available for integration in PD: two-phase method, three-phase method, and path tracing method.

Method

The effectiveness of multi-phase methods and path tracing in early design stages, aiming to assist PBD, is examined. The selected simulation methods are those available in tools integrated with PD software. The comparative analysis considers a 3D base model case with two variations: static glazing and dynamic glazing. The multi-phase methods were explored based on Subramaniam's application diagram (Subramaniam, 2017), namely: the two-phase method as the faster method for static materials and the three-phase as more suitable method for dynamic materials. Path tracing was designed to simulate both. In addition to traditional settings, the paper proposes simulation simplifications to accelerate analysis cycles, compatible with the design process demands. The widely validated Daysim/Radiance software is adopted for comparative analysis, which is divided into three steps.

1. Base-case and the 3D parametric system;
2. Daylight simulation settings;
3. Comparative performance-based analysis.

The first step involves selecting the base case and the design variation system, which includes a building with a curtain wall facade and six glazing and shading solutions, as well as an urban environment, all established within a unique PD system. The second step establishes simulation settings, including simplifications for each method, to test faster simulation options while still obtaining useful results. The third step conducts a comparative analysis in the context of PBD to evaluate accuracy compared to a reference method. All cases use Rhinoceros 3D and Grasshopper 3D to generate models and simulations. The Honeybee + plugin was used for multi-phase simulations, Honeybee for Daysim simulations, and ClimateStudio for path tracing simulations.

Selection of the base-case

The base case is a nine-story non-residential building with highly glazed facades, open plan, and structural technical core in a hypothetical urban context for Miami, Florida (latitude: 25.77 degrees north) (Figure 1a,b). The goal is to replicate the initial stage of architectural design.. Miami was chosen for its sunny climate and demand for solar protection strategies in facades due to high temperatures. No specific zone from Miami's building code was specified, opting for a generic base case.

The facade has 70% glazed area (fully glazed facade without structure). The Parametric model generates multiple horizontal shading options, and six were chosen: two regular horizontal brise-soleil (SD1, SD2), three parametric twisted horizontal brise-soleil (SD3, SD4, SD5), and a solution without shading device for reference (NSD). Figure 2 displays each shading device (SD) with their respective window obstruction percentages (Wo) and average vertical shading angle (VSA).[1] The SD3 and SD5 have the same average VSA, but opposite geometries. The shading system was modeled using a triangular mesh with coplanar cells. To assess fast simulation methods for the combinations, external shading was paired with seven glazing solutions: six static and one electrochromic.

Daylighting simulations

This section describes the simulation settings for each method and tool used. Table 1 summarizes methods and strategies. The glass choice and tools used impacted the settings strategies: two-phase in Honeybee is not suitable for annual dynamic glass simulations, while path tracing in ClimateStudio can handle both, do not allow climate data manipulation. The simplifications tested in this context were the reduction of climate data file for the two-phase method, the simplification of BSDF file for the three-phase method, and the reduction of ray samples in path tracing method. In this case, two numbers of ray interactions: 4028 and 1280. Simulations were also performed without simplifications.

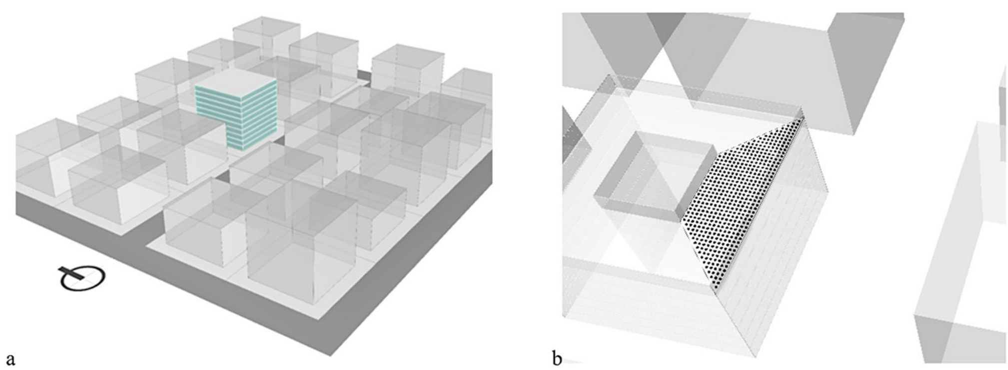


Figure 1. a. Base-case with urban obstruction. b. Seventh floor with sensors used in simulations.

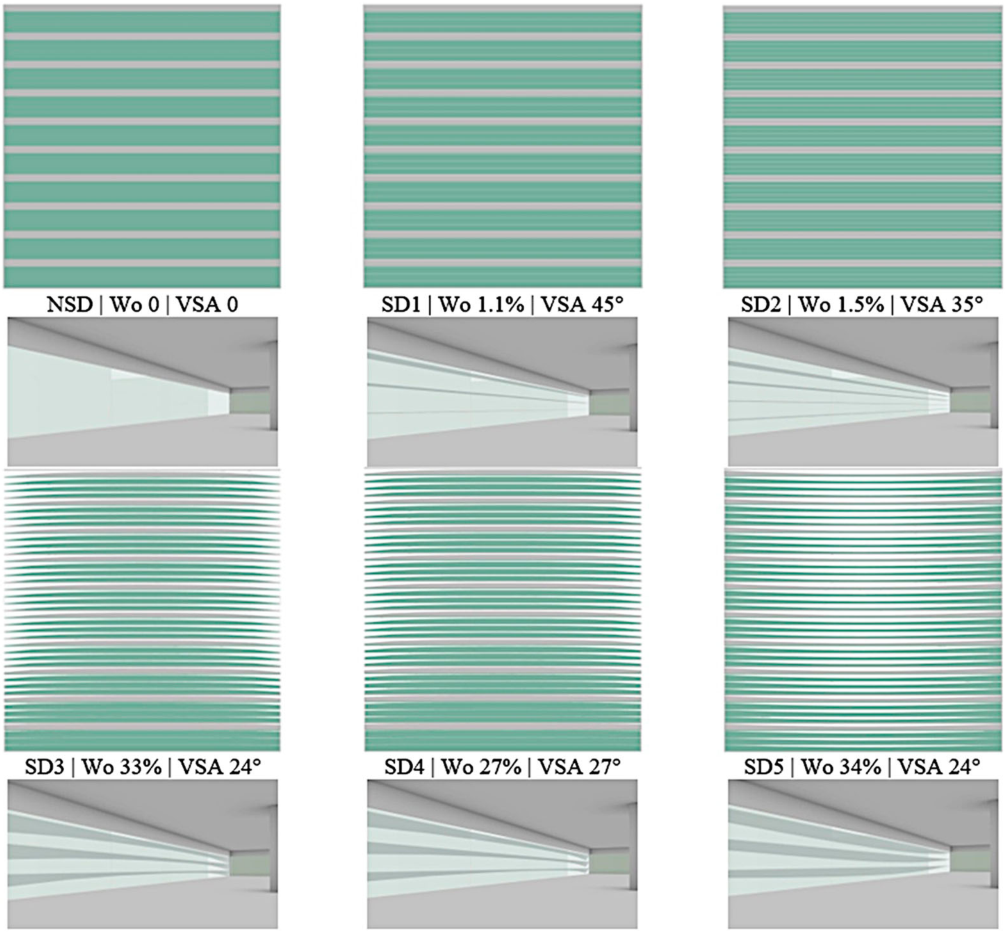


Figure 2. External SD combinations considered. NSD has no SD. SD1, SD3, SD4 and SD5 has 4 fins per floor. SD2 has 6 fins. Solutions SD3, SD4 and SD5 has a twisted effect along facade.

Simulation settings

Analysis concentrated on the 7th floor of building, with internal venetian blinds covering windows on the north, east, and west sides year-round, while those on the south side are fully open. 492 analysis sensors were positioned in a regular grid of 1.2 × 1.2 m, at a height of 0.75 m in the southern part of the floor, in all simulations. The grid size was defined through convergence testing (Figure 1b).

Table 1. Simulation's summary.

Solution	Method	Settings	Reference	combinations conducted
SD combinations + 05 static glasses	Two-phase	Complete climate data	Daysim	36
	Two-phase-sim	Climate data reduction		36
	Path tracing	-ab and -lw recommended by the tool		36
	Path tracing-sim	Paths, -ab and -lw reduction		36
SD combinations + electrochromic glass	Three-phase	BSDF describing SD and glass		6 (four states)
	Three-phase-sim	BSDF describing only glass. SD is in the 3D modeling		6
	path tracing	-ab and -lw recommended by the tool		6
	path tracing-sim	Paths, -ab and -lw reduction		6

Optical properties of materials used are provided in Table 2. Glass optical properties were sourced from WINDOW 7.7 software's database (Berkeley Lab, 2019). Table 3 shows the settings used for each method. Each method has its own characteristics, and parameter selection considered previous studies and convergence tests. Path tracing considered recommended default settings of the tool used (Brembilla et al., 2019; Reinhart & Breton, 2009; Subramaniam, 2018; Solemma, 2020).

The electrochromic glass schedule was fixed for the tested simulation methods based on Daysim results: each SD solution produced a specific routine to maintain the sensor's illuminance near the window within the range of 300–2000 lux. For the electrochromic glass in the three-phase method, two BSDF characterizations were considered: a simplified version describing only the glass states, and a complete configuration describing both the glass states and the SD solution. WINDOW 7.7 software was utilized, and the SD description used the average VSA for characterization.

Climate data reduction

To expedite the analysis, the reduction of the climate file was considered, following a strategy also employed by Negendahl and Nielsen (2015). An experiment examined the reduction of climate data while maintaining annual characteristics for daylight simulations in non-residential building designs. The selected time range based on occupancy was from 06:00 to 18:00, and the number of days was reduced in three samples:

1. from 365 to 182 days (half of the days of year),
2. from 365 to 121 days (a third of the days of year),
3. from 365 to 91 days (a quarter of the days of year).

Histograms for global horizontal radiation (GHR), direct horizontal radiation (DHR), and diffuse global radiation (DGR) were analyzed. A maximum 5% difference between populations was allowed. Monthly averages of hourly data were used to compare the complete and reduced file, considering average, maximum, and minimum values. Option 1 had a maximum error of 0.6%, option 2, a maximum error of 1.7%, and option 3, a maximum error of 5.1%. Option 2 was selected as it maintains the original climate data characteristics while reducing file size. Figure 3 shows histograms for complete climate data set and reduced climate data (one-third of the days) for global horizontal radiation. Climate data reduction was automated using the Grasshopper for Rhino 3D plugin.

Comparison analysis

The optimization in a PBD establishes variations based on numerical indexes that represent the design problem. In this research, spatial daylight autonomy (sDA) is used as performance index, since it is widely used in literature, and it is recommended in IES LM-83 (LM, 2013). For the validation approach, two analyzes were conducted:

Table 2. Optical properties in simulations. The SD uses a translucent material.

Construction	Refl	Tvis (diff)	Glazing system	Tvis
Ceiling	0.70	0	Glass 01	0.696
Interior walls	0.50	0	Glass 02	0.530
Floor	0.20	0	Glass 03	0.381
External ground	0.10	0	Glass 04	0.173
External buildings	0.40	0	Glass 05	0.071
Internal shading	0.70	0	Electrochromic glass states	0.597
External SD	0.65	0.20		0.173
				0.055
				0.009

Table 3. Radiance parameters set for methods.

Method	Parameters
Daysim (DS)	ab 7 -ad 2048 -ar 128 -as 2048 -aa 0.2 -lw 0.005
Two-phase (2Ph)	ab 6 -ad 2048 -ar 2048 -aa 0.01 -lw 4e-07
Two-phase with simplifications (2Phsim)	ab 6 -ad 2048 -ar 2048 -aa 0.01 -lw 4e-07
Three-phase (3Ph)	Vmtx -ab 10 -ad 20000 -ar 512 -lw 5e-07 Dmtx -ab 4 -ad 1024 -ar 512 -lw 1e-06
Three-phase with simplifications (3Phsim)	Vmtx -ab 10 -ad 20000 -ar 512 -lw 5e-07 Dmtx -ab 4 -ad 1024 -ar 512 -lw 1e-06
Path tracing (PT)	ab 7 -ad 1 -as 4028 -lw 0.005[a]
Path tracing with simplifications (PTsim)	ab 6 -ad 1 -as 1280 -lw 0.01

[a]Radiance parameters used were recommended by ClimateStudio manual.

1. The $sDA_{50,\,X}$ results were compared between the test methods and reference (Daysim). Seven sDA illuminance thresholds were considered: 200, 300, 500, 1000, 1500, 2000 and 2500 lux. The occupancy interval was between 6:00 and 18:00.
2. According to sDA results, representative cases were selected to compare values for each sensor in the room. The representative cases chosen were those with the largest differences and those that presented average differences. The error was computed for each sensor.

Each method was analyzed considering the average time per simulation, a scatter plot with correlation results, and a box plot with statistical indexes. The root mean square deviation was also calculated to characterize results. Two metrics were used to evaluate the comparison: an architectural approach validation, which quantified the normalized deviation of the set of solutions relative to the reference set of results, and a second metric. The first was performed using Equation 1:

$$D\% = \left| \frac{a_n}{a_{n+\dots} - a_0} - \frac{b_n}{b_{n+\dots} - b_0} \right| \quad (1)$$

Where:

D% is the difference between the location observed in each software for the same pair of cases. D % shows the relative deviation for a case in two normalized populations, being one considered as reference. a_0 and b_0 are the initial reference outputs from Daysim/Radiance and the methods tested, respectively. $a_{n+\dots}$ and $b_{n+\dots}$ are the final domain reference outputs from Daysim/Radiance and the methods tested, respectively. a_n and b_n are the output tested from Daysim and the methods compared. The two populations are normalized, and the tested values are compared relatively rather than absolutely. Despite this, when the population domain is the same in both populations tested,

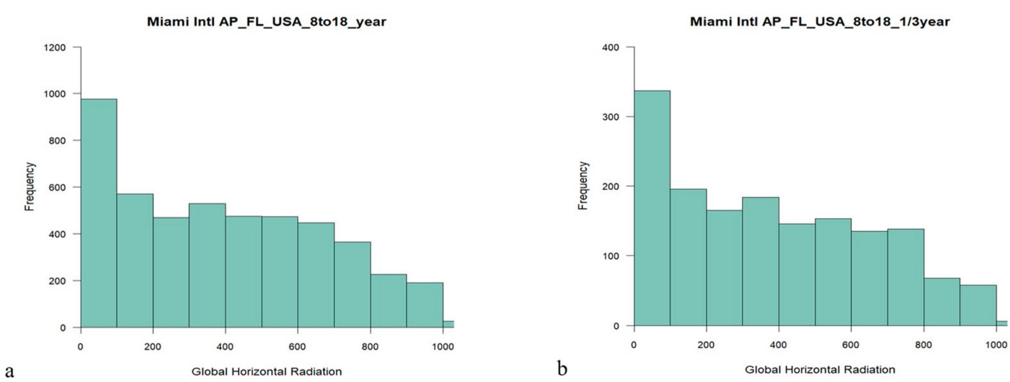


Figure 3. histograms showing GHR data for Miami climate. The histogram 'a' shows data for the interval between 8:00 am and 6:00 pm every day of the year. The 'b' option shows a sampling of one-third of the days for the same time interval.

that is, $a_{n+\ldots}$ and $b_{n+\ldots}$ is the same and a_0 and b_0, this equation produces similar results than the classical error approach, that is, the absolute difference of results tested. This is a variation of the equation proposed by Queiroz, Westphal, and Pereira (2020).

Secondly, the absolute order of results (Do) was calculated for each population tested, which represents the difference between the position of the test result in the set and the position of the reference result. This method helps to determine if the deviation (D%) was significant enough to alter the position of the results in the tested population when compared to the reference population. In PBD processes, it is crucial to identify which set of solutions is the best compared to others, and therefore any numerical deviations should not affect the position of the case in the set relative to the reference.

$$D_o = |O_{an} - O_{bn}| \tag{2}$$

Where:

D_o is the difference between the result position observed in each software for the same pair of cases. D_o shows the deviation between two sets, which is, from an architectural point of view, an important characteristic during the first stages of design. In that moment, the design team is interested in establishing a comparative between variations of the design tested. With this metric it is possible to quantity if the results are classified in a similar way than Daysim.

Results

This section debate simulation time and result differences for each method, which are important for optimizing architectural models. Results will be divided into two parts: static glass and electrochromic glass simulations. Table 4 shows average simulation durations for each method. The two-phase method with simplifications was the fastest for static glass, being up to 0.11 min/simulation faster than path tracing with simplifications. For electrochromic glass, path tracing with simplifications was faster by an average of 1.51 min/simulation compared to three-phase method with simplifications.

Static glass

Both methods tested had high correlations, but two-phase method showed higher correlations for sDA in static glass simulations, with an r^2 of 0.99 for both simulation strategies. The path tracing method had correlation results with r^2 of 0.94 and 0.95 (Figure 4). Figure 5 shows boxplots of deviation results calculated using equations 1 and 2. The two-phase method had better results, with an average D% of 1.50% for solutions without simplifications and 1.55% for solutions with simplifications, and a root mean square deviation of 3.38% without simplifications and 3.35% with simplifications. Simulations with simplifications had higher order deviation, with 75% of the data population having Do below 5 positions.

Table 4. Time of simulations performed for each method.

Method	Average time per simulation (min)	Total time (min)
DS – static glass	26.96	970.87
DS – electrochromic glass	772.35	4634.10
2Ph	1.54	46.20
2Phsim	0.87	26.10
3Ph	6.14	36.86
3Phsim	2.65	15.92
PT – static glass	2.62	78.66
PTsim – static glass	0.98	30.11
PT – electrochromic glass	2.80	16.82
PTsim – electrochromic glass	1.14	6.86

Path tracing method had an average D% of 6.97% for solutions without simplifications and 6.57% for solutions with simplifications, and a root mean square deviation of 12.45% without simplifications and 11.68% with simplifications (Figure 5). Like the two-phase method, simulations with simplifications showed higher order deviation, with 75% of the data population having Do below 10 positions. The two-phase method had maximum deviations of 11%, while the path tracing method reached deviations of up to 27% when considering outliers. When comparing the two-phase method with path tracing, the root mean square differences were up to 9.07%. This indicates that, for the configurations used, the two-phase method produces results that are more like Daysim than the path tracing method.

To demonstrate the types of errors that can arise from simplified settings, two cases were chosen to compare the Daylight Availability (DA) results for each of the 492 sensors distributed throughout the room. Two criteria were established for the selection process: an outlier case with the largest deviation previously observed, and a representative case that was near the average value of the calculated deviations, both based on the simplified models. This step involved examining the errors (the absolute difference between the reference data and the tested data) for the sensors in the room. Table 5 presents a detailed analysis of the evaluated cases for the simplified configurations.

The results show that, In the highest deviation case, path tracing had a 21.00% root mean square deviation for daylight autonomy (DA) values (300 lux threshold), while the two-phase model had a 5.6% root mean square deviation with simplifications. This means that the path tracing had an average error of 15.40% higher than the two-phase model (Figure 6). In the case of average deviations, path tracing had a 9.00% root mean square deviation for DA values (1500 lux), while the two-phase model had a root mean square deviation of 8.56%. This results in a difference of only 0.31% between both (Figure 7).

Electrochromic glass

Dynamic glazing simulations employed state change routines compatible with Daysim outcomes. Daysim simulations were used to ensure illuminance levels between 300 and 2000 lux. The resulting

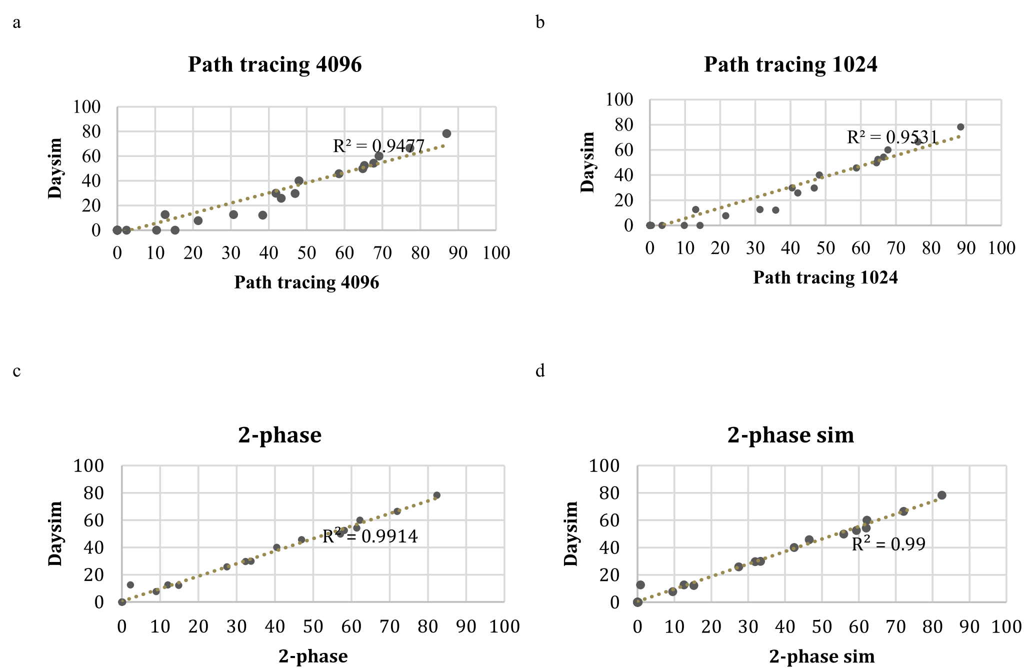


Figure 4. Correlation between methods tested and the Daysim results.

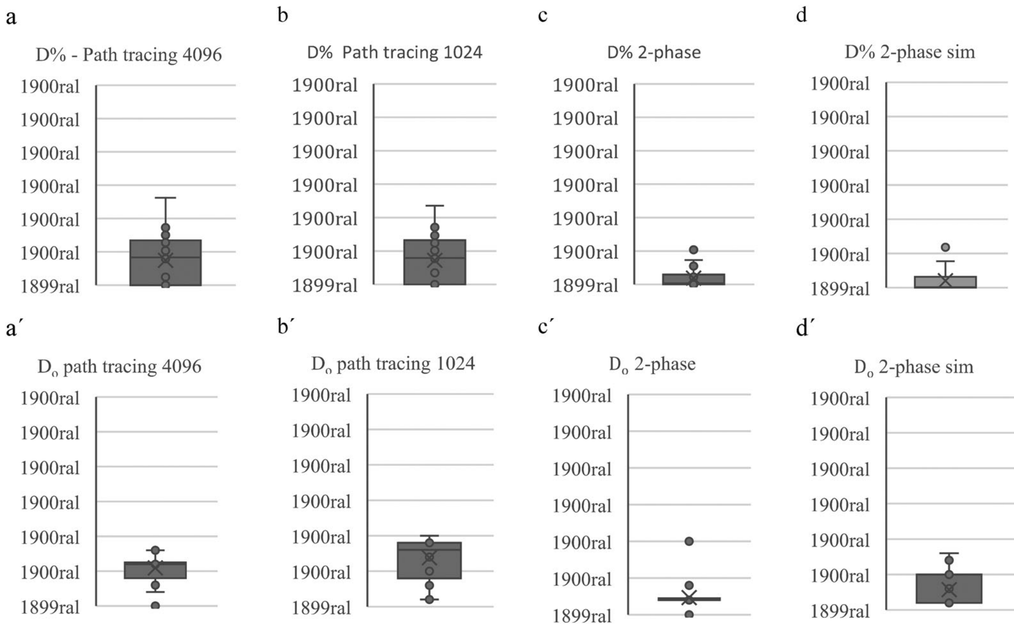


Figure 5. Boxplots for the sDA deviation results for each level of illuminance tested.

Table 5. representative solutions selected for electrochromic glazing.

Method	Maximum outlier case	sDa Deviation	Representative average case	sDA Deviation
Simplified path-tracing	SD2 Tvis 0.173 sDA 300	35%	NSD Tvis 0.697 sDA 1500	10.16%
Simplified 2-phase	SD2 Tvis 0.173 sDA 300	7.5%	NSD Tvis 0.697 sDA 1500	4.3%

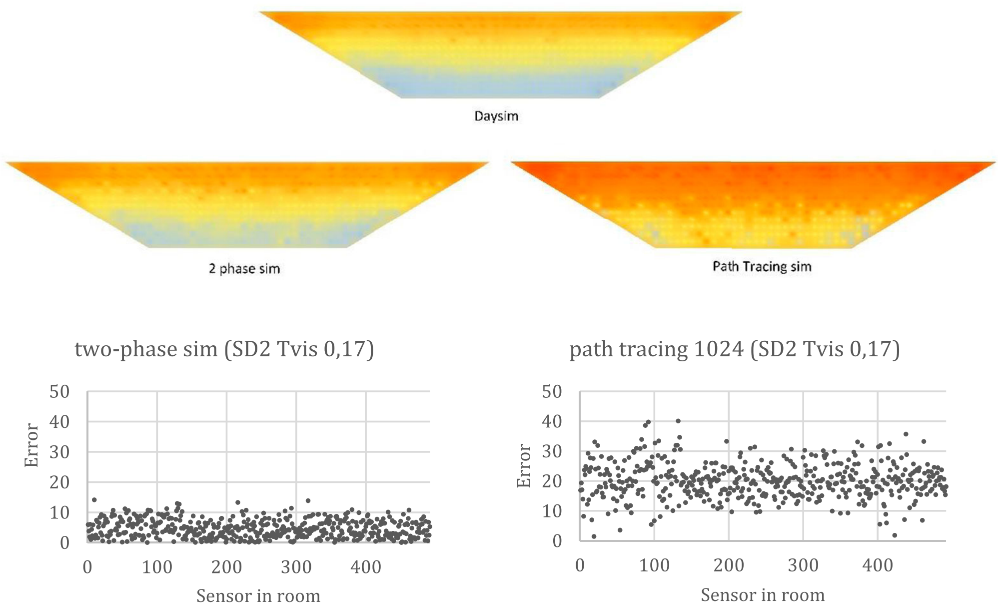


Figure 6. Error for DA results in each sensor in the room.

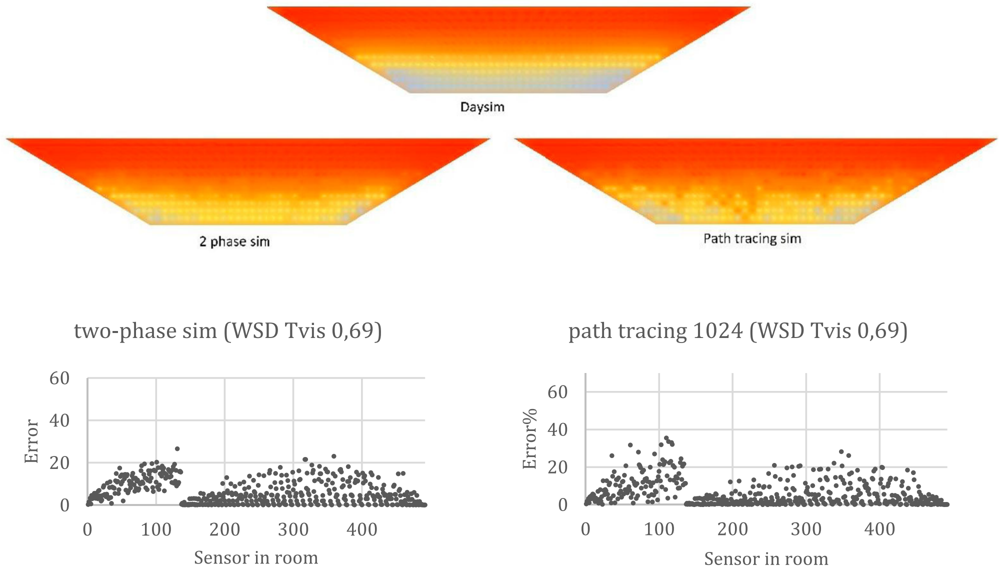
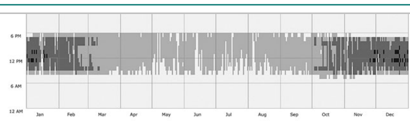
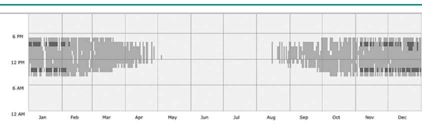
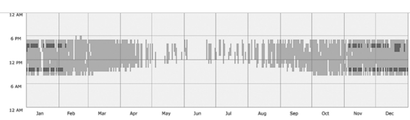
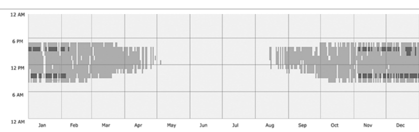
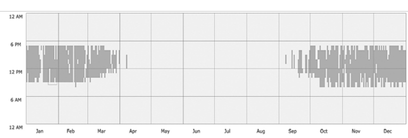
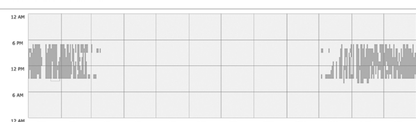


Figure 7. Error for DA results for each sensor in the room.

routines for each shading solution were used for simulations with both the two-phase and path tracing methods, resulting in the consistent operation of the electrochromic window across all simulation methods. The schedules generated by Daysim for each shading solution are presented in Table 6.

The study found that path tracing method and the complete three-phase method using full configuration of the Bidirectional Scattering Distribution Function (BSDF) showed higher correlations in sDA values. The path tracing method achieved r^2 values of 0.91 and 0.90 for non-simplified and simplified versions, respectively, while the complete three-phase method had an r^2 of 0.93. However, the simplified three-phase method had a lower correlation coefficient of 0.81 (as shown in Figure 8). The simplified three-phase method had most deviations (D%) below 20%, but with outliers that reached 66%, resulting in a root mean square deviation of 19.72%. The largest deviations were observed in the case of sDA50.500 for solutions with external SD.

Table 6. Electrochromic state schedule for each SD solution.

NSD		SD3	
SD1		SD4	
SD2		SD5	

State 3 (full tint) State 2 State 1 State 0 (clear)

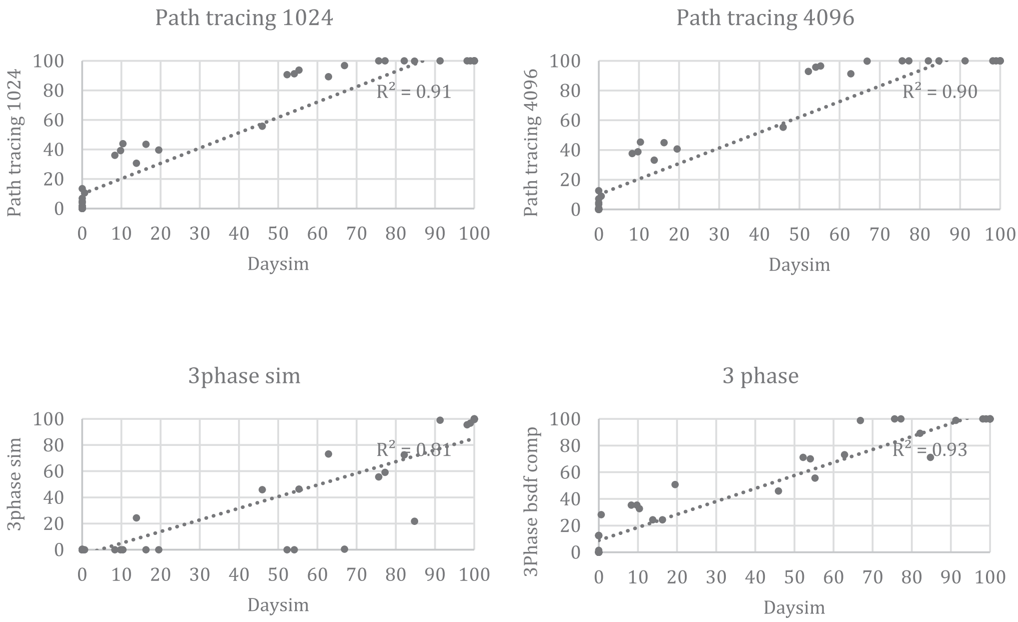


Figure 8. Correlation between methods tested and Daysim results.

For the complete three-phase method, the average value of D% was 9,32% for the solution, with a root mean square deviation of 12.90%. Most of the deviations (D%) were below 32%. It is important to note that the complete BSDF configuration was not sensitive to the parametric variation of the SD geometry. The BSDF settings were determined using Window7 software, considering an average value of VSA. Therefore, shading solutions SD3 and SD5 achieved similar results despite being geometrically distinct, as they have the same average VSA.

The path tracing method yielded an average D% of 11,2% for the solution without simplifications and 11,3% for solutions with simplifications. For the solutions with simplifications, most of the deviations were below 40%, with a root mean square deviation of 10.29%. The path tracing method without simplifications produced most deviations below 41% with a root mean square deviation of 11.11% (as shown in Figure 9).

Like the static glasses, certain results were chosen for comparison among the Daylight Autonomy (DA) values obtained for each sensor in the room. The same criteria were utilized for the selection of cases. In other words, for the electrochromic windows, the results with the maximum deviations and those representative of the mean of the population were chosen for detailed analysis. Table 7 displays the selected cases along with their respective sDA levels and corresponding D% values.

The results indicate that in the selected cases with the highest deviations, path tracing had a root mean square deviation of 32.50%. The three-phase model with simplifications had a root mean square deviation of 20.90%, while the three-phase model without simplifications had a root mean square deviation of 25.97% (Figure 10). In cases with average deviations, the path tracing had a root mean square deviation of 27.85% for the DA values. The three-phase model with simplifications had a root mean square deviation of 21.56%, and the three-phase model without simplifications had a root mean square deviation of 11.03% (Figure 10).

Discussion

The paper achieved its goal by comparing daylighting simulation methods with focus on expediting PBD analysis. It assesses both result deviations and experimentally evaluates the potential

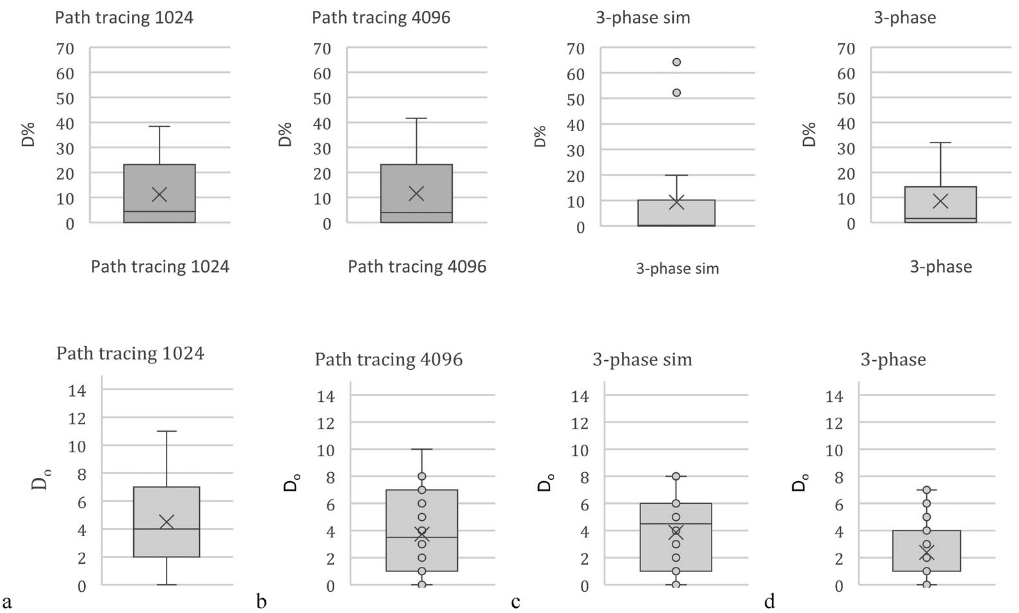


Figure 9. Boxplots for the sDA deviation results for each level of illuminance tested.

displacement of data position. The results complement previous studies by Subramaniam (2018) and Brembilla et al. (2019), including path tracing method performance compared to Daysim, while also incorporating simplifications in settings to enhance efficiency. It also expands information by including discussions of dynamic glass simulation within context of design. These findings assist architects in selecting daylighting simulation methods for PD, contributing to the debate on simplifications in the simulation processes during early design stages, and providing insights into errors and simplification strategies for professionals interested in the subject. The following sections will address the results for static and dynamic glazing.

Static glazing

The two-phase method with simplifications was faster and more accurate than path tracing method for static glazing simulations, while maintaining accuracy even when reducing the climate file. It exhibited errors below 10% and yielded similar results to Daysim, with a speedup of up to 10 times. The version with reduced climate data maintained accuracy and was 56% faster than the version with the complete climate file. This complements the conclusions of Subramaniam (2018) on the accuracy of this method and introduces the possibility of simplifications maintaining accuracy.

The results also demonstrated that the two-phase method had higher correlations and smaller differences with Daysim across all tested metrics (DA and sDA). This implies that during the design alternatives evaluation in PBD processes, the two-phase method would yield similar responses to Daysim when compared to path tracing, using recommended parameters from the

Table 7. representative solutions selected for electrochromic glazing.

Method	Maximum outlier case	sDa Deviation	Representative average case	sDA Deviation
Simplified path-tracing	SD1 sDA 500	33.94	SD2 sDA 500	15.04
Simplified 3-phase	SD2 sDA 500	64.23	SD4 sDA 300	17.69
Complete 3-phase	SD 5 sDA 500	31.91	SD2 sDA 500	13.62

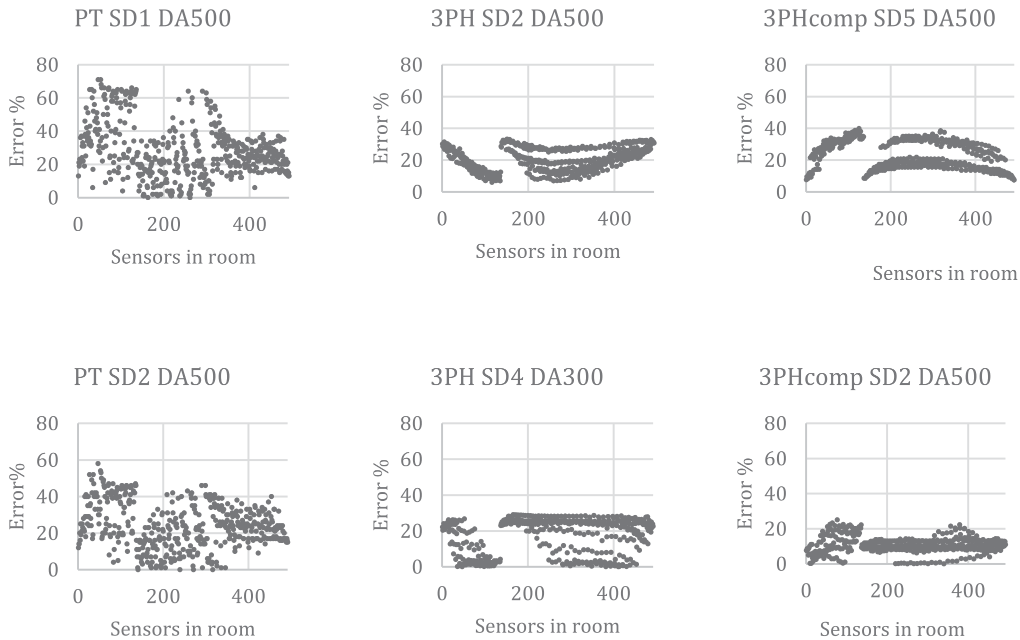


Figure 10. Error for DA results for each sensor in the room.

tool and parameters proposed in the paper. This adds information to the previous study by Brembilla et al. (2019), including the design context approach and the path tracing comparison as well.

For static glazing simulations, all methods presented similar outliers. The case with greater deviations was the combinations with SD2, which has the highest number of fins. Among the sDA illuminance threshold levels, the combinations with SD and thresholds of 300 and 500 lux were the ones that produced the greatest deviations. It is relevant to note because they are important thresholds for daylighting design.

Electrochromic glazing

For dynamic glazing, path tracing was fast and suitable for PBD, allowing better integration with PD compared to the three-phase method. However, the non-simplified three-phase configuration was more accurate than the Path tracing but restricts variations within the parametric model.

The simplified three-phase method had good correlations with Daysim, but higher sDA outliers, resulting in worse averages than path tracing. Path tracing had higher overall deviations but better deviation averages. The complete three-phase method had the smallest deviations but couldn't be used in PBD with the tested toolset due to interface limitations in creation of BSDF files for each façade variation.

BSDF creation and integration within PD models are crucial for PBD processes. The SD settings were established separately in Window 7, and it is not possible to parametrically adjust the SD for BSDF creation. Therefore, it is recommended to use the described process when there are inflexible window solutions in the PD model and when the SD is geometrically regular.

Outliers varied between methods. Path tracing had largest deviations in sDA ranges of 500 and 1000 lux for SD models. In the simplified three-phase method, the greatest deviations were in SD and sDA solutions of 500 lux. The complete three-phase method had the largest deviations in the SD 5 case. Results were insensitive to SD5 and 4 curvature variation, as average VSA was used in the BSDF configuration. Note that SD4 and 5 had the same average VSA but different geometries.

Conclusion

This paper investigates fast daylight simulation methods in PBD, with emphasis on integrating with PD and simplifications for accelerated analysis. It compares three fast simulation methods to Daysim, testing accuracy and deviations using variations of a single parametric model. Six facade solutions with five static glasses and one dynamic glass with four stages were examined. The results provide guidance to use daylight simulation in PBD processes.

The results indicate that annual climate data sampling can be used to accelerate simulation in the context of PBD. The two-phase method maintained accuracy below 10% for static glazing also with simplifications. Path tracing with simplifications had deviations exceeding 20% (as defined in previous studies, see 1.3) compared to Daysim. In Climatestudio, path tracing was the most effective for dynamic PBD solutions due to time and interoperability. Despite path tracing having larger deviations than the three-phase method, the latter needs improved interoperability with PD tools.

A limitation of this study is the limited sampling due to the extended simulation time in Daysim, considering the parameters used. Also, only Daysim was used as a reference. Future research can explore more examples, metrics, and real data. Other areas of investigation may include defining additional parameters for simplifications and benchmarking acceptable errors criteria in early-stage design simulations.

In conclusion, this paper provides insights into integration of daylighting simulation methods with parametric models in PBD. The comparative analysis of three fast simulation techniques, along with the proposed metrics for evaluating simulation outcomes and deviations during the design generation phase, is a contribution to architects engaged in daylighting simulation and PBD processes.

Note

1. VSA (Vertical Shading Angle) is formed by two planes: one vertical at the base of the glass and the other formed by the farthest end of the horizontal SD to the base of the glass.

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