



**Customer Strategies for Responding to Day-Ahead Market Hourly  
Electricity Pricing:  
Appendices**

Prepared for the  
California Energy Commission

Principal Authors

C. Goldman, N. Hopper and R. Bharvirkar  
Lawrence Berkeley National Laboratory

B. Neenan, R. Boisvert, P. Cappers, D. Pratt, K. Butkins and L. Scholle  
Neenan Associates

Ernest Orlando Lawrence Berkeley National Laboratory  
1 Cyclotron Road, MS 90R4000  
Berkeley CA 94720-8136

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Prepared By: Charles A. Goldman  
Lawrence Berkeley National Laboratory  
Berkeley, California

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**Prepared For:**  
**California Energy Commission**  
Public Interest Energy Research (PIER)  
Program

Dave Michel  
**Contract Manager**

Mark Rawson  
**Program Area Team Lead**

Martha Krebs  
**Deputy Director**  
**ENERGY RESEARCH AND DEVELOPMENT**  
**DIVISION**

B.B. Blevins  
**Executive Director**

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**Appendix A. 2004 SC-3A Customer Survey**

1. Please confirm your contact information.

1. Name \_\_\_\_\_  
2. Organization \_\_\_\_\_  
3. Title \_\_\_\_\_  
4. Address \_\_\_\_\_  
\_\_\_\_\_  
5. Phone \_\_\_\_\_ 6. Fax \_\_\_\_\_  
7. Email \_\_\_\_\_

**The following questions pertain to your facilities that receive electricity service from Niagara Mohawk Power Corporation (NMPC) under the SC-3A rate classification.**

2. On a normal summer weekday, during which of the following time periods is your facility's electricity use highest? (CHECK ONLY ONE)

- 1. 8 a.m. – 12 noon
- 2. 12 noon – 6 p.m.
- 3. 6 p.m. – 10 p.m.
- 4. 10 p.m. – 8 a.m.
- 5. Do not know

3. Which of the following best describes how often you monitor the next day's hourly electricity prices? (CHECK ONLY ONE)

- 1. Routinely, most days **Skip to Question 5**
- 2. Weekly
- 3. Only during periods of hot weather
- 4. Only during NYISO emergency program events - EDRP and/or ICAP/SCR
- 5. Rarely
- 6. Other (please specify): \_\_\_\_\_
- 7. Do not know

4. Why do you not monitor prices more frequently? (CHECK ONLY ONE)

- 1. Unaware that prices change hourly
- 2. Limited resources to do so
- 3. Limited technology to do so
- 4. My electric service contract with a competitive supplier (ESCO) makes monitoring hourly prices irrelevant
- 5. Other (please specify): \_\_\_\_\_
- 6. Do not know

**Questions 5 through 12 refer to your facility's experience over the past five years during summer weekdays.**

5. If you have reduced electricity use or turned on on-site generation in response to high hourly electricity prices, how high were prices when you responded? (check only one)

\$0.10/ kWh	\$0.20/ kWh	\$0.50/ kWh	\$0.75/ kWh	\$1.00/ kWh	Have not responded to prices	Do not know
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

6. If you have reduced electricity use or turned on on-site generation in response to NYISO emergency program events, why have you done so? (check all that apply)

- 1. To earn EDRP or ICAP/SCR curtailment incentive payments
- 2. To avoid paying penalties for not responding to ICAP/SCR events
- 3. My organization considers it a civic duty to help keep the electric system secure
- 4. NYISO emergencies coincide with high SC-3A prices
- 5. Other (please specify): \_\_\_\_\_
- 6. My facility has not responded to NYISO emergency events
- 7. Do not know

7. If you have reduced electricity use or turned on on-site generation in response to public appeals and/or mandates to reduce electricity consumption, why have you done so? (CHECK ALL THAT APPLY)

- 1. My organization is required or expected to comply with public appeals
- 2. My organization considers it a civic duty to respond to public appeals

- 3. Public appeals coincide with high SC-3A prices
- 4. Other (please specify): \_\_\_\_\_
- 5. My facility has not responded to public appeals
- 6. Do not know

8. When you have reduced electricity use or turned on on-site generation, how has your facility changed its electricity use (check all that apply):

- 1. Electricity use was **shifted**: consumption was reduced and made up at another time (e.g., equipment use was rescheduled to later in the day or the next day)
- 2. Electricity use was **foregone**: consumption was reduced and not made up at another time
- 3. Turned on **on-site generation**
- 4. I have not reduced or shifted electricity use nor turned on on-site generation
- 5. Do not know

9. In the table below, please indicate which specific equipment or end-uses have been affected when you reduced electricity or turned on on-site generation and how (CHECK ALL THAT APPLY):

<i>Equipment and End-Uses</i>	Actions Undertaken to Reduce Electricity Use					
	Shifted	Foregone	Turned on on-site generation	Have not curtailed	Do not have this equipment	Do not know
Lighting	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Air conditioning	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Plug loads (e.g., office equipment, vending machines)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Process equipment/production lines	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Water pumping	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Refrigeration	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

10. Over the past 5 years, has your facility experienced major changes in electricity consumption? (CHECK ALL THAT APPLY)

- 1. Yes - My facility has increased production over this time period
- 2. Yes – My facility has decreased production over this time period

- 3. Yes – My facility has invested in equipment to reduce overall electricity consumption
- 4. Yes – Other changes (please describe): \_\_\_\_\_
- 5. No
- 6. Do not know

11. If some portion of your load has been **shifted**, which of the following time periods was it most often rescheduled to? (CHECK ONLY ONE)

- 1. The day before the curtailment
- 2. Earlier or later the same day as the curtailment
- 3. The day after the curtailment
- 4. Some later time
- 5. My facility has not shifted load
- 6. Do not know

12. If some portion of your load has been **foregone**, which best describes the impact on your facility's operations? (CHECK ONLY ONE)

- 1. No impact
- 2. Slight inconvenience or employee discomfort
- 3. Significant inconvenience or employee discomfort
- 4. Business operations must be adjusted
- 5. Other (please specify:) \_\_\_\_\_
- 6. My facility has not foregone load
- 7. Do not know

13. What barriers has your organization experienced in responding to high hourly electricity supply prices? (CHECK ALL THAT APPLY)

- 1. Insufficient time or resources to pay attention to hourly prices
- 2. Managing electricity use is not a priority
- 3. The cost/inconvenience of responding outweighs the savings
- 4. Institutional barriers in my organization make responding difficult
- 5. My organization's management views these efforts as too risky
- 6. Inflexible labor schedule

- 7. Negative previous experience with day-ahead hourly pricing
- 8. Flat-rate or time-of-use (TOU) contract makes responding unimportant
- 9. Other (please specify): \_\_\_\_\_
- 10. No barriers have been encountered
- 11. Do not know

14. How does your facility intend to mitigate SC-3A price variability in the next two to three years? (CHECK ALL THAT APPLY)

- 1. Secure a time-of-use or flat-rate contract for electricity supply from an alternative supplier (ESCO)
- 2. Secure a financial hedge so that my facility does not have to worry about high prices
- 3. Continue to reduce load when prices are high, as I have in the past
- 4. Adjust operations to allow for greater capability to respond to high SC-3A prices
- 5. Invest in control and other load management technologies to enhance my ability to respond to high SC-3A prices
- 6. Invest in on-site generation to adjust electricity use
- 7. My facility does not intend to respond to SC-3A price variability
- 8. Do not know

15. **In the future**, how high would hourly SC-3A electricity prices have to be for your facility to reduce electricity use below normal levels or turn on on-site generation? (CHECK ONLY ONE)

\$0.10/ kWh	\$0.20/ kWh	\$0.50/ kWh	\$0.75/ kWh	\$1.00/ kWh	\$2.00/ kWh	Would not change use at any price	Do not know
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

**The following questions ask about energy management technologies installed at facilities served by NMPC, specifically:**

Energy Management Control Systems (EMCS): control systems that optimize operations of end-use equipment, usually HVAC, through a series of sensors, communicators and controllers

Peak Load Management Devices: devices that control the electric demand of HVAC equipment, lighting, process loads, motors or drives

Energy Information Systems (EIS): integrated systems of software, data acquisition hardware, and communication systems used to manage electricity use over a variety of end-uses in a single facility or across several remotely managed facilities

16. Has your facility installed an Energy Management Control System (EMCS) or peak load management devices?

- 1. Yes
- 2. No **Skip to Question 19**
- 3. Do not know **Skip to Question 19**

17. For what purpose(s) are your facility's EMCS and/or peak load management devices used? (CHECK ALL THAT APPLY)

- 1. To respond to high hourly prices
- 2. To reduce overall electricity bills
- 3. To reduce peak-demand charges
- 4. Facility/process control automation
- 5. Other (please specify): \_\_\_\_\_
- 6. Do not know

18. During which summers have you used your EMCS and/or peak load management devices for these purposes? (CHECK ALL THAT APPLY)

- 1. Summer of 2000
- 2. Summer of 2001
- 3. Summer of 2002
- 4. Summer of 2003
- 5. Summer of 2004
- 6. Do not know

19. Has your organization installed an Energy Information System (EIS) in the past five years?

- 1. Yes
- 2. No **Skip to Question 22**
- 3. Do not know **Skip to Question 22**

20. For what purpose(s) is your facility's EIS used? (CHECK ALL THAT APPLY)

- 1. To respond to high hourly prices



- 2. To reduce overall electricity bills
- 3. To reduce peak-demand charges
- 4. Facility/process control automation
- 5. Other (please specify): \_\_\_\_\_
- 6. Do not know

21. During which summers have you used the EIS for these purposes? (CHECK ALL THAT APPLY)

- 1. Summer of 2000
- 2. Summer of 2001
- 3. Summer of 2002
- 4. Summer of 2003
- 5. Summer of 2004
- 6. Do not know

22. Does your facility have on-site generation (e.g., self-generation, cogeneration or emergency generators)?

- 1. Yes
- 2. No **Skip to Question 24**
- 3. Do not know **Skip to Question 24**

23. For what purpose(s) is (are) your facility's on-site generator(s) used? (CHECK ALL THAT APPLY)

- 1. To respond to high hourly prices
- 2. To reduce overall electricity bills
- 3. To reduce peak-demand charges
- 4. Emergency backup / reliability
- 5. Cogeneration
- 6. Other (please specify): \_\_\_\_\_
- 7. Do not know

**The following questions pertain to how you purchase electricity at your facilities served by NMPC under the SC-3A rate classification.**

24. On average, what percent of your facility's total annual operating costs do your energy costs (e.g. electricity, natural gas, fuel oil, etc.) account for? (check only one)

- 1. Less than 1%
- 2. Between 1% and 3%
- 3. Between 4% and 6%
- 4. Between 7% and 10%
- 5. Between 11% and 20%
- 6. Greater than 20%
- 7. Do not know

25. On average, what percent of your facility's total annual operating costs do your electricity costs account for? (check only one)

- 1. Less than 1%
- 2. Between 1% and 3%
- 3. Between 4% and 6%
- 4. Between 7% and 10%
- 5. Between 11% and 20%
- 6. Greater than 20%
- 7. Do not know

26. Has your facility ever purchased electricity from an alternative supplier (ESCO)?

- 1. Yes
- 2. No *Skip to Question 29*
- 3. Do not know *Skip to Question 29*

27. During which of the following summers did your facility buy electricity from an ESCO under a **time-of-use** or **flat-rate** contract? (check all that apply)

- 1. Summer of 2000
- 2. Summer of 2001
- 3. Summer of 2002
- 4. Summer of 2003
- 5. Summer of 2004

- 6. My facility was not on this type of rate during these time periods
- 7. Do not know

28. During which of the following summers did your facility buy electricity from an ESCO under a contract **in which prices change hourly?** (check all that apply)

- 1. Summer of 2000
- 2. Summer of 2001
- 3. Summer of 2002
- 4. Summer of 2003
- 5. Summer of 2004
- 6. My facility was not on this type of rate during these time periods
- 7. Do not know

29. In the last five years, if you have **not** bought electricity exclusively from ESCOs, please indicate why not (CHECK ALL THAT APPLY):

- 1. Could not find a hedged (flat-rate) contract
- 2. Could not find an ESCO willing to serve my organization
- 3. ESCO offers have been too expensive
- 4. The savings offered by ESCOs have not been enough to justify the switch
- 5. Institutional barriers in my organization make switching difficult
- 6. Prefer NMPC's prices
- 7. Prefer NMPC's reputation
- 8. Prefer NMPC's service
- 9. Unavailability of long-term contracts
- 10. Contract(s) with NYPA limit(s) my organization's interest
- 11. Believe that contracts with NYPA prevent me from choosing an ESCO
- 12. Other (please specify): \_\_\_\_\_
- 13. My organization has bought electricity exclusively from ESCOs since 1999
- 14. Do not know

30. In the future, what would prompt you to buy electricity from an ESCO? (CHECK ALL THAT APPLY)

- 1. More interest by ESCOs in serving my facility

- 2. Better priced ESCO flat-rate or time-of-use offerings than those currently available
- 3. Better priced ESCO Day-Ahead Market indexed offerings than those currently available
- 4. More information or education on how to evaluate ESCO offers
- 5. More interest/support from my organization's management
- 6. Higher forecasted SC-3A electricity prices
- 7. More volatile forecasted SC-3A electricity prices
- 8. Other (please specify): \_\_\_\_\_
- 9. Nothing could induce my organization to switch (why?):  
\_\_\_\_\_  
\_\_\_\_\_
- 10. Do not know

31. Currently, the default SC-3A commodity price varies from hour to hour but is provided to you on a day-ahead basis. Suppose that in the future the default SC-3A commodity price was instead provided to you at the beginning of each hour and was effective for the load consumed in that hour (e.g. no advance notice of prices). What would you do? (CHECK ONLY ONE)

- 1. Continue buying commodity service from NMPC
- 2. Continue buying commodity service from an ESCO
- 3. Switch to an ESCO for an alternative commodity service
- 4. Consider offers from an ESCO for an alternative commodity service
- 5. Do not know

32. During which of the following summers did your facility purchase a financial product that hedged electricity price volatility (e.g. a contract for differences, swap, etc.)? (check all that apply)

- 1. Summer of 2000 **Skip to End**
- 2. Summer of 2001 **Skip to End**
- 3. Summer of 2002 **Skip to End**
- 4. Summer of 2003 **Skip to End**
- 5. Summer of 2004 **Skip to End**
- 6. My facility did not have a financial hedge during these time periods
- 7. Do not know

33. Which of the following have influenced your decision not to buy a financial hedge?  
(CHECK ALL THAT APPLY)

- 1. Not sure what a financial hedge is or why I would need one
- 2. Could not find an institution that offered a financial hedge
- 3. Offered hedges were too expensive
- 4. Institutional barriers in my organization make procuring financial hedges difficult
- 5. My organization is comfortable managing risk without a financial hedge
- 6. My facility already had a flat-rate or TOU supply contract
- 7. Other (please specify): \_\_\_\_\_
- 8. Do not know



## **Appendix B. The Generalized Leontief Demand Model, Theoretical and Empirical Specifications and Interpretations**

### B.1 Introduction

The purpose of this appendix is to provide an overview of the conceptual approach and empirical model used to quantify the response in electricity usage by industrial and commercial customers to hourly varying, day-ahead electricity prices in Niagara Mohawk's SC-3A electricity rate. The conceptual model is based on the modern economic theory of the derived demand for inputs by profit maximizing and/or cost minimizing firms. This conceptual foundation is reported in detail by Goldman, *et al.* (2004); it is therefore only summarized in this appendix. While relying on the same conceptual model, the empirical demand specification for this current study is substantially different. It is based on a flexible Generalized Leontief (GL) cost function, rather than the more restrictive Constant Elasticity of Substitution (CES) specification.

By allowing price response to vary with the level of prevailing prices, this flexible demand model facilitates the understanding of RTP participants' response to RTP prices in two important ways. First, we are able to structure a specific statistical test for the hypothesis that a firm exhibits no price response. Second, the GL demand specification allows estimates of demand response to vary across price levels. Therefore, for those firms that are price responsive, we can also test the hypothesis that a firm's willingness to shift electricity between high-and low-priced periods increases as the prices in the high-priced periods rise.<sup>1</sup> Once separate demand models are estimated for each customer, it is possible to test this second hypothesis by pooling the individual customer estimates of demand response and estimating two additional models. In the first of them, we model estimates of demand response as a function of the ratio of peak to off-peak prices, including additional variables that are available for all customers. In the second model, we are also able to quantify the separate effects on average price responsiveness of additional customer-specific characteristics and circumstances that were collected through a self-administered customer survey.

After a brief review of the electricity demand model, the GL model is described in detail, along with a strategy for empirical estimation and a geometric, intuitive interpretation of the range in possible demand response accommodated by the flexible specification. Finally, the econometric issues related to model specification and the testing of important hypotheses are addressed.

### B.2 The Electricity Demand Model

The model for electricity demand in this study is consistent with the modern economic theory of the firm. It provides the theoretical underpinnings for the initial empirical

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<sup>1</sup> In contrast to most other studies of RTP customer demand response, we define daily peak and off-peak pricing periods rather than treat electricity in every hour as a distinct commodity. Several alternative peak-period specifications are employed in our empirical estimation in order to test which best characterizes customer behavior.

evaluations of RTP-type services by Caves, *et al.* (1984), and more recently by King and Shatrawka (1994) and Schwarz, *et al.* (2002).<sup>2</sup> According to this theory, firms are assumed to maximize profits (or minimize the cost of producing a given level of output), and electricity usage is modeled according to a sequential, three-level profit or cost function that is assumed to be separable in electricity usage.<sup>3</sup> At the first level, weekday electricity usage is allocated between peak and off-peak time periods, which reflect differences in the price of electricity, the value of electricity, or both. The second level allocates monthly usage between weekdays and weekends. The third, and final, level determines overall electricity expenditures as a proportion of total costs, reflecting the relative demand for electricity in relation to all other inputs in the production process.

In this study, we focus the demand model specification on the first stage – the allocation of daily electricity usage between high-price (peak) and low-price (off-peak) hours.<sup>4</sup> To facilitate an analysis of electricity demand response to RTP prices from this perspective, it is necessary to develop an appropriate definition of the electricity commodity.

### B.2.1. Defining the Electricity Commodity

Because of the continuous nature of electric service and usage, defining the hours that constitute the peak and off-peak periods in electricity demand models is generally treated as an empirical question, driven by the prices customers face and the circumstances by which they use and value electricity. Studies of price response to time-of-use (TOU) rates typically utilize pooled data for customers participating in different TOU rates, or data are pooled across several treatments, where prices or the definition of the peak period vary by the experimental design (Caves, *et al.* 1984; Patrick 1990; Braithwait 2000).

To establish the definition of distinct electricity commodities in one of the most detailed studies ever undertaken, Caves, *et al.* (1987) identified six separate commodities for customers facing a six-hour peak-pricing period of 9 A.M. – 12 noon and 1 P.M. – 4 P.M. These peak hours were then further divided into two separate commodities—one two-hour commodity (11 A.M. – 12 Noon and 1 P.M. – 2 P.M.) and one four-hour commodity (9 A.M. – 11 A.M. and 2 P.M. – 4 P.M.). Other hours in the day were aggregated into four

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<sup>2</sup> The model is conceptually similar to the consumer demand model discussed by Braithwait (2000).

<sup>3</sup> For a production function or utility function to be weakly separable in any partition of its arguments, the marginal rate of substitution between any two inputs or goods in a separable subset is independent of all inputs or goods that are not in the subset (Chambers 1988, pp. 45-46). In other words, any function in  $n$  variables,  $f(x) = F(x_1, \dots, x_n)$ , that is separable in a partition  $x^1$  through  $x^m$ , where  $x^1$  is a vector representing a subset of the  $n$  variables, can be written as  $f(x) = F(f^1(x^1), \dots, f^m(x^m))$ . Each of the sub-functions can be treated as an aggregate input or consumption bundle—essentially a production or utility function in and of itself. Therefore, it is legitimate to think of production or consumption occurring in two steps. To use the example of a production function, inputs in the sub-vector are combined to create the aggregate inputs in the first step. In the second step, these aggregate inputs are used to produce the output from the macro production function. The practical implication is that choice of cost minimizing input levels within any sub-function depends only on prices for those inputs in the sub-function. Thus, input demands and price response elasticities can be derived from the sub-function alone.

<sup>4</sup> Caves, *et al.* (1984) estimate a demand model that includes all three stages of electricity demand. It is perhaps the only study that looks at all three stages of electricity demand, and one of only a handful of studies that consider more than just the within-day energy demand.



separate commodities, all priced the same. They argued that this sub-aggregation of the peak is needed to characterize “needle peaking” behavior. Extending this structure to RTP-type programs with hourly prices would require 24 separate electricity commodities, one for each hour of the day.

This specification would be warranted if industrial and commercial customers could in fact adjust usage on an ongoing basis to changing hourly prices. However, there is compelling evidence that firms implicitly characterize the day as being comprised of a peak and an off-peak period (Neenan, *et al.* 2003). While the exact specification of the peak hours is firm specific, common business practices, driven in large part by traditional rate structures, support utilizing a single specification to capture most of the variation in usage.

Consistent with this line of reasoning, we analyze the price response behavior of NMPC’s RTP customers by estimating the demand model for several alternative specifications of the peak period that differ in length. To gain insights into which hourly aggregates firms view as distinct commodities, we focus the discussion on a specification that best represents the data.

### B.2.2 The Model of Daily Demand for Electricity in Peak and Off-Peak Periods

Since the focus is on the first stage in the decision process described above, our empirical demand model deals only with the allocation of daily electricity usage between high-price (peak) hours and low-price (off-peak) hours. It is generally thought that electricity use in these two periods are valued differently by the firm. In contrast, it may also be the case that peak and off-peak electricity inputs are complementary to the firm and could in fact be demanded in nearly fixed proportions. To formulate this demand model, we define a firm’s production function that is separable in electricity inputs as:

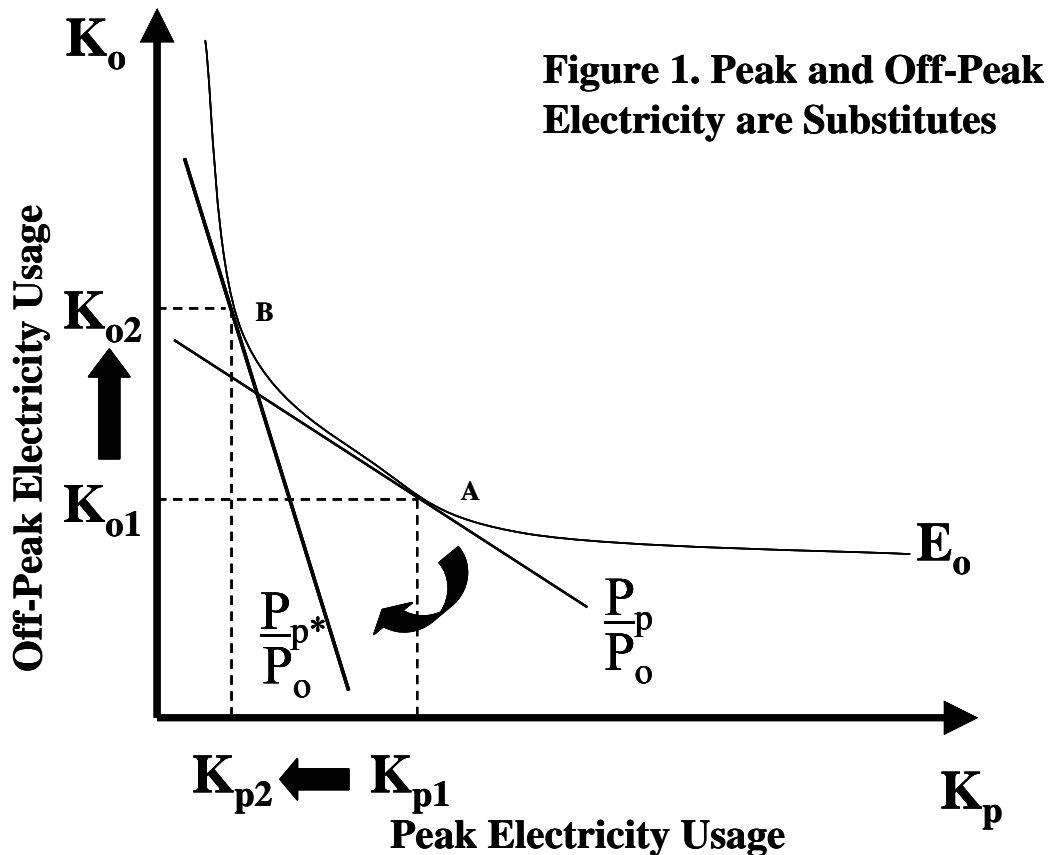
$$(1) \quad Q = F(x_1, x_2, \dots, x_n, E(k_p, k_o)),$$

where  $Q$  is the output of the firm,  $x_i$  are inputs other than electricity and  $k_p$  and  $k_o$  are electricity used in peak and off-peak periods, respectively.

Electricity is assumed to be separable from other production inputs. Therefore, this sub-function  $E = E(k_p, k_o)$  represents an aggregate electricity input; a firm can produce a given level of output by combining different amounts of peak and off-peak electricity that yield a given level of the electricity aggregate, say  $E_0$ , needed by the firm to produce its output. In considering the use of peak and off-peak electricity, there are four cases that should be distinguished. The particular case that applies to any individual customer depends not only on the technical aspects of certain production processes, but also on behavioral considerations. The situation can also vary depending on circumstances, such as when certain firms decide to reduce what might be termed “discretionary” energy use. In this case, the rate at which firms substitute off-peak for peak electricity could be called “state dependent”. These four cases are described graphically here. In later sections, these

individual cases are related directly to the estimated parameters of the GL demand models.

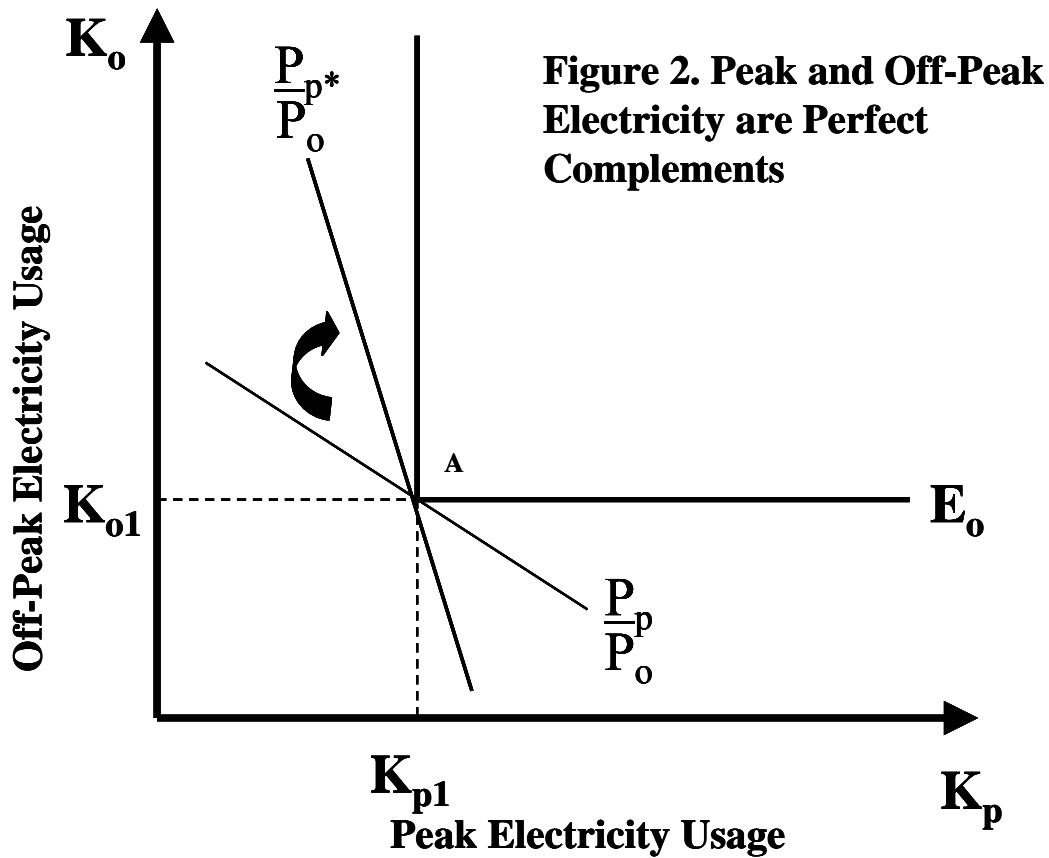
Case 1 is where peak and off-peak electricity are substitute inputs in production, depicted in Figure 1. This case is what most would think of as the normal situation regarding a firm's ability to substitute peak and off-peak electricity.



The curve  $E_0$  in Figure 1 represents those combinations of peak electricity ( $K_p$ ) and off-peak electricity ( $K_o$ ) that produce an energy aggregate,  $E_0$ , needed to support the firm's desired (and constant) output. At an initial ratio of peak to off-peak electricity prices (given by the price line in Figure 1 labeled  $P_p/P_o$ ),<sup>5</sup> the firm would minimize the cost of producing  $E_0$  by using  $K_{p1}$  and  $K_{o1}$  of peak and off-peak electricity, respectively. This is point A in Figure 1. If there is an increase in the peak period price of electricity to  $P_p^* > P_p$ , the price line gets steeper and if the firm is to continue to produce  $E_0$ , the minimum cost way of doing so is by using more electricity off-peak and less on-peak (e.g.  $K_{p2} < K_{p1}$  and  $K_{o2} > K_{o1}$ ). This is at point B in Figure 1, and it is the increase in the peak price of electricity that leads to a decrease in the ratio of peak to off-peak electricity usage. It is this change in the ratio of peak to off-peak electricity use that measures the firm's price responsiveness. This change in input intensity is related to the slope of the curve,  $E_0$ . The

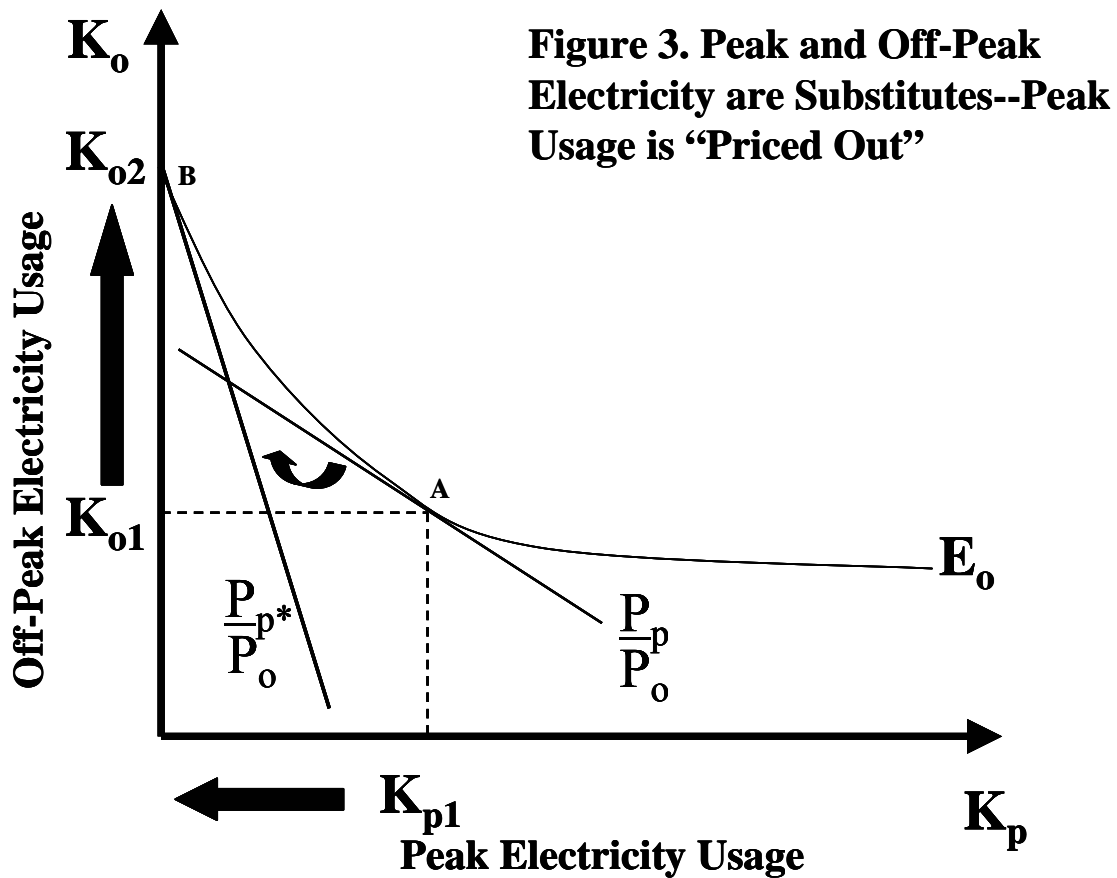
<sup>5</sup> For this given set of prices,  $P_p$  and  $P_o$ , the price line represents all combinations of  $K_p$  and  $K_o$  that can be purchased for a fixed budget.

measure of this change in the ratio of input use in percentage terms is commonly called the elasticity of substitution, and it is often denoted by  $\sigma$ . In our case,  $\sigma$  measures the percentage change in the ratio of peak to off-peak electricity use to a one percent change in the ratio of off-peak to peak electricity. As the curvature or slope of  $E_0$  becomes more pronounced, a firm's price responsiveness, as measured by the elasticity of substitution falls, and as the curve  $E_0$  becomes flatter, the price responsiveness increases. Finally, in this particular case, we have drawn the curve  $E_0$  so that it never crosses either axis. Thus, regardless of how high the peak price rises relative to the off-peak price, production always requires some peak electricity. Technically, this is the case where  $0 < \sigma < \infty$ .



In this study, one extreme case, where  $\sigma = 0$ , is of particular interest, and it is depicted in Figure 2. In this case, there is no possibility for substituting peak for off-peak electricity regardless of the relative prices of peak and off-peak electricity. This means that output can only be produced using these two electricity inputs in fixed proportions, and peak and off-peak electricity are called perfect complements. The equal output curve,  $E_0$ , is the rectangle (Figure 2). At point A in the figure, the electricity aggregate,  $E_0$ , is produced using  $K_{p1}$  and  $K_{o1}$  units of peak and off-peak electricity, respectively. The fixed proportions nature of production is reflected in the rectangular curve  $E_0$  in the following way. If  $K_p$  is increased above the level  $K_{p1}$  while holding  $K_o$  at  $K_{o1}$ , we would move to the right horizontally along the curve  $E_0$ . Since we remain on the curve  $E_0$ , the electricity aggregate and output are constant, and the extra peak electricity would be of no use to the

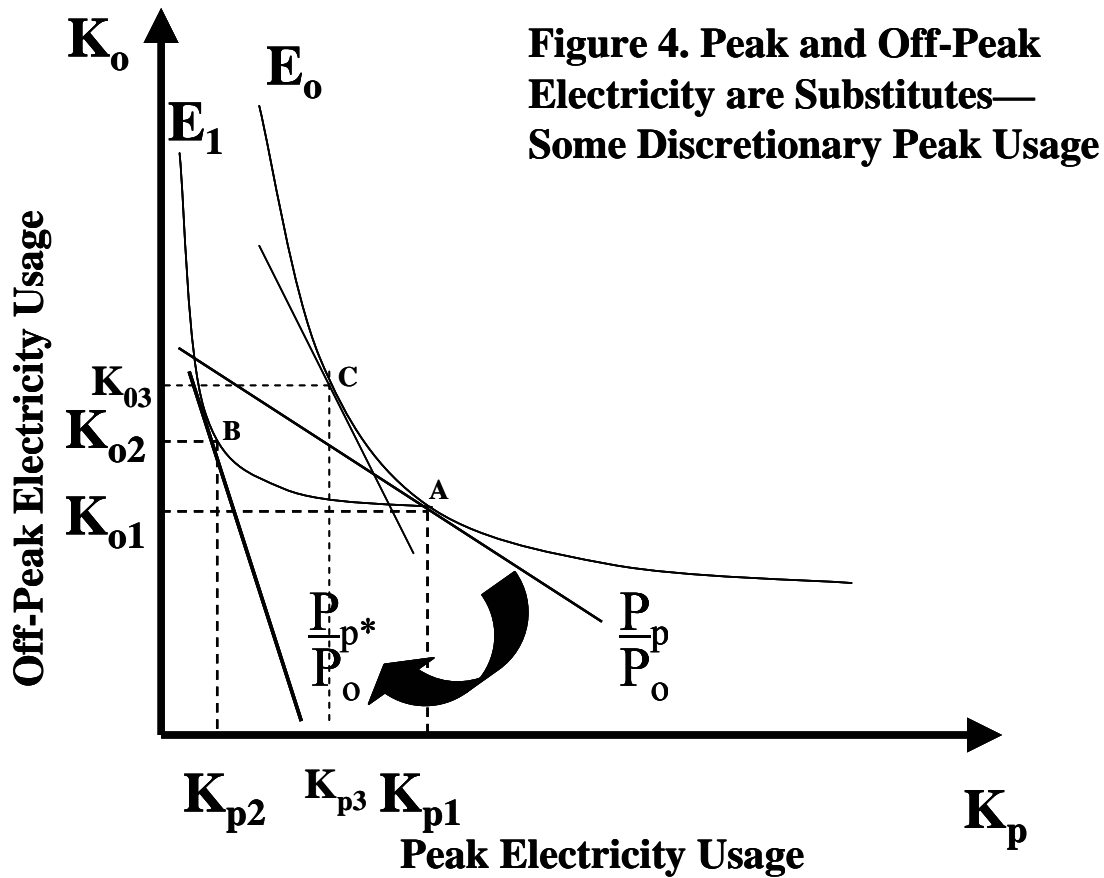
firm. A similar argument can be made for trying to increase output by increasing the amount of off-peak electricity without any increase in peak usage.<sup>6</sup>



The third case is depicted in Figure 3. As in Case 1 above, peak and off-peak electricity are substitute inputs in production, and the curve  $E_0$  in Figure 3 still represents those combinations of peak electricity ( $K_p$ ) and off-peak electricity ( $K_o$ ) that produce an energy aggregate,  $E_0$ , needed to support the firm's desired (and constant) output. At an initial ratio of peak to off-peak electricity prices (given by the price line in Figure 3 labeled  $P_p/P_o$ ), the firm would minimize the cost of producing  $E_0$  by using  $K_{p1}$  and  $K_{o1}$  of peak

<sup>6</sup> The other extreme, where  $\sigma = \infty$ , is of little interest in the study of the substitutability of off-peak for peak electricity. We include a short discussion here for completeness only. This is the case where peak and off-peak electricity are called perfect substitutes. In this case, rather than the  $E_0$  curve being convex to the origin as it is in Figure 1, the  $E_0$  curve is instead a straight line with a negative slope that intersects both the horizontal and vertical axes. In this case, a certain number of units of off-peak electricity can always be substituted for one unit of peak electricity to keep the electricity aggregate at  $E_0$  and output constant. The flatter this straight line is, the fewer is the number of units of off-peak electricity needed to substitute for one unit of peak electricity. If the ratio of peak to off-peak prices happens to be the same as the slope of this line, then any point on the line represents a minimum-cost combination of peak and off-peak electricity to keep  $E$  at  $E_0$ . However, if the price-ratio line becomes steeper, representing a relative increase in price of peak electricity, then only off-peak electricity is used in production. Alternatively, if the price-ratio line becomes less steep, representing an increase in the off-peak price, then only peak electricity would be used in production.

and off-peak electricity, respectively. This is point A in Figure 3. In contrast to the situation in Figure 1, we see that in this case the  $E_0$  cuts the vertical axis at point B. Although off-peak electricity is substituted for peak electricity as the price of peak electricity rises, there is a price (say  $P_p^*$ ), at which peak electricity is “priced out of the market”, and peak usage drops to zero (point B in Figure 3). There are perhaps a small number of firms with production processes that accommodate such dramatic substitution possibilities and are able to forgo all electricity use during peak periods when peak prices are extremely high. However, we focus on this case because it may depict very well the behavior of firms with significant on-site generation. Regardless of the price of peak electricity these firms may still require peak electricity as an input, but as the price of peak electricity rises to a certain level, the demand for peak power from the *grid* falls to zero (point B in Figure 3). The firm continues to receive its off-peak power from the *grid*, but now relies completely on its on-site generation to supply its peak electricity needs.<sup>7</sup>



**Figure 4. Peak and Off-Peak Electricity are Substitutes—Some Discretionary Peak Usage**

The fourth case of interest is also more descriptive of a behavioral response to high peak electricity prices than it is of a pure technical relationship usually depicted by input demand models. This case is depicted in Figure 4, and it is the case where demand for peak electricity depends on prevailing circumstances (“state dependent” if you will). For

<sup>7</sup> As is seen below, this situation is accommodated within the framework of the GL model, but it cannot be modeled by the CES function.

example, there are some firms that when prices of peak electricity reach a certain point, they shed some “discretionary” load (e.g., maintain output but ask customers and employees to forgo normal comfort levels so lights and/or air conditioning can be turned down). Thus, by shedding discretionary load as the price of peak electricity rises, this firm moves from point A on the  $E_0$  curve to point B on the  $E_1$  curve, in effect shedding more peak load (from  $K_{p1}$  to  $K_{p2}$ ) than would be possible if production levels were to be maintained at normal comfort levels for customers and employees (e.g., by moving up the  $E_0$  curve to point C with a peak usage of  $K_{p3} > K_{p2}$ ).

More is said about the range in elasticities of substitution below. However, based on this general discussion, it is clear that in order to estimate firm’s change in peak and off-peak electricity use in response to price changes, we need only specify a functional form for  $E = e(k_p, k_o)$  and estimate it empirically. Perhaps the most critical factor in selecting a functional form is that it be able to reflect a wide range in price responsiveness as measured by the elasticity of substitution. As is seen below, both the CES and the GL models have this capacity. However, the GL model has the added flexibility of not imposing the same elasticity of substitution for a firm regardless of the firm’s output level, electricity usage, and the price of electricity.

#### B.2.2.1 The CES Specification

In much previous literature, including two recent studies of customer demand response in new competitive markets by Neenan, *et al.* (2003) and Charles River Associates (2004), a constant elasticity of substitution (CES) functional form was used. It has the form:

$$(2) \quad E = [ \delta (k_p)^{-\rho} + (1-\delta) (k_o)^{-\rho} ]^{-1/\rho}$$

In this function,  $E$  is an aggregate electricity input that exhibits constant returns to scale (Moroney 1972; and Ferguson 1969). That is, if electricity use is increased by the same proportion in both peak and off-peak periods, the value of the energy aggregate increases by that same proportion. The parameter  $\delta$  reflects the natural peak Kwh intensity of production. The parameter  $\rho$  is a transformation of the elasticity of substitution between peak and off-peak electricity use,  $\sigma = 1/(1 + \rho)$ .<sup>8</sup> This elasticity of substitution is constant regardless of the levels of energy use or levels of output.

Neenan *et al.* (2003) have shown that this elasticity of substitution is also a convenient measure of the price responsiveness of electricity demand between peak and off-peak periods.<sup>9</sup> In particular, they show that the ratio on peak to off-peak electricity use is a function of the inverse of the price ratio (the ratio of off-peak to peak price) and the parameters  $\delta$  and  $\sigma$ . That is:

<sup>8</sup>The algebra needed to derive this relationship, along with the derivation of the elasticity of substitution, is found in Ferguson (1969, pp. 103-04) and is not repeated here.

<sup>9</sup>This involves solving the first-order conditions to the constrained optimization problem for minimizing the cost of producing a given output for the factor demands and substituting them back into the direct cost function. This procedure allows one to write the indirect cost-minimizing cost function in terms of output and input prices only.

$$(3) \quad k_p / k_o = \{ [\delta / (1 - \delta)] [p_o / p_p] \}^\sigma.$$

One attractive feature of this model is that we can multiply the right-hand-side of this equation by an appropriate error term ( $\varepsilon$ ), take the logarithms of both sides, and obtain an unbiased, minimum-variance estimate of  $\sigma$  using ordinary least squares (OLS) regression:

$$(4) \quad \ln [k_p / k_o] = \sigma \ln [\delta / (1 - \delta)] + \sigma \ln [p_o / p_p] + \ln \varepsilon.$$

Furthermore, from this transformed, logarithmic relationship, it is clear that the elasticity of substitution, the parameter  $\sigma$ , measures the proportional change in the ratio of electricity use in peak and off-peak periods due to a percentage change in the inverse price ratio (e.g.,  $\partial \ln [k_p / k_o] / \partial \ln [p_o / p_p] = \sigma$ ). For this production function to be well behaved, Ferguson (1969) shows that the elasticity of substitution must lie between the extremes discussed above (e.g.,  $\infty \geq \sigma \geq 0$ ).<sup>10</sup> To reiterate from above, if  $\sigma = 0$ , then peak and off-peak electricity must be used in fixed proportions. The higher  $\sigma$  is, the more responsive energy use is to changing in relative prices between peak and off-peak periods. For example, if  $\sigma < 1$ , then as the price ratio changes by one percent, the ratio of peak to off-peak energy use changes by less than one percent. For  $\sigma > 1$ , the ratio of energy use changes by more than one percent as the inverse price ratio changes by one percent.

While this CES model is easy to estimate in logarithmic form and one of its parameters has a natural interpretation as a demand response elasticity, one potential disadvantage of the CES specification is that the elasticity of substitution is constant—invariant with respect to initial peak relative to off-peak electricity usage or to the initial relative prices. This is inconsistent with the view by some that when peak usage is high relative to non-peak usage, a customer may find it more difficult to shift load in response to a change in relative prices. It is also thought by some that a firm's ability to shift load to off-peak declines as more and more load is shifted. In addition, the CES specification is inconsistent with the view that a customer's willingness to respond to price increases at times of very high peak prices.<sup>11</sup> Since we are interested in studying these two issues, we must turn to an alternative, more flexible Generalized Leontief (GL) specification. In doing so, however, we encounter some challenges in estimation.

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<sup>10</sup>This relationship shows that  $\sigma$  is the proportional change in the use of electricity in the peak period relative to the off-peak period (holding output, in this case the electricity aggregate, constant), as the inverse price ratio increases or decreases by one percent (Ferguson 1969, pp. 103-04).

<sup>11</sup> A nested extension of this CES form was used originally by Herriges, *et al.* (1993) to characterize customer demand for electricity, where consumption within days is weakly separable from consumption across days. Schwarz, *et al.* (2002) also adopted a modification of this model to obtain intra-day and inter-day price responses. Despite this particular feature, there are two primary reasons we gave no further consideration to this nested CES model. First, this model still forces intra-day response elasticities to be constant; thus the model does not have the needed flexibility to allow for daily differences in intra-day response elasticities. Second, we focus our attention on intra-day price response because in the responses to our customer survey, the vast majority of the customers indicated that they did not shift from one day to another.

### B.2.2.2 The Generalized Leontief Specification

This model of electricity demand is based on the Indirect Generalized Leontief Cost Function. To begin its development, one must, as in the case of the CES model, specify a firm's production function that is separable in electricity inputs as given in equation (1) above (i.e.,  $Q = F(x_1, x_2, \dots, x_n, E(k_1, \dots, k_n))$ , where  $Q$  is output,  $x_i$  are inputs other than electricity and  $k_1$  through  $k_n$  are electricity used in periods 1 through  $n$ , respectively). Because production is assumed to be separable in electricity inputs, we can specify the function  $F$  as above, where the electricity inputs can be combined according to an aggregator function  $E$ . This is essentially being able to specify a sub-function within  $F$ . Any combination of  $k_p$  and  $k_o$  that yields the same value for  $E$  is equally productive in producing  $Q$ . It is the nature of this sub-function that determines the substitutability of electricity among different periods of the day.

Appealing to duality theory (Shephard 1970), we can also, in theory, specify the indirect cost functions associated with both the production function  $Q$  and the sub-function  $E$  above.<sup>12</sup> Because of the assumption that the function is separable in electricity inputs, we are again only concerned with the indirect cost function associated with the electricity aggregate's sub-function. From that sub-function, we can derive expressions for the elasticity of substitution among electricity use during different times of the day.

If we assume that the underlying aggregator function for  $E$  is linear homogeneous in the electricity inputs ( $k_i$ ) and that the indirect cost function  $C$  is a flexible generalized Leontief (GL) function, then we have for peak and off-peak periods:<sup>13</sup>

$$(5) \quad C = E \left\{ \sum_i \sum_j d_{ij} (p_i p_j)^{1/2} \right\};$$

$E$  is a measure of effective electricity as given by the electricity sub-function. This function is linear homogeneous in all prices, which is a requirement for a well behaved indirect cost function. That is, if all prices are changed in the same proportion, then  $C$ , the cost of producing the electricity aggregate, changes in the same proportion as well. We also require that  $d_{ij} = d_{ji}$ , for symmetry.

Since we are interested in a model that can capture price response for only two periods, (peak,  $p$ , and off-peak,  $o$ ), the corresponding GL model is given by:<sup>14</sup>

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<sup>12</sup> This involves solving the first-order conditions to the constrained optimization problem for minimizing the cost of producing a given output for the factor demands and substituting them back into the direct cost function. This procedure allows one to write the indirect cost-minimizing cost function in terms of output and input prices only.

<sup>13</sup> Diewert (1974) shows that if the generalized Leontief function (or any cost function) can be decomposed in this form, then the underlying aggregator function for  $E$  reflects a constant returns to scale technology. Put differently, this implies that total cost of the electricity aggregate is equal to a unit cost function (the term in  $\{ \}$  in equation (6)) multiplied by the level of the energy aggregate. Furthermore, this specification implies that the "isoclines" are linear rays out of the origin and that for a given ratio of peak to off-peak electricity prices, the ratio of peak to off-peak electricity use will be invariant with respect to the output level. Obviously, when the price ratio changes, the ratio of input use will change as well.



$$(6) \quad C = E \{d_{pp} p_p^{1/2} p_0^{1/2} + d_{p0} p_p^{1/2} p_0^{1/2} + d_{0p} p_0^{1/2} p_p^{1/2} + d_{00} p_0^{1/2} p_0^{1/2}\}$$

### B.2.2.3 Deriving the Elasticity of Substitution

Since the elasticity of substitution in this model is not constant as it is in the CES model, it is not equal to one of the parameters in equation (6). It must be derived in the following way. From Shepherd (1970), we know that optimal demands for peak and off peak electricity can be determined by differentiating (6) with respect to each price:

$$(7) \quad \partial C / \partial p_p = k_p = E [d_{pp} + d_{p0} (p_0 / p_p)^{1/2}].$$

$$(8) \quad \partial C / \partial p_0 = k_0 = E [d_{00} + d_{p0} (p_p / p_0)^{1/2}]$$

To develop a measure for the price responsiveness in the GL indirect cost function, we must also begin by deriving what is known as the Allen (1938) partial elasticities of input substitution, which, for any indirect cost function such as the one in equation (6), are equal to:<sup>15</sup>

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<sup>14</sup> In conducting this analysis, there were a number of other second-order flexible forms that might have been used in the empirical specification. One such commonly used flexible form, the translog (TL) model (Boisvert 1982 and Chambers 1988) would have avoided estimating any equations that are non-linear in the parameters. The TL model relies on estimating a set of electricity cost share equations that are linear in the model parameters and does not require observations on the electricity aggregate. While this TL form was particularly attractive from an estimation perspective, this alternative was not pursued because as Caves and Christensen (1980 a, b) point out, the TL model does not perform well when substitution elasticities are likely to be small, or when there are likely to be small shares or large relative differences among shares. This is partly true because the translog has the Cobb-Douglas form (a form that embodies considerable input substitution possibilities) as a special case. Patrick and Wolak (2001) found this to be problematic in an application of customer demand for electricity under real time pricing, and Diewert and Wales (1987) encountered similar difficulties in an earlier study using aggregate U.S. industrial data. Patrick and Wolak (2001) argue that the GL model is superior to the TL model because it has a fixed-coefficient Leontief technology as a limiting case, and therefore, it can reflect rather modest substitutions possibilities. However, these authors also note that if one imposes global concavity, the GL model loses some of its flexibility—in particular all inputs must be substitutes. To circumvent these difficulties related to both the TL and GL models, Patrick and Wolak (2001) and Taylor *et al.* (2005) employ a Generalized McFadden (GM) cost function that is “...second-order flexible, yet suited to capture small positive and negative elasticities of substitution between electricity demands across load periods within a day” (Patrick and Wolak, 2001, p. 27). Despite these theoretical advantages to the GM model, the computational burden is increased substantially as well. Their need to accommodate both positive and negative elasticities of substitution results from their specification of more than two demand periods. However, our empirical model specifies only two demand periods. In this case, any model in which global concavity is either assumed, or imposed, requires that the inputs be substitutes. Therefore, the primary issue that led to their selection of the GM model is of no concern in our present application of the GL indirect cost model.

<sup>15</sup> As discussed originally by Allen (1938, pp. 508-09), the partial elasticity measures the degree to which the demand for factor *j* changes as the price of factor *i* changes. If  $\sigma_{ij} > 0$ , and the price of factor *i* increases, then the use of factor *j* increases, thereby taking part in the replacement of factor *i* in production. The two factors are said to be *competitive*. If, on the other hand,  $\sigma_{ij} < 0$ , the two factors are *complements*, and as the price of one of them rises, the demand for both falls. Competitiveness between factors is, on the whole the more general case; one factor cannot be complementary with all others. In the two-input case, direct elasticity of substitution (which measures the percentage change in factor intensities as the inverse price ratio changes by one percent) is equal to the Allen partial elasticity of substitution.

$$(9) \quad \sigma_{p0} = C_{p0} / [C_p C_0],$$

where the subscripts refer to the first- and second-order partial derivatives of C with respect to electricity inputs  $k_p$  and  $k_0$ . Fortunately, in the two-input case such as we have here, the direct elasticity of substitution (which measures the percentage change in factor intensities as the inverse price ratio changes by one percent, holding a firm's output constant) is also equal to the Allen partial elasticity of substitution. Thus, although denoted somewhat differently, the measure of price response given by equation (9) is directly comparable to that derived from the CES model above.

Evaluating equation (9) for the GL cost function given in equation (6), we have:

$$(10) \quad \sigma_{p0} = \frac{1}{2} [C d_{p0} (p_p / p_0)^{-1/2}] / [E a_p a_0],$$

where  $a_p = k_p / E$  and  $a_0 = k_0 / E$  are the peak and off-peak energy proportions of the electricity aggregate, which are derived by dividing the electricity demand equations in equations (7) and (8) by E. That is:

$$(11) \quad a_p = k_p / E = d_{pp} + d_{p0} (p_0 / p_p)^{1/2}$$

$$(12) \quad a_0 = k_0 / E = d_{00} + d_{p0} (p_p / p_0)^{1/2}.$$

Finally, although they do not have a terribly meaningful interpretation in this application, we also need to estimate the Allen own partial elasticities of substitution, so that we can check to see if the underlying function is well behaved. These are given by (i and j = p and 0):

$$(13) \quad \sigma_{ii} = -\frac{1}{2} [C \sum_{j \neq i} d_{ij} (p_j)^{-1/2} p_i^{-3/2}] / [E a_i^2].$$

#### B.2.2.4 Estimating the Generalized Leontief Function

Normally, to estimate the parameters of this cost GL function, one need only assume an additive error structure associated with the input share equations (11) and (12), and then estimate them as a system of equations where there are across-equation restrictions to ensure symmetry of the parameters. Unfortunately, because E in our case is the energy aggregate and cannot be observed directly, it is impossible to employ this strategy. However, using full information maximum likelihood (FIML) methods within PROC MODEL in SAS, one can estimate the parameters of the cost function (6) from the ratio  $a_p$  and  $a_0$ . That is, we can estimate:

$$(14) \quad [a_p / a_0] = [k_p / k_0] = [d_{pp} + d_{p0} (p_0 / p_p)^{1/2}] / [d_{00} + d_{p0} (p_p / p_0)^{1/2}],$$

In this form, the equations are extremely non-linear in the parameters, and it might be best to take the logarithms of both sides for estimation purposes. We have the following logarithmic specification:

$$\ln [ a_p / a_0 ] = \ln [ k_p / k_0 ] = \ln \{ [ d_{pp} + d_{p0} (p_0 / p_p)^{1/2} ] / [ d_{00} + d_{p0} (p_p / p_0)^{1/2} ] \}.$$

This strategy will not get rid of the non-linearities, but it will convert each equation into the differences between two logarithms within which there are coefficients imbedded. Whether SAS deals with that kind of non-linearity better than these quotients is an empirical question. Within PROC MODEL, we can also impose the symmetry restrictions on  $d_{ij}$ , and force the adding up restrictions to  $\sum_i \sum_j d_{ij} = 1$  to normalize to a unit  $E_0$  curve.

### B.2.2.5 Calculating Elasticities of Substitution

Regardless of which transformation is easiest to estimate, we can use the results to calculate the elasticities of substitution from equations (10) and (13). This is done by substituting the estimated parameters, denoted  $d_{ij}^*$  (for  $i$  and  $j = p$  and  $0$ ), from equation (14) or (15) into equations (11) and (12) to calculate  $(a_p)_{fit}$  and  $(a_0)_{fit}$  at each data point. In turn, these estimated expressions are substituted into equation (6) to obtain estimates of  $(C/E)_{fit}$ . Finally, these estimated expressions are substituted into equations (10) and (13) to obtain for each data point estimates of:

$$(16) \quad \sigma_{p0} = 1/2 [(C/E)_{fit} d_{p0}^* (p_p p_0)^{-1/2}] / [(a_p)_{fit} (a_0)_{fit}],$$

$$(17) \quad \sigma_{pp} = -1/2 [(C/E)_{fit} d_{pp}^* (p_0^{1/2} p_p^{-3/2})] / [(a_p)_{fit}^2], \text{ and}$$

$$(18) \quad \sigma_{00} = -1/2 [(C/E)_{fit} d_{0p}^* (p_p^{1/2} p_0^{-3/2})] / [(a_0)_{fit}^2].$$

As stated above, in this two-input case, the cross Allen partial elasticity of substitution (17) is equivalent to the direct elasticity of substitution which measures the proportional change in the ratio of peak to off-peak electricity use due to a one percent change in the inverse price ratio (Ferguson 1969).<sup>16</sup> By examining equations (16) through (18), one major advantage of using the GL function is apparent. In contrast to the CES model, the elasticity of substitution for the GL model can now vary from observation to observation; it can also vary with price ratios, the energy aggregate, and the cost minimizing input levels. The trade-off necessary to gain this flexibility is that there is now no guarantee that the function is well behaved--e.g.,  $\infty > \sigma_{0p} \geq 0$  (Ferguson 1969).

### B.2.2.6 Interpreting the Coefficients in the GL Demand Model: Their Relationship to $\sigma_{0p}$

Whether or not the cost function is well behaved depends on the estimated values of the model parameters,  $d_{pp}$ ,  $d_{0p}$ , and  $d_{00}$ .<sup>17</sup> A sufficient condition for the GL function to be

<sup>16</sup> This relationship shows that  $\sigma$  is the proportional change in the use of electricity in the peak period relative to the off-peak period (holding output, in this case the electricity aggregate, constant), as the inverse price ratio increases by one percent (Ferguson 1969, pp. 103-04).

<sup>17</sup> For the most general case, where the parameters  $d_{00}$ ,  $d_{pp}$  and  $d_{0p}$  are allowed to take on any possible value, the resulting cost function need not be non-negative for all factor prices. Thus, in this case, it is necessary to ensure that at each data point the estimated cost function is monotonically increasing and strictly quasi-concave in input prices. To do this, we must verify that the fitted values for all the input-

well behaved is that all parameters be non-negative (Diewert 1974). Perhaps more important for our purposes, a necessary condition for the function to be well behaved is that  $d_{0p}$ , must be greater than or equal to zero. There are three cases of empirical interest, given this restriction on  $d_{0p}$  (Diewert, pp.503-504), and they can be related to the types of demand response depicted in Figures 1 and 2 above.

Case 1: If  $d_{pp}$ ,  $d_{0p}$ , and  $d_{00}$  are all non-negative, then peak and off-peak electricity are substitutes and the substitution possibilities are characterized by the curve in Figure 1.

Case 2: If  $d_{0p}$  takes on an extreme value of zero, then for the function to be well-behaved,  $d_{pp} \geq 0$  and  $d_{00} \geq 0$ , with at least one strict inequality. The significance of this case is that the GL model reduces to the ordinary two-factor Leontief production function that is characterized by fixed factor proportions as in Figure 2 above. Here, peak and off-peak electricity are perfect complements and the firm has no opportunities for input substitution. Since this situation may indeed characterize a number of firms, it is important that this GL model encompasses this possibility. Because this situation is captured by one of the coefficients in the model, we can develop a formal test of the hypothesis of fixed proportion electricity demand. The details of the test are discussed below.

Case 3: For this case, we have  $d_{pp} < 0$ ,  $d_{0p} > 0$ , and  $d_{00} > 0$ . Here, we can still trace out the curve representing input combinations that can keep output constant, but there is a price of peak electricity,  $p_p^*$  that makes the peak to off-peak price so large that peak electricity is no longer used. This happens when  $p_0/p_p < d_{pp}^2/d_{00}^2$ . This situation is depicted in Figure 3, where the constant electricity aggregate curve  $E_0$  intersects the vertical axis. This situation cannot be modeled by the CES function. While it is perhaps unlikely that many firms can forgo all electricity use during peak periods when peak prices are extremely high, the fact that the GL model can identify this type of behavior suggests that this flexible model is capable of capturing the entire range of price responsive behavior.

### B.3 Empirical Specification of the GL Model and Econometric Considerations

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output equations are positive and that the  $n \times n$  matrix of the  $\sigma_{ij}$  substitution elasticities is negative semi-definite at each observation (Berndt, 1991, p. 465 and p. 493).

The test to verify that this matrix is negative semi-definite is related to a well-known adding up condition for the Allen (1938, pp.504-05) partial elasticities of substitution. In the multi-factor case, this adding up condition ensures that the substitution relationships are more numerous and more important than the complementary relationships.

For the two-factor, the inputs must be substitutes: the adding up relationship implies:  $S_p \sigma_{0p} = -S_0 \sigma_{00}$  and  $S_0 \sigma_{0p} = -S_p \sigma_{pp}$ , where the  $S$ 's are corresponding cost shares. Berndt (1991) shows that the right hand side of the second relation is equal to the negative of own price elasticity of demand for peak electricity, holding output and all other input prices constant. The left-hand term is the corresponding cross-price elasticity of off-peak demand with respect to peak price, holding output and all other input prices constant. Thus, as the peak price is increased, the percentage increase in off-peak electricity usage equals the percentage decrease in peak electricity usage. It follows that if total actual off-peak demand is greater than total on-peak demand, then as peak price is increased, the increase in total off-peak demand will be larger than the decrease in peak demand. Thus, total daily electricity usage will increase.

In the 2004 analysis of Niagara Mohawk’s SC-3A customers, a CES response function was estimated to provide empirical estimates of elasticities of substitution between peak and off-peak electricity (Goldman, *et al.* 2004). In that study, responses from a survey were used to determine if the elasticities of substitution differed systematically by customer characteristic. Since the CES model restricts elasticities of substitution to be constant for any individual firm, the most efficient way to identify the importance of these firm characteristics was to estimate one model for all customers by pooling the data across customers and accounting for the effects of specific firm characteristics by introducing a series of additional variables that are the product of the price ratio and a series of 0-1 “dummy variables which are assigned a value of 1 if the firm has the particular characteristic. This model estimation was accomplished in a straightforward fashion in SAS.

Because of the non-linear nature of the equations to be estimated for the GL model, it is not possible to account for these firm effects by including a series of 0-1 “categorical” variables into a model that is estimated using the data pooled across all customers. Furthermore, even if such a strategy were possible, it would not exploit the full flexibility of the GL model. Therefore, to exploit the flexibility of the GL model, where elasticities of substitution can differ across days for a single firm, we employ the following estimation strategy:

- 1) Estimate a demand model for each customer;
- 2) Calculate the substitution elasticities for each data point for each firm;
- 3) Pool the estimated elasticities for all firms and all data points; and
- 4) Estimate a second model from this pooled data set where the daily elasticities of substitution between peak and off-peak electricity are a function of daily peak to off-peak electricity price ratios and a variety of individual firm characteristics.
- 5) Estimate a third model to relate the average elasticities of substitution by firm to a number of individual firm characteristics.

In estimating the elasticities of substitution between peak and off-peak electricity use, we exploit the flexibility of the GL demand model by estimating a separate model for each firm.<sup>18</sup> This entire estimation strategy is similar to the one used by Taylor and Schwarz (1990). In his study, Patrick used a GL demand model to estimate household demand response by customer, and used a second regression to relate customer characteristics to the degree of price responsiveness.

### B.3.1 Demand Model Specification

To conduct our estimation, we specify the following GL model for each individual firm as follows:

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<sup>18</sup> Other efforts to model demand response have also relied on estimated demand models for individual firms, although the functional forms have been different than the one used here (e.g., Schwartz *et al.*, 2002, Patrick and Wolak 1997). In several cases, the analyses were based on functional forms that did not allow the substitution elasticities to vary across days for each firm (e.g. Caves, *et al.* 1984 and Charles River Associates 2004).

$$(19) \quad \ln [ k_{pt,f} / k_{ot,f} ] = (w_f) W_{tf} + \{ \ln [ (h_{pf}) (H_{tf}) + (d_{pp,f}) + (d_{p0,f}) ( p_{0t,f} / p_{pt,f} )^{1/2} ] \} - \{ \ln [ (h_{of}) (H_{tf}) + (d_{00,f}) + (d_{0p,f}) ( p_{pt,f} / p_{0t,f} )^{1/2} ] \} + \varepsilon_{t,f},$$

where, for each weekday,  $t$ , and firm,  $f$ , we define:  $k_{pt,f}$  = actual peak kWh;  $k_{ot,f}$  = actual off-peak kWh;  $p_{pt,f}$  = average hourly peak price / kWh; and  $p_{0t,f}$  = average hourly off-peak price/kWh.

The effect of weather is accounted for in two distinct ways. The variable  $W_{tf}$  is a continuous variable reflecting cooling degrees (e.g., the difference between the average peak period temperature and 65). This variable enters as an intercept shifter, thus controlling for differences in peak to off-peak usage during days that are unusually warm. The second weather variable is  $H_{tf}$  is another weather index. It is a binary (0,1) variable to distinguish hot ( $H_{tf} = 1$ ) from cool days ( $H_{tf} = 0$ ). We used the average Temperature Heat Index (THI) derived by the National Weather Service during the peak period hours to distinguish hot (Avg. THI  $\geq 85$ ) from cool (Avg. THI  $< 85$ ) days.<sup>19</sup> By including this variable in the model, we account for differences in the model parameters on hot vs. cool days. This happens because the estimated coefficients on this variable affect the size of the estimated parameters of the model (e.g.  $d_{00,f}$  and  $d_{pp,f}$ ), but only on hot days. In turn, these parameters affect the size of the estimated elasticities of substitution. The process by which these coefficients are changed on hot days is discussed in the next section.

In the estimation,  $d_{p0,f} = d_{0p,f}$  to ensure the required symmetry among estimated coefficients. We also require that  $d_{00,f} + d_{pp,f} + d_{0p,f} + d_{p0,f} = 1$  and  $h_{pf} = h_{of}$ , which normalizes the coefficients to reflect a one unit curve for the energy aggregate.<sup>20</sup>

### B.3.2 Calculating Elasticities of Substitution by Firm

Since we do not pool the data across firms, we have separate estimates for the coefficients,  $d_{00,f}$ ,  $d_{pp,f}$ ,  $d_{0p,f}$ , and  $d_{p0,f}$  for each firm,  $f$ . We use the estimates of these individual demand model parameters to derive a set of firm-specific estimates of substitution elasticities according to equations (16), (17), and (18) and the procedures outlined in the previous section. It is important to be quite specific about how the effect of hot days is captured in these estimates of the elasticities of substitution. As above, let a \* denote an estimated parameter. From equation (19), we can now think of two of the parameters to be estimated in this model as  $d_{pp,f}^* = (h_{pf}) (H_{tf}) + (d_{pp,f})$  and  $d_{00,f}^* = (h_{of}) (H_{tf}) + (d_{00,f})$ . Thus, for hot days (when  $H_{tf} = 1$ ), the coefficients that are substituted into equations (16, 17, and 18) for calculating the elasticities of substitution are  $d_{pp,f}^* = (h_{pf}) + (d_{pp,f})$  and  $d_{00,f}^* = (h_{of}) + (d_{00,f})$ . In contrast, for cool days (when  $H_{tf} = 0$ ), the estimated coefficients being used in equations (16, 17, and 18) for calculating the elasticities of substitution are simply  $d_{00,f}^* = d_{pp,f}$  and  $d_{pp}^* = d_{00,f}$ .

<sup>19</sup> The weather index is constructed from temperature and dew point values for five weather stations located in the utility's service territory. See Goldman, *et al.* (2004) for details on the construction of the index.

<sup>20</sup> Although the parameter estimates changed if this adding up condition were set to a number different from 1, the estimates of the elasticities of substitution were invariant to the specification.

### B.3.3 Some Econometric Issues in Estimating the GL Function

There are two important econometric issues that were of some concern in estimation of the GL function. The first has to do with whether or not the prices in this demand model (equation (19)) are endogenously determined with peak and off-peak loads. The second has to do with whether or not there is autocorrelation between the error terms in equation (19), and in the subsequent equations for steps 4 and 5 of the overall estimation strategy. The final issue is the extent to which there is heteroskedasticity in the error structures of the equations for steps 4 and 5 of the overall estimation strategy. Each issue is discussed in turn.

#### B.3.3.1 Exogenous Prices and Loads

In equation (19), the logarithm of the ratio of peak to off-peak load is the dependent variable, whereas the off-peak and peak prices of electricity appear on the right hand side as explanatory variables, often called regressors. Most econometric methods assume that the regressors in a model are exogenous, or independent of each other, and are also not simultaneously determined with the dependent variable. In our case, this would mean that prices are being determined simultaneously along with peak and off-peak load. If the regressors are not exogenous, then by including an endogenous variable (that is correlated with the equation's error term) on the right hand side of the equation, the estimated residuals of the model are affected, and the parameter estimates are no longer unbiased, and their standard errors are no longer consistent (minimum variance). This leads to an underestimation of the standard errors and could lead to the t-ratios being biased upward. The consequence would be to reject some null hypotheses when it was unwarranted.

This issue can arise in any economic model of aggregate supply and demand when, as is often believed, prices and quantities are simultaneously determined. However, the issue is not thought to be a serious one when the model is dealing with the input demand at the firm level. When firms are small relative to the total market, it is reasonable to assume that they are price takers.<sup>21</sup> A second reason for believing that this issue is not important in this case is the fact that in making their energy use decisions, these SC-3A customers face 24-hour prices determined the day before in the day-ahead market. Thus, while their demand response decisions to high prices may affect prices on the same day in the real time market, their decisions can have no effect on the prices in the day ahead market that were determined the day before. Since customers face prices on which to base their decisions on hourly electricity usage that are established the day before, the price

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<sup>21</sup> In a price dependent zonal supply model of the new unregulated New York electricity market, Cappers (2004) argues that real time load is highly correlated with the error term, and its inclusion on the right-hand side of the model would affect the estimated residuals. For this reason, he employed the method of instrumental variables by estimating real time load as a function of a heat index and some cyclical variables for each zone. The estimated loads, now uncorrelated with the supply equation's error term, were then used as regressors in the supply models.

variables used in the firm-level electricity demand model specified here are truly exogenous.<sup>22</sup>

### B.3.3.2 Autocorrelated Error Terms

Standard regression models are generally based on the assumption that the error term of any observation is not influenced by the error term of any other observation. Formally, this means that the error terms are uncorrelated, and  $E(\varepsilon_i, \varepsilon_j) = 0$  for  $j \neq i$ . However, for each of our customers, we have a time series of data--the daily observations of peak and off-peak prices and loads are arrayed chronologically, and there is good reason to think that relative loads are likely to persist from day to day. Alternatively, it may also be the case that due to high prices, or other disturbances, that the effects on peak to off-peak demand may continue for a few days. In each of these cases, there may be some systematic relationship between the error terms. That is,  $E(\varepsilon_i, \varepsilon_j) \neq 0$  for  $j \neq i$ . For this case, Gujarati (1995) shows that although the OLS estimators are unbiased and consistent, they are no longer minimum variance. In addition, even though the OLS estimators remain unbiased and consistent, the confidence intervals derived from them using the variances corrected for autocorrelation are likely to be wider than when using a GLS estimator. Thus, in our analysis, we test for autocorrelation, and apply the appropriate correction if warranted. This appropriate correction for a model that is non-linear in the parameters is accommodated within PROC MODEL in SAS.

### B.3.3.3 Accounting for Heteroskedasticity

Another usual assumption in regression analysis is that disturbance terms for all observations in the model are independently and identically distributed (i.e.,  $E(\varepsilon_i^2) = \sigma^2$ , for all  $i$ ). In this application, some firms may exhibit zero or near zero estimated price responsiveness over all days, while others may exhibit quite a large average price response, but the daily variation around the mean price response may be rather large as well. Therefore, it may be reasonable to expect that the variance of the disturbance term in the equation (from step 4 of the estimation strategy) in which the estimated elasticities of substitution (pooled across all customers) are regressed on the ratio of prices and some limited firm characteristics are indeed not equal across firm or across observations within firms. That is, it may be that the elasticities of substitution are proportional to those factors that make a firm more or less price responsive on a particular day.

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<sup>22</sup> The modeling framework and rationale is similar to that used by Neenan, *et al.* (2003) in estimating prices in the day-ahead market. In the present case, quantities demanded by firms are determined by exogenous day-ahead prices, as set out in the terms of the SC-3A contract between Niagara Mohawk and the SC-3A customers. Similarly, an analogous situation applies in the day ahead electricity market in New York due to the nature of the market rules. In the day ahead market, Load Serving Entities (LSEs) and other market participants are required to submit a fixed bid load, the load they are willing to purchase in the day ahead market at any price. Once this load for all customers has been received, the NYISO determines hourly prices in the day that minimize the cost of meeting fixed bid load by solving its day ahead unit commitment algorithm. Thus, these fixed bid loads are truly exogenous at the time the day ahead prices are determined.



This characteristic of the error structure in a regression model is known as heteroskedasticity (Gujarati 1995). Although the coefficients remain unbiased, the estimators of the parameters are no longer minimum variance. That is, unless heteroskedasticity is recognized, the t-ratios associated with the coefficients may be biased, and in turn so are the t-ratios used for testing hypotheses about the significance of the effects of certain variables. Therefore, in both the second and third equations in the analysis (the regressions using pooled data to explain the effect of price levels on the elasticities of substitution and differences in average substitution elasticities), we test for heteroskedasticity using a generalized White's test (Greene 1990, pp. 399-428). In the cases where there is heteroskedasticity, we use the generalized method of moments to derive the appropriate correction and re-estimate the model. The tests and the re-estimations are accomplished within SAS.

#### B.3.4 Elasticity of Substitution Regression Model Specification

Once we have the estimated daily elasticities of substitution from each of the individual demand models, we pool the data across customers in order to test the hypotheses that price responsiveness is related to the level of electricity prices and other individual firm characteristics.

The basic regression equation specifies that the elasticities of substitution are a function of the peak to off-peak price ratio, the percent current use of maximum demand and business class. These variables are also specified in the model as intercept and/or slope shifters to account for the interaction among these characteristics. The model is specified as for day t and firm f):<sup>23</sup>

$$(20) \quad \sigma_{tf} = \alpha + \beta_1 [ p_{pt,f} / p_{0t,f} ] + \beta_2 \text{Man}^* [\% \text{MaxD}_{t,f}] + \beta_3 \text{Man}^* [ p_{pt,f} / p_{0t,f} ] \\ + \beta_4 \text{Gov/Ed}^* [\% \text{MaxD}_{t,f}] + \beta_5 \text{Gov/Ed}^* [ p_{pt,f} / p_{0t,f} ] + \beta_6 \text{PW}^* [\% \text{MaxD}_{t,f}] \\ + \beta_7 \text{PW}^* [ p_{pt,f} / p_{0t,f} ] + \beta_8 \text{C\&R}^* [\% \text{MaxD}_{t,f}] + \beta_9 \text{C\&R}^* [ p_{pt,f} / p_{0t,f} ] + \\ u_{tf},$$

where  $[ p_{pt,f} / p_{0t,f} ]$  = ratio of peak to off-peak electricity prices on day t for firm f; Man = 0-1 variable—1 for manufacturing firms, 0 otherwise; Govt/Ed = 0-1 variable—1 for government/education firms, 0 otherwise; PW = 0-1 variable—1 for public works firms, 0 otherwise; C&R = 0-1 variable—1 for commercial and retail firms, 0 otherwise;  $\% \text{MaxD}_{t,f}$  = % peak use on day t for firm f is of maximum demand for firm f in the year for which day t is an observation; and  $u_{tf}$  is a random error term.

##### B.3.4.1 Interpreting the Coefficients

The important hypotheses about the effects on the elasticity of substitution of electricity of the ratio of peak to off-peak prices and firm characteristics are reflected in the expected signs of the coefficients for each of the variables in equation (20). These

<sup>23</sup> In the empirical analysis, a first-order autoregressive process is incorporated into the model to correct for autocorrelation of the residuals. The reason for this correction is discussed in the previous section in conjunction with estimating the GL demand model.

specific hypotheses are discussed along with the empirical results below. In this section, we instead discuss in more general terms how to interpret the coefficients.

In general, the interpretation of the estimated coefficients in equation (20) is consistent with those in any linear regression model. For continuous variables, such as the peak to off-peak price ratio, the regression coefficient,  $\beta_1$ , reflects the marginal change in the elasticity of substitution,  $\sigma$ , due to a unit increase in the variable (e.g., an increase in the peak to off-peak price ratio from 1.5 to 2.5). Formally, we have:

$$(21) \quad \partial \sigma_{tf} / \partial [p_{pt,f} / p_{ot,f}] = \beta_1.$$

Since there are some categorical 0-1 variables used to identify any systematic differences in price responsiveness among firms in various categories, such as business class, the coefficient  $\beta_1$  actually measures the effect of changes in the price ratio on the elasticities of substitution for a reference group of customers. In our model, this reference group of firms contains firms in the health care industry. By including these categorical variables as interaction terms with the price ratio, the effects of the price ratio on the elasticities of substitution are allowed to differ for other groups of customers from that of the reference group. For manufacturing firms, for example, the elasticity of substitution differs from that of the reference group of firms (e.g. firms in the health care industry in our case) by an amount  $\beta_2$ . For government/education firms the elasticity of substitution differs from that of the reference group of firms (e.g. firms in the health care industry in our case) by an amount  $\beta_5$ .

Since some of the variables in the model are formulated by multiplying a 0-1 categorical variable by a more conventional, continuous variable, it is perhaps useful to highlight the interpretation of the coefficients associated with these variables. We can illustrate by considering the combined effect of the peak to off-peak price ratio and the percent peak use is of maximum peak demand on the elasticity of substitution,  $\sigma$ , for manufacturing firms. These combined effects are captured in the following three terms. That is, the elasticity of substitutions for manufacturing firms differ from that of the reference group (e.g., firms in the health care industry) by an amount equal to:

$$(22) \quad \beta_2 \text{Man}^* [\% \text{MaxD}_{t,f}] + \beta_3 \text{Man}^* [p_{pt,f} / p_{ot,f}],$$

To determine the total amount by which  $\sigma$  for manufacturing firms differs from other firms, one need only evaluate expression (22) for ( $\text{Man} = 1$ ) and the appropriate values of  $[\% \text{MaxD}_{t,f}]$  and  $[p_{pt,f} / p_{ot,f}]$  for the day of interest,  $t$ , and manufacturing firm of interest,  $f$ . Normally, these combined effects are summarized by calculating expression (22) at the mean and the extremes of these two continuous variables. Expression (22) also reveals the changes in  $\sigma$  due to marginal changes in both  $[\% \text{MaxD}_{t,f}]$  and  $[p_{pt,f} / p_{ot,f}]$ . That is, the  $\sigma$  for manufacturing firms is also increased by an amount  $\beta_2$  as peak electricity use increases by 1% relative to maximum peak demand and by an amount  $\beta_3$  as the ratio of peak to off-peak price increases by one unit. For  $\text{Man} = 1$ , these two relationships are derived formally by:

$$(23) \quad \partial \sigma_{tf} / \partial [\% \text{MaxD}_{t,f}] = \text{Man} * \beta_2, \text{ and}$$

$$(24) \quad \partial \sigma_{tf} / \partial [p_{pt,f} / p_{0t,f}] = \beta_1 + \text{Man} * \beta_3.^{24}$$

We can isolate the differences in the elasticity of substitution for government and educational firms (Govt/Ed = 1) in a similar way by examining the coefficients in the following expressions:

$$(25) \quad \beta_4 \text{ Gov/Ed} * [\% \text{MaxD}_{t,f}] + \beta_5 \text{ Gov/Ed} * [p_{pt,f} / p_{0t,f}];$$

$$(26) \quad \partial \sigma_{tf} / \partial [\% \text{MaxD}_{t,f}] = \text{Govt/Ed} * \beta_4, \text{ and}$$

$$(27) \quad \partial \sigma_{tf} / \partial [p_{pt,f} / p_{0t,f}] = \beta_1 + \text{Govt/Ed} * \beta_5.^{25}$$

Summarily, we can isolate the differences in the elasticity of substitution for public works firms (PW = 1) by examining the coefficients in the following expressions:

$$(28) \quad \beta_6 \text{ PW} * [\% \text{MaxD}_{t,f}] + \beta_7 \text{ PW} * [p_{pt,f} / p_{0t,f}];$$

$$(29) \quad \partial \sigma_{tf} / \partial [\% \text{MaxD}_{t,f}] = \text{PW} * \beta_6, \text{ and}$$

$$(30) \quad \partial \sigma_{tf} / \partial [p_{pt,f} / p_{0t,f}] = \beta_1 + \text{PW} * \beta_7.^{26}$$

Finally, we can isolate the differences in the elasticity of substitution for commercial and retail firms (C&R = 1) by examining the coefficients in the following expressions:

$$(31) \quad \beta_8 \text{ C\&R} * [\% \text{MaxD}_{t,f}] + \beta_9 \text{ C\&R} * [p_{pt,f} / p_{0t,f}];$$

$$(32) \quad \partial \sigma_{tf} / \partial [\% \text{MaxD}_{t,f}] = \text{C\&R} * \beta_8, \text{ and}$$

$$(33) \quad \partial \sigma_{tf} / \partial [p_{pt,f} / p_{0t,f}] = \beta_1 + \text{C\&R} * \beta_9.^{27}$$

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<sup>24</sup> The coefficient  $\beta_1$  is in this equation for the marginal effect of price to reflect the effect of the price ratio on the elasticity of substitution for health care firms, the reference group. The coefficient  $\beta_3$  measures the differential effect of the price ratio for the manufacturing group, but the combined effect is the sum of the two coefficients.

<sup>25</sup> The coefficient  $\beta_1$  is in this equation for the marginal effect of price to reflect the effect of the price ratio on the elasticity of substitution for health care firms, the reference group. The coefficient  $\beta_5$  measures the differential effect of the price ratio for the government/education group, but the combined effect is the sum of the two coefficients.

<sup>26</sup> The coefficient  $\beta_1$  is in this equation for the marginal effect of price to reflect the effect of the price ratio on the elasticity of substitution for health care firms, the reference group. The coefficient  $\beta_7$  measures the differential effect of the price ratio for the public works group, but the combined effect is the sum of the two coefficients.

<sup>27</sup> The coefficient  $\beta_1$  is in this equation for the marginal effect of price to reflect the effect of the price ratio on the elasticity of substitution for health care firms, the reference. The coefficient  $\beta_9$  measures the differential of the price ratio for the manufacturing group, but the combined effect is the sum of the two coefficients.

### B.3.5 Average Elasticity of Substitution Regression Specification

The regression specified in equation (20) is designed to test some important hypotheses about the effect of the prices and relative usage on firms' elasticities of substitution. However, although the GL model allows for elasticities of substitution for a firm to vary by day, we are also interested in how firm characteristics affect the average ability of firms to respond to price. We can accomplish this by again by pooling data for the average elasticities of substitution across firms, and regressing the averages on individual firm characteristics, particularly those derived from the survey. The model has the form:

$$(34) \quad \sigma_f = \alpha + \beta_1 \text{C\&R} + \beta_2 \text{Man} + \beta_3 \text{Health} + \beta_4 \text{Gov/Ed} + \beta_6 \text{EDRP} + \beta_8 \text{EMCS} + \beta_9 \text{MON-F} \\ + \beta_7 \text{SCR} + \beta_{10} \text{GEN} + \beta_{11} \text{ELECT-INCR} + \beta_{12} \text{Avg-P-NP-KW} + u_{if},$$

where, C&R = 0-1 variable—1 for commercial and retail firms, 0 otherwise. Man = 0-1 variable—1 for manufacturing firms, 0 otherwise; Govt/Ed = 0-1 variable—1 for government/education firms, 0 otherwise; Health = 0-1 variable—1 for health care firms, 0 otherwise; EDRP = 0-1 variable--1 for firms in EDRP, 0 otherwise; SCR = 0-1 variable--1 for firms in SCR, 0 otherwise; EMCS = 0-1 variable--1 for firms with EMCS equipment, 0 otherwise; MON-F = 0-1 variable--1 for firms that monitor load frequently, 0 otherwise; EDRP = 0-1 variable--1 for firms with on-site generation, 0 otherwise; ELECT-INCR = 0-1 variable--1 for firms indicating that electricity usage has risen over the sample period, 0 otherwise; EDRP = a continuous variable for the average peak to off-peak load.

### B.4 Aggregate Demand Response

The theoretical discussion and mathematical and empirical models in this appendix relate primarily to methods by which one can estimate a customer's load reduction in response to changes in the ratio of peak to off-peak electricity prices. Other models are specified to identify the effect of customer characteristics on both average levels of price responsiveness and day-to-day changes in price responsiveness. From the information generated using these methods, it is also possible to estimate the aggregate demand response. This aggregate demand response has important policy significance, as it provides an indication of the response forthcoming from the entire group of customers during times of high prices, which are also often coincident with those times when the system is near capacity.

To generate this aggregate demand response, we begin by estimating each firm's peak period load reduction for a change in the peak period price of electricity by using that firm's average estimated elasticity of substitution. As shown in Footnote **Error!** **Bookmark not defined.**, the "adding up" conditions on the Allen partial elasticities of substitution that are required of a well-behaved indirect GL cost function allow one to derive an estimate for the own-price elasticity of demand for constant output and other

input price levels. For a given percentage change in the peak price relative to its average level (the term in [ ] ), the firm f's peak period load reduction,  $DR_{p,f}$ , would be:

$$(35) \quad DR_{p,f} = \{ (\sigma_f * S_{0,f}) * [ (p_{p,f}^* - \text{Avg. } p_{p,f}) / \text{Avg. } p_{p,f}] \} * \text{Avg. } k_{p,f},$$

where  $\sigma_f$  is firm f's average elasticity of substitution for peak and off-peak electricity usage,  $S_{0,f}$  is firm f's share of total electricity cost spent on off-peak electricity, and  $\text{Avg. } k_{p,f}$  is firm f's average peak electricity usage.

To derive an aggregate level of demand response, we sum each firm's level of demand response for the same change in the peak price. By simulating each firm's demand response over a wide range of peak period prices, it is possible to create an aggregate demand response curve that indicates the estimated amount of demand response that the analyzed SC-3A customers would provide during the peak period for any given level of peak price.

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## Appendix C. Empirical Estimates of NMPC’s SC-3A Customers’ Response to Day-Ahead Market Electricity Prices

In this Appendix, we present the full set of empirical results from this study of SC-3A customers’ price response. The main findings are also included and discussed in Chapter 3 of the main report and policy implications are discussed in Chapter 5. For those interested in additional detail, this appendix provides it.

There are three primary sets of empirical results that correspond to the three estimated equations described in Appendix B. The first involves estimates of substitution elasticities between peak and off-peak electricity use derived from separate demand models estimated for each customer. The second and third sets of results are derived from heuristic regression models. The first regresses daily elasticities against prices, price ratios and load conditions to answer questions about the character of price response. The second regresses customer-average elasticities against customer-level characteristics and circumstances.

A detailed breakdown of the customer accounts included in the three stages of this study is provided in Table 1. While load and price information were available for 146 SC-3A accounts, only the 119 accounts that were known to have faced hourly varying prices were included in the analysis (see Appendix B for the rationale behind this decision). Of these 119 accounts, only 55 answered the survey and could be included in the second regression model.

**Table 1. SC-3A Customer Accounts, 2000-2004**

Business Class	Customers Paying SC-3A Prices								
	All SC-3A Customers		Total				With Complete Survey Responses		With Complete Survey Responses - % of Total on SC-3A Rates
	Number of Accounts	Peak Demand (MW)	Number of Accounts	Peak Demand (MW)	Number of Accounts	Peak Demand (MW)	Number of Accounts	Peak Demand (MW)	
Commercial / Retail	17	55	17	49	8	24	47	49	
Gov't / Education	44	206	34	166	16	82	47	49	
Health Care	17	78	8	38	2	5	25	13	
Manufacturing	46	233	44	221	23	127	52	57	
Public Works	22	70	16	40	6	15	38	38	
<b>Totals</b>	<b>146</b>	<b>642</b>	<b>119</b>	<b>514</b>	<b>55</b>	<b>253</b>	<b>46</b>	<b>49</b>	

### C.1 Estimates of Customer Price Responsiveness

We analyzed the price response behavior of NMPC’s RTP customers by estimating the GL demand model (from equations (19) and (16) in Appendix B) for several alternative specifications of the peak period that differ in length and by time of day. We focused on summer weekdays, and initially defined six separate peak periods, as follows:

- two five-hour peaks, 11:00 a.m. to 4:00 p.m. and 12:00 noon to 5:00 p.m.;
- two four-hour peaks, 12:00 noon to 4:00 p.m. and 1:00 p.m. to 5:00 p.m.; and
- two three-hour peaks, 1:00 p.m. to 4:00 p.m. and 2:00 p.m. to 5:00 p.m.

The load-weighted average and range in firm-level elasticities of substitution of peak for off-peak electricity for the 119 accounts are provided in Table 2; the firms are grouped according to business class. Recall that the elasticity of substitution is defined as the percentage change in the ratio of peak to off-peak electricity usage due to a one percent change in the ratio of off-peak to peak prices.

**Table 2. Load-Weighted Elasticities of Substitution of Off-Peak for Peak Electricity by Customer Class and Peak-Period Definition**

Business Class	Number of Accounts	Peak Periods								
		11:00 a.m. to 4:00 p.m.			12:00 noon to 4:00 p.m.			1:00 p.m. to 4:00 p.m.		
		Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Commercial / Retail	17	0.01	0.02	0.02	0.06	0.07	2.06	0.05	0.07	1.93
Gov't / Education	34	0.01	0.02	0.03	0.04	0.06	2.99	0.06	0.08	1.75
Health Care	8	0.00	0.01	0.01	0.03	0.03	0.03	0.03	0.03	0.03
Manufacturing	44	0.00	0.07	0.09	0.12	0.12	0.13	0.12	0.12	0.14
Public Works	16	0.00	0.09	4.63	0.01	0.02	0.02	0.01	0.01	0.02
<b>All Accounts</b>	<b>119</b>	<b>0.01</b>	<b>0.04</b>	<b>0.35</b>	<b>0.07</b>	<b>0.08</b>	<b>1.33</b>	<b>0.07</b>	<b>0.09</b>	<b>0.88</b>

Business Class	Number of Accounts	Peak Periods								
		12:00 noon to 5:00 p.m.			1:00 p.m. to 5:00 p.m.			2:00 p.m. to 5:00 p.m.		
		Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Commercial / Retail	17	0.01	0.03	0.04	0.05	0.06	1.43	0.05	0.06	1.49
Gov't / Education	34	0.00	0.04	0.05	0.07	0.08	1.21	0.09	0.10	0.42
Health Care	8	0.00	0.04	0.05	0.03	0.04	0.04	0.03	0.04	0.04
Manufacturing	44	0.00	0.07	0.09	0.12	0.12	0.14	0.15	0.16	0.18
Public Works	16	-0.04	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02
<b>All Accounts</b>	<b>119</b>	<b>0.00</b>	<b>0.05</b>	<b>0.06</b>	<b>0.08</b>	<b>0.09</b>	<b>0.63</b>	<b>0.10</b>	<b>0.11</b>	<b>0.38</b>

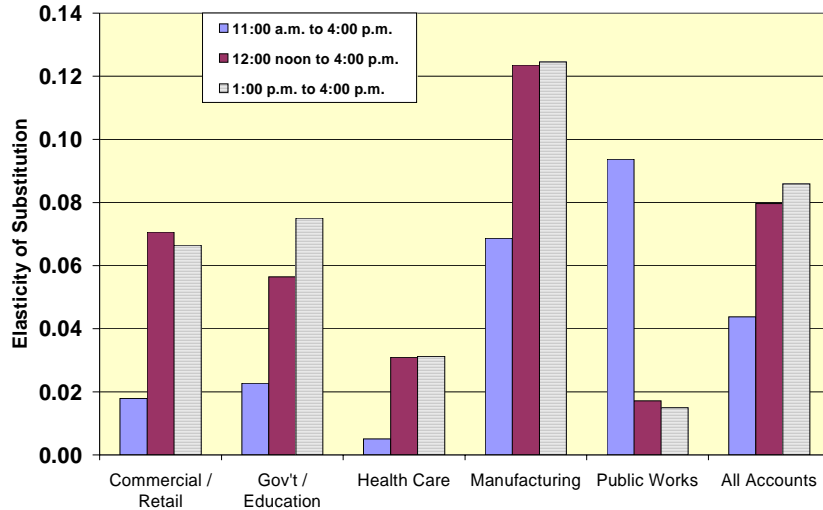
Note: Firm-level individual elasticities of substitution summarized here are estimated from demand model (equation (19)) and equation (16).

There are several important patterns to note. For the 119 customers as a group, the average price responsiveness is remarkably similar regardless of the peak period. The load-weighted average elasticity of substitution ranges between 0.04 and 0.09 in all but one of the peak periods. For the 2-5 p.m. peak period, the average elasticity of substitution is slightly higher (0.11). On average and regardless of the definition of the peak period, differences by business class are more pronounced. They are generally highest for manufacturing, ranging from a low of 0.07 for manufacturing for the 11:00 a.m. to 4:00 p.m. peak to 0.16 for the 2-5 p.m. peak. The elasticities for the government/education sector range from a low of 0.02 to a high of 0.10 for these same peak periods. On average, the price responsiveness of commercial/retail customers is somewhat below that for government/education customers. This difference is largest for the 2-5 p.m. peak period (average elasticity is 0.06 vs. 0.10).

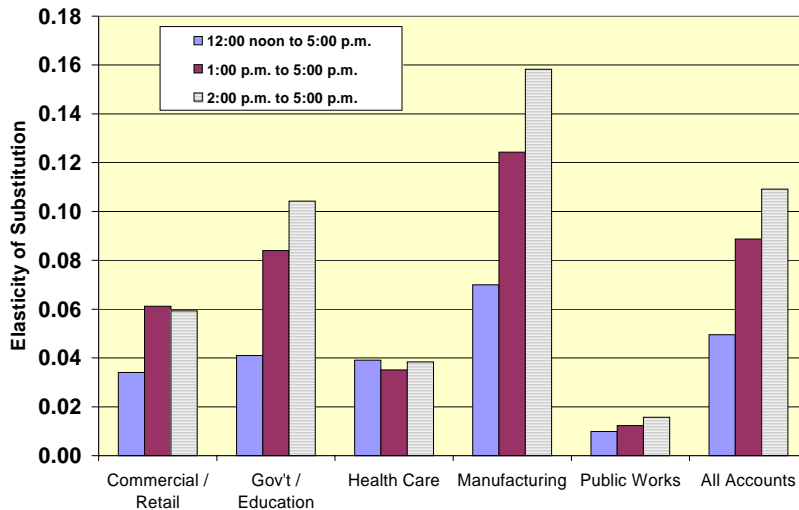
There are, however, also some systematic differences as the length of the peak period increases (Table 2 and Figures 1 and 2). First, there is quite a bit of consistency in the elasticities of substitution for peak periods of 3 and 4 hours in length, regardless of when the peak ends, although the elasticities are generally nearly the same or somewhat lower for all business classes for the 4-hour peak. With the exception of healthcare (for the peak

periods ending at 5:00 p.m.), and public works (for the peak ending at 4:00 p.m.), price response in all other sectors falls dramatically in moving from the 3-or 4-hour peak to the 5-hour peak. As the peak period expands in length, there are fewer adjacent hours in which to shift load to minimize the effect on output.

For all subsequent analyses and results, the 2-5 p.m. peak period was used.



**Figure 1. Average Elasticities of Substitution for Peak Periods Ending at 4:00 p.m.**

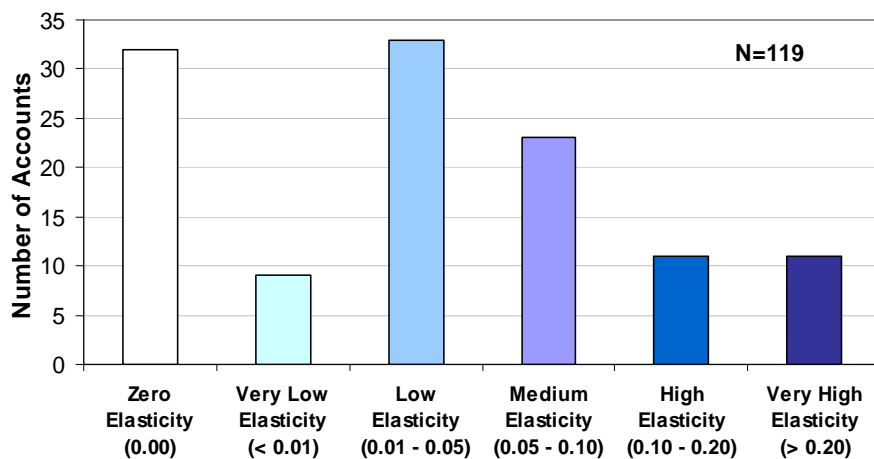


**Figure 2: Average Elasticities of Substitution for Peak Periods Ending at 5:00 p.m.**

Among the strengths of the GL demand model are its ability to accommodate variations in elasticities across firms and days and its ability to identify customers that are completely non-responsive (zero elasticities). As evidenced by the ranges in the load-

weighted elasticities of substitution by business class and peak period (Table 2), the differences among customers' elasticities are substantial.

The distribution of customers' average elasticities are shown by customer account and non-coincident peak load in Figures 3 and 4. Thirty-two of the 119 firms (about 28%) have zero elasticities of substitution (e.g. they have rectangular constant output curves and are not price responsive, Case 2 in Appendix B). Another nine customers (about 8%) have elasticities averaging less than 0.01, and 33 (about 28%) have elasticities of substitution between 0.01 and 0.05. Just under 20% (33 firms) have elasticities of substitution between 0.05 and 0.10. The remaining 18% are quite price-responsive, with elasticities of substitution above 0.10.<sup>28</sup> It is the firms in these final two groups that account for the vast majority of SC-3A customers' aggregate price response.<sup>29</sup>



**Figure 3. Distribution of Accounts by Elasticity of Substitution**

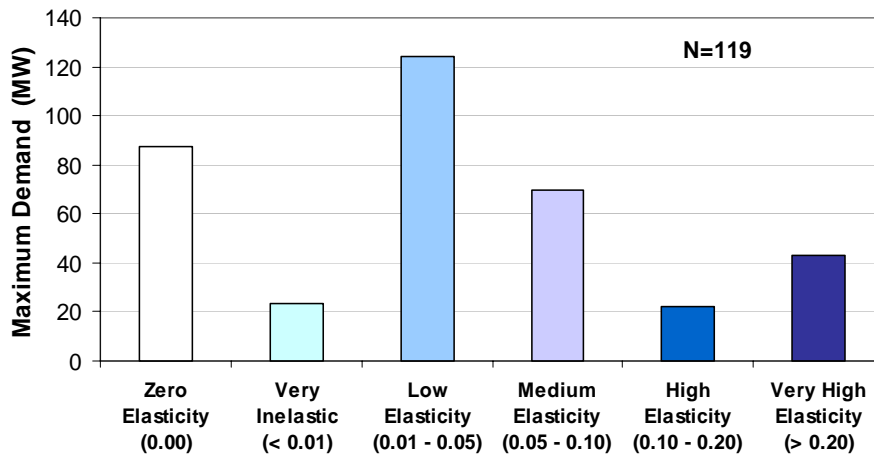
As emphasized in Appendix B (Case 3), it is through the use of this GL methodology that we are also able to identify firms, perhaps with on-site generation, where peak electricity may well be “priced out of the market” at very high peak prices. Of the 119 customers, we found four government/education and one commercial/retail customer with this characteristic. The ratios of peak to off-peak prices at which their usage of peak electricity is “priced out of market” range from lows of 7 and 15, to highs of 95 and well over 100. As might be expected, all five of these customers have on-site generation, and they are among the 11 customers with elasticities of substitution greater than 0.20 (Figure 3).

Firms with zero elasticities of substitution account for about 24% (87 MW) of the 119 SC-3A customers' combined non-coincident maximum demand (Figure 4). Those with

<sup>28</sup> Because of the flexibility of the GL model, it is necessary to check to see if estimated demand functions are well-behaved at each data point for all firms that have something other than rectangular isoquants. Using procedures discussed in Section A.2.2.6, we performed these checks and found that at each data point the estimated cost function is monotonically increasing and strictly quasi-concave in input prices.

<sup>29</sup> To identify firm characteristics that are related to an ability to respond to price and to the lack of an ability to do so, we do not eliminate non-price responsive firms from the subsequent analysis.

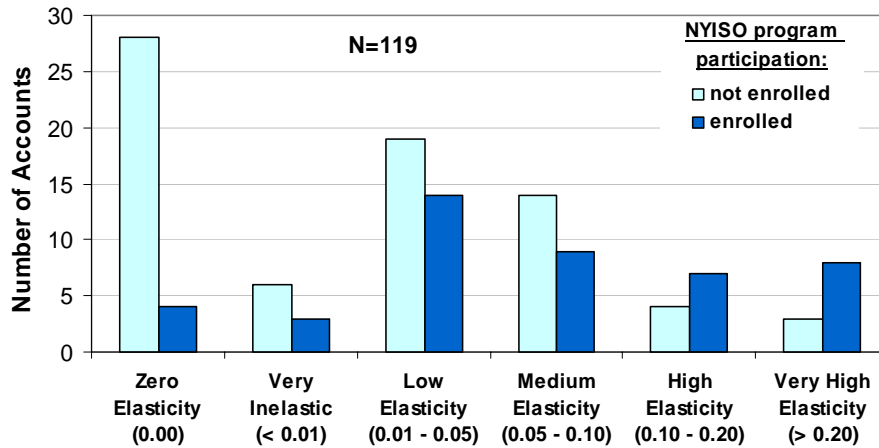
elasticities of substitution between zero and 0.01 represent 6% (23 MW) of maximum demand. Another 34% (124 MW) is accounted for by firms with elasticities of substitution between 0.01 and 0.05. The remaining 18% (65 MW) of maximum demand is accounted for by the final two groups of relatively price responsive customers (elasticities of substitution above 0.10). Firm electricity usage is thus distributed over these price response categories in proportions similar to firm numbers. This is contrary to the notion that price response is positively related to the firm's usage level.



**Figure 4. Distribution of Load by Elasticity of Substitution**

Some subject customers, in addition to facing daily prices, were enrolled in NYISO demand response programs, which offer inducements to reduce load on very short (two-hour) notice. Figure 5 displays the distribution of NYISO program participants and non-participants by substitution elasticity category.<sup>30</sup> Not surprisingly, the firms with the highest elasticities of substitution show disproportionately high levels of participation. However, some customers with relatively low elasticities of substitution (under 0.05) or even zero elasticities participated in the NYISO programs. This apparent anomaly is discussed in Chapters 3 and 5.

<sup>30</sup> Beginning in 2001, the NYISO demand response programs have been offered every summer. Customers generally enroll for six-month terms.



**Figure 5. Distribution of Accounts by Elasticity of Substitution and NYISO DR Program Participation**

To control for the effects of weather on electricity use and price responsiveness, two variables were included in the GL demand model (equation (19) in Appendix B). Table 3a shows the number of customers in each business category for which the coefficients on these two weather variables are significantly different from zero, along with the number of firms with “significant” coefficients that are positive and negative. Overall, the weather intercept shifter was significant for 58 (49%) of the 119 firms but the slope shifter, which indicates a direct effect of weather on price response, was significant for only 32 (27%) of the 119 firms. Including the intercept weather effect, to measure the coincidence of high prices and high peak to off-peak loads, was critical as it provided a “correction” for almost half of the firms. The coefficient was positive and significant for 52 of these 58 firms. For them, the ratio of peak to off-peak load is higher during hot days than on cool days. For six firms, the coefficient was negative and significant, indicating that the ratio of peak to off-peak usage was lower during hot days. This particular effect of weather on peak to off-peak usage also differs substantially across business sectors. This effect was statistically significant for a larger percentage of firms for those sectors wherein firms conduct service businesses from office buildings or in a campus setting. The coefficients on the intercept terms are significant for more than 65% of firms in the healthcare, government/education, and commercial/retail sectors. In contrast, less than 25% of the manufacturing and 40% of the public works firms appear to be sensitive to weather in the sense that there is a change in peak to off-peak usage during hot days.

**Table 3a. Weather Variables Included in GL Model**

Business Class	Weather Intercept			Weather Slope		
	Statistically Sig.		Statistically Insig.	Statistically Sig.		Statistically Insig.
	Positive	Negative		Positive	Negative	
Commercial/Retail	12	1	4	3	4	10
Government/Education	22	1	11	0	8	26
Health Care	6	0	2	1	3	4
Manufacturing	7	3	34	2	7	35
Public Works	5	1	10	0	4	12
<b>Totals</b>	<b>52</b>	<b>6</b>	<b>61</b>	<b>6</b>	<b>26</b>	<b>87</b>

The effect of weather on price response intensity is both less extreme and less diverse across business sectors. For only two sectors (commercial/retail and healthcare) do more than 40% of the firms have statistically significant “slope” variable coefficients. A quarter or less of firms in the other sectors exhibit a relationship between weather and price response. Although weather appears to be an important factor affecting price response for these firms, the signs on these estimated parameters cannot be interpreted directly. In some cases, the average elasticity is higher for firms for which this particular weather variable is statistically significant, and lower for others.

Although it is impossible to disentangle the separate effects of these two weather variables on the estimated elasticities of substitution, the net effect of both can be determined by calculating the average elasticities of substitution for each firm for both “hot” and “cool” days (i.e. the weather intercept is set at its daily value, but “hot” days are defined as those days when the binary slope variable for weather takes on a value of “1”; “cool” days are when this variable takes on a value of “0”). Table 3b shows the load-weighted average elasticities of substitution for hot and cool days by business class.

**Table 3b. Impact of Weather on Price Response by Business Sector**

Business Class	Avg. Elasticity of Substitution	
	Cool	Hot
Commercial / Retail	0.05	0.10
Government / Education	0.10	0.12
Health Care	0.04	0.04
Manufacturing	0.16	0.15
Public Works	0.02	0.02
<b>Total</b>	<b>0.109</b>	<b>0.113</b>

It is evident that the inclusion of these weather variables in the GL model specification has a definite effect on the elasticities of substitution. The difference in the overall load weighted average elasticities of substitution across all firms is relatively small—0.113 for hot days compared to 0.109 on cool days. However, this is not true across all business classes. For example, when comparing hot days with cool days, the sectors with heavy reliance on cooling loads (commercial/retail and government/education) exhibit a marked

increase in their elasticities of substitution. Manufacturing firms, on the other hand, appear to become less price responsive on hot days, but only slightly so. There is no appreciable difference in average elasticities of substitution for the other two business classes for hot and cool days.

## C.2 Factors Affecting Price Response

This section discusses the results of two additional regression equations that investigate factors that affect daily and customer-average elasticities. The first includes all 119 customers that have faced SC-3A rates sometime during the study period. In contrast, the second regression uses data collected from the survey as explanatory variables and therefore includes only the 55 customers for which survey responses are available. For purposes of comparison, and in an attempt to understand how general our results are, we also estimate the first regression using data from only the 55 customers.

**Table 4. Characteristics of Modeled Customers**

Business Class	Customers Paying SC-3A Prices			With Complete Survey Responses		
	Number of Accounts	Peak Demand (MW)	Load-Wgt. Avg. Elasticity	Number of Accounts	Peak Demand (MW)	Load-Wgt. Avg. Elasticity
Commercial / Retail	17	49	0.06	8	24	0.03
Gov't / Education	34	166	0.10	16	82	0.13
Health Care	8	38	0.04	2	5	0.05
Manufacturing	44	221	0.16	23	127	0.06
Public Works	16	40	0.02	6	15	0.01
Totals	119	514	0.11	55	253	0.07

Table 4 contains information about the 119 firms and the sub-set of 55 firms. This sub-set represents about 46% of the 119 firms, and about 49% of the peak demand. The healthcare and public works sectors are under represented in the sub-set of 55. The load-weighted average elasticities of substitution for the sub-sample are 36% lower than the larger group's average, and the sub-sample exhibits significantly lower average elasticities for the commercial/retail and manufacturing sectors, and higher elasticities for the government/education sector than the respective larger samples.

### C.2.1 Factors Affecting Daily Elasticities of Substitution

The first of the two regression equations uses the price ratio and the firm's usage as a percentage of maximum demand as variables to explain day-to-day differences in the estimated firm-level elasticities of substitution. By including interactive terms made up of the product of the price ratio and the business sector and the demand percentage, the effects of these factors on price response by business activity are estimated. The definitions of the variables used in this regression are presented in Table 5. Although this discussion (and the discussion in Chapter 3) focus on the results of the regression model based on data for all 119 firms, a model based on data from the 55 firms is also reported in Table 6.



**Table 5. Definitions of Explanatory Variables**

Variable	Definition
Intercept	
Public Works	0-1 Variable, 1 if firm is in Public Works, 0 otherwise
Comm/Retail	0-1 Variable, 1 if firm is Commercial or Retail, 0 otherwise
Manufacturing	0-1 Variable, 1 if firm is in Manufacturing, 0 otherwise
Gov/Ed	0-1 Variable, 1 if firm is in Government or Education, 0 otherwise
Public/Health	0-1 Variable, 1 if firm is in Public Health, 0 otherwise
% Max Dmnd	Daily peak electricity usage as % of corresponding year's max demand
Price Ratio	Ratio of Peak to Off-Peak Prices
Event	0-1 Variable, 1 if an NYISO EDRP event was declared, 0 otherwise
EDRP	0-1 Variable, 1 if firm participates in NYISO's EDRP, 0 otherwise
SCR	0-1 Variable, 1 if firm participates in NYISO's ICAP/SCR program, 0 otherwise
Increased Energy	0-1 Variable, 1 if survey respondent said energy use increased over the 2000-2004 period, 0 otherwise
Monitor Prices Freq.	0-1 Variable, 1 if survey respondent said the firm monitors electricity prices frequently
Use EMCS	0-1 Variable, 1 if survey respondent said the firm uses its EMCS equipment, 0 otherwise
Have On-Site Gen	0-1 Variable, 1 if survey respondent said the firm has on-site generation, 0 otherwise

For each sample there are two regressions shown in Table 6, labeled “GLS” (generalized least squares) and “GMM” (generalized method of moments) models. Ultimately, it is the “GMM” models that are of interest, because they include a correction for autocorrelation. In effect, we can think of the estimation process as involving two steps. The first is to estimate the “GLS” model. This model includes an AR(1) process to deal with the autocorrelation that was apparent from examining the data and is due to a time-dependent persistence of similar elasticities of substitution from one day to the next. The high degree of fit, as measured by the R<sup>2</sup> values for both the “GLS” models are the result of this correction for autocorrelation. Once this correction has been done, any variations around this strong persistent trend due to the explanatory variables in the model are reflected in the resulting parameter estimates.

**Table 6. Estimated Daily Elasticity of Substitution Regression Model Results**

Variable	Survey Sub-Sample (55)				Full Sample (119)			
	GLS Model		GMM Model		GLS Model		GMM Model	
	Parm. Est.	t-Stat	Parm. Est.	t-Stat	Parm. Est.	t-Stat	Parm. Est.	t-Stat
Pk Off-Pk Price Ratio (PR)	-0.0034	-0.69	-0.0040	-5.49	-0.0025	-1.06	-0.0028	-8.92
Commercial/Retail * PR	0.0004	0.07	0.0005	0.63	0.0184	6.15	0.0199	4.22
Manufacturing * PR	0.0018	0.35	0.0023	2.27	0.0014	0.55	0.0016	2.34
Gov't/Education * PR	0.0162	3.13	0.0185	3.90	0.0228	8.69	0.0241	3.44
Public Works * PR	0.0014	0.25	0.0019	2.30	0.0009	0.31	0.0010	2.80
Daily % of Max Demand (MD)	-0.0680	-13.79	-0.0702	-1.92	-0.0375	-10.29	-0.0513	-1.99
Commercial/Retail * MD	0.0621	6.55	0.0651	1.79	0.0511	8.00	0.0646	2.43
Manufacturing * MD	0.0659	11.42	0.0674	1.85	0.0352	8.74	0.0487	1.89
Health Care * MD	0.0692	3.66	0.0718	1.97	0.0378	3.44	0.0514	2.00
Public Works * MD	0.0676	9.29	0.0695	1.91	0.0373	6.95	0.0509	1.97
AR(1)	0.4348	56.04	0.4341	5.48	0.4674	87.62	0.4657	8.82
R-Squared	0.98		0.98		0.99		0.99	
Durbin Watson Statistic	2.14		2.14		2.13		2.13	
White's Test Statistic	5,516				13,022			

While the results of the “GLS” models looked extremely encouraging, we were, as suggested in Appendix B, concerned about heteroskedasticity of the error structure because of the wide variation in daily elasticities of substitution for some firms. Thus, we performed a White’s test for heteroskedasticity and found the White’s statistic to be significantly different from zero in both “GLS” Models. The second step of the estimation, therefore, was to correct for this problem of heteroskedasticity by re-estimating the model using the generalized method of moments (hence the “GMM” name on the second models). While the procedure correcting for heteroskedasticity does generally change the estimated coefficients, the effect is often expected to be minor. This proved to be the case in both models. However, the primary purpose in correcting for heteroskedasticity is to ensure that the standard errors on the coefficients (and therefore the t-statistics) are unbiased. Based on this correction, most of the t-ratios increased, while some others declined. In the final “GMM” models, all but one estimated coefficient (for the sample of 55) are all significantly different from zero. It is also encouraging that the corresponding estimated coefficients for each of the “GMM” models have the same sign and are of similar magnitude. The overall performance of the “GMM” model for the sample of 119 does slightly outperform the one for the sample of 55, particularly in terms of the size of the t-ratios. One possible interpretation of these results is that by including more firms in the sample, we are able to measure the effects of the variables on price responsiveness with greater statistical precision.

Since many of the variables are interaction terms, an effective way to interpret the results is to estimate the changes in the elasticities of substitution for a particular change in one of the explanatory variables—in this case the price ratio and the percent of maximum demand. Using the estimated coefficient for the “GMM” model for the 119 customers from Table 6, the effects of changes in the price ratio and in usage relative to maximum demand are summarized in Table 7 for the full sample of 119 customers and in Table 8 for the sub-set of 55 survey respondents.

**Table 7. Marginal Changes in Elasticities of Substitution by Business Class: 119 Customer Sample**

Business Class	Number of Accounts	Average Elasticity	Elasticity of Substitution for a Marginal Change in:			
			Peak to Off-peak Price Ratio <sup>a</sup>		Proportion Use of Max Demand <sup>b</sup>	
			Elasticity	% Change from Avg.	Elasticity	% Change from Avg.
Commercial/Retail	17	0.115	0.132	14.8%	0.116	1.2%
Gov't/Education	34	0.159	0.180	13.4%	0.154	-3.2%
Health Care	8	0.035	0.032	-8.1%	0.035	0.0%
Manufacturing	44	0.087	0.086	-1.4%	0.087	-0.3%
Public Works	16	0.018	0.017	-9.5%	0.018	-0.2%
Total	119					

<sup>a</sup> A marginal change in the peak to off-peak price is having the price ratio change from 2 to 3.

<sup>b</sup> A marginal change is use increasing from say 0.6 of maximum demand to 0.7 of maximum demand

**Table 8. Marginal Changes in Elasticities of Substitution by Business Class: 55 Customer Sub-Sample**

Business Class	Number of Accounts	Average Elasticity	Elasticity of Substitution for a Marginal			
			Peak to Off-peak Price Ratio <sup>a</sup>		Proportion Use of Max Demand <sup>b</sup>	
			Elasticity	% Change from Avg.	Elasticity	% Change from Avg.
Commercial/Retail	8	0.054	0.051	-6.3%	0.054	-0.9%
Gov't/Education	16	0.181	0.196	8.0%	0.174	-3.9%
Health Care	2	0.052	0.048	-7.6%	0.052	0.3%
Manufacturing	23	0.060	0.058	-2.8%	0.059	-0.5%
Public Works	6	0.021	0.019	-10.0%	0.021	-0.3%
Total	55					

<sup>a</sup> A marginal change in the peak to off-peak price is having the price ratio change from 2 to 3.

<sup>b</sup> A marginal change is use increasing from say 0.6 of maximum demand to 0.7 of maximum demand

For each business sector, the number of accounts and the un-weighted average the elasticity of substitution values are reported by sector in the left-hand part of the Table 7. To the right of these values, Table 7 is separated into two sections. The first section provides for each sector the impact on that sector's average elasticity of substitution (and the % change for the average) for a unit increase in the price ratio, say from a ratio of 2:1 to 3:1. This provides an indication of the extent to which elasticities of substitution larger for higher peak prices than for lower ones.<sup>31</sup> A positive percentage change indicates that price response increases as the peak price increases; a negative percentage change indicates that price responsiveness falls as the peak price increases.

The commercial/retail and government/education sectors both exhibit increased price responsiveness at higher peak prices: the former increases by 14.8% and the latter by 13.4% in response to a 50% increase in the peak price (holding the off-peak price constant). These firms can be expected to decrease peak usage more at very high market prices than at moderately high prices. Healthcare and public works customers, on the other hand, show the opposite result; their price response drops by 8.1% and 9.5%, respectively, as the price ratio increases in our example by 50%. The manufacturing sector's price response appears to be nearly immune to changes in the nominal peak price.<sup>32</sup>

<sup>31</sup> Although it is the price ratio that is changing, one can still use these results to infer something about changes in price responsiveness as the peak price changes, *ceteris paribus*. This is due to the fact that the price ratio can increase when the peak price increases while there is no change in the off-peak price. To illustrate, one might use an example of off-peak price at \$0.05/kwh and a peak price of \$0.10/kwh. This gives a price ratio of 2. If the peak price increases to \$0.15/kwh, then the price ratio goes from 2:1 to 3:1. The same would be true of initial prices of \$0.10/kwh off peak and \$0.20/kwh on peak, with a peak price increase up to \$0.30/kwh.

<sup>32</sup> The data in Table 8 can be interpreted in a manner similar to those in Table 7. It is encouraging that with the exception of the commercial/retail sector, the signs and the relative magnitudes of the percentage changes in the elasticities of substitution are quite similar to those from Table 7. It is important to note that the sign of the effects differ between the two samples only for the commercial/retail sector, and this is the one sector in which there is a statistically insignificant coefficient on one interaction term involving the

The remaining section of Table 7 shows the impact on the sector's average elasticity of substitution of a 10% increase in the amount of load a customer uses on a given day relative to its maximum (summer) peak demand. A positive value indicates that the ability to reduce load in response to increases in the peak to off-peak price ratio increases as the firm approaches its summer peak demand. A negative percentage change suggests the opposite – that customers are less able to reduce load in response to relative increases in peak price as they approach their peak demand.

The impact of the size of the firm's load relative to its peak usage level on the response elasticity is very small for all sectors but the government/education sector. In that sector, firms reduce their ability to respond to relative price increases by just over 3% as their load increases from 60% to 70% of their peak summer demand.<sup>33</sup>

#### C.2.1.1 Impact of Nominal Prices on Daily Elasticities

In specifying the regression models displayed in Table 6, the daily elasticities of substitution for all firms were regressed on the peak-to-off peak price ratio and the proportion of maximum demand at which the firm was operating on a given day. By specifying this equation in terms of the price ratio, we maintain consistency with the GL functional form used to estimate the elasticities of substitution. This model specification essentially constrains the size of the effect on the elasticity of substitution from a change in the peak price to be the same as for a change in the off-peak price, but one effect is the inverse of the other. In so doing, the marginal effects reported in Tables 7 and 8 in a pure sense reflect changes in the elasticities of substitution as the price ratio changes.

However, these marginal changes could also be interpreted in terms of changes in just the peak price, if one assumes that the off-peak price remained unchanged. Even though this is, from a technical point of view, also a correct interpretation, it is sometimes difficult to convey the equivalence of a change in the price ratio if it is due only to a change in the peak price. Therefore, a model that relates daily differences in the elasticities of substitution directly to the peak prices was also estimated. Results from these regressions and their marginal effects are reported for both customer samples in Tables 9, 10, and 11. The relative performances of the four models in Table 9 are very similar to those in Table 6 in the sense that those based on data for the 119 customers are slightly more robust than for the sample of 55 customers. Furthermore, the correction for heteroskedasticity improves these results as well. Therefore, it is sufficient to focus this discussion on fourth model in Table 9: the “GMM” model for the full, 119 customer sample.

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price ratio. This insignificant coefficient appears in the model based on the sub-sample of 55 firms. Thus, by being able to include more firms in this regressions, we were able to isolate this effect with greater statistical precision.

<sup>33</sup> Taken in aggregate, these results seem counterintuitive in that government/education customers are more responsive on hot days and as prices rise, but are less responsive as they approach their maximum demand. This can be rationalized by observing the lack of coincidence of high prices, hot days, and high loads for these customers - a finding that runs counter to conventional wisdom for this class of customers.

**Table 9. Estimated Daily Elasticity of Substitution Regression Results Using Nominal Peak Prices**

Variable	Survey Sub-Sample (55)				Full Sample (119)			
	GLS Model		GMM Model		GLS Model		GMM Model	
	Parm. Est.	t-Stat	Parm. Est.	t-Stat	Parm. Est.	t-Stat	Parm. Est.	t-Stat
Peak Price (PR)	-0.0034	-0.69	-0.0068	-2.36	-0.0052	-0.46	-0.0060	-4.25
Commercial/Retail * PR	0.0004	0.07	0.0000	-0.01	0.0231	1.74	0.0272	3.25
Manufacturing * PR	0.0018	0.35	0.0012	0.39	0.0024	0.20	0.0034	1.79
Gov't/Education * PR	0.0162	3.13	0.0424	2.92	0.0803	6.60	0.0886	2.40
Public Works * PR	0.0014	0.25	0.0006	0.18	0.0010	0.08	0.0014	0.83
Daily % of Max Demand (MD)	-0.0680	-13.79	-0.0708	-1.92	-0.0396	-10.81	-0.0534	-2.04
Commercial/Retail * MD	0.0621	6.55	0.0649	1.76	0.0573	8.92	0.0719	2.65
Manufacturing * MD	0.0659	11.42	0.0680	1.84	0.0373	9.22	0.0509	1.93
Health Care * MD	0.0692	3.66	0.0697	1.88	0.0388	3.54	0.0523	1.99
Public Works * MD	0.0676	9.29	0.0702	1.90	0.0394	7.31	0.0530	2.02
AR(1)	0.4348	56.04	0.4365	5.50	0.4693	88.08	0.4675	8.84
R-Squared	0.98		0.98		0.99		0.99	
Dubin Watson Statistic	2.14		2.14		2.13		2.13	
White's Test Statistic	5,516				14,012			

In comparing this model with the corresponding one in Table 6, we see that the signs on the corresponding terms are the same across both. Since the peak price is in the numerator of the price ratio, the direction of change in the elasticity of substitution is the same for a change in each corresponding variable containing a price term. The same is true for each of the variables containing a “Max Demand” component. It is only the magnitude of the effects that differ. These differences are best seen by comparing the results from Tables 7 and 10.<sup>34</sup> The differences in the magnitudes of the effects of the “Max Demand” variables across models are very small. However, this is not true for the variables containing the price ratio (from Table 7) and the peak price (from Table 10). Although the direction of the change is the same, the magnitudes of the effects are generally smaller in the model that includes only the peak price (Table 10).

**Table 10. Marginal Changes in Elasticities of Substitution for Peak Price Regression: 119 Customer Sample**

Business Class	No. of Accounts	Average Elasticity	Elasticity of Substitution for a Marginal Change in:			
			Peak Price <sup>a</sup>		Proportion Use of Max Demand <sup>b</sup>	
			Elasticity	% Change from Avg.	Elasticity	% Change from Avg.
Commercial/Retail	17	0.115	0.117	1.8%	0.117	1.6%
Gov't/Education	34	0.159	0.241	5.2%	0.154	-3.4%
Health Care	8	0.035	0.029	-1.7%	0.035	-0.3%
Manufacturing	44	0.087	0.084	-0.3%	0.087	-0.3%
Public Works	16	0.018	0.014	-2.5%	0.018	-0.2%
<b>Total</b>	<b>119</b>					

<sup>a</sup> A marginal change in the peak price from say \$0.20/kWh to \$0.30/kWh

<sup>b</sup> A marginal change is use increasing from say 0.6 of maximum demand to 0.7 of maximum demand

<sup>34</sup> Similar comparisons, for the sub-sample of 55 customers, can be made by examining the results in Tables 8 and 11. Since these results are included primarily for purposes of completeness, there is no need to discuss them in detail.

**Table 11. Marginal Changes in Elasticities of Substitution for Peak Price Regression: 55 Customer Sub-Sample**

Business Class	No. of Accounts	Average Elasticity	Elasticity of Substitution for a Marginal Change in:			
			Peak Price <sup>a</sup>		Proportion Use of Max Demand <sup>b</sup>	
			Elasticity	% Change from Avg.	Elasticity	% Change from Avg.
Commercial/Retail	8	0.054	0.053	-1.3%	0.054	-1.1%
Gov't/Education	16	0.181	0.185	2.0%	0.174	-3.9%
Health Care	2	0.052	0.051	-1.3%	0.052	-0.2%
Manufacturing	23	0.060	0.059	-0.9%	0.059	-0.5%
Public Works	6	0.021	0.020	-2.9%	0.021	-0.3%
<b>Total</b>	<b>55</b>					

<sup>a</sup> A marginal change in the peak price from say \$0.20/kWh to \$0.30/kWh.

<sup>b</sup> A marginal change is use increasing from say 0.6 of maximum demand to 0.7 of maximum demand

There are several possible explanations for this result, but the most plausible is that the models in Table 9 are actually mis-specified. Since peak and off-peak electricity use can be viewed as substitute inputs in production processes, economic theory suggests that any model that attempts to explain relative changes in electricity use between these two inputs should include the prices of both inputs. We find that when the peak price rises, off-peak prices also rise, but by a smaller amount. Because of this correlation, the coefficients on the variables involving peak price terms in the models in Table 9 are biased because the models exclude a term that is correlated with peak price.<sup>35</sup> Thus the effect of the change in the peak price reflected in Tables 7 and 8 dominates the offsetting effect of a relatively smaller increase in the off-peak price. Because there is no way to control for the relative changes in peak to off-peak prices in the model in Table 9, the true effect of changes in the peak price is somewhat understated. For these reasons, we argue that the correct model specifications and measures of marginal effects on the elasticities of substitution are in Tables 6, 7 and 8, even though the interpretation of the results in terms of changes in the peak price are not as straightforward as in the models in Tables 9, 10, and 11.

### C.2.3 Factors Affecting Customer-Average Elasticities of Substitution

In the third empirical model, we identify key customer characteristics that affect average elasticities of substitution by firm.<sup>36</sup> We do so by regressing these average elasticities on a variety of variables that describe customer circumstances. Since a number of these variables were obtained through the survey, the sample of firms included in this analysis is limited to the 55 customers that provided answers to the appropriate survey questions. A comparison of this subset of firms with the 119 used to estimate the other two equations indicates that, with the exception of healthcare and public works, other sectors are proportionally represented both in terms of customer numbers and maximum demand.

<sup>35</sup> See Goldberger 1990, p. 189-90, for a discussion of omitted variable bias.

<sup>36</sup> See Table 5 for the definitions of the variables.

Table 12 presents the estimated parameters for the third equation designed to identify systematic differences in average elasticities across firms associated with firm characteristics and circumstances. The  $R^2$  indicates that the explanatory variables included in this equation account for about a third of the variation in the average elasticities of substitution for the 55 firms. White's statistic indicates that the errors terms do not exhibit heteroskedasticity, so no correction was required.

**Table 12. Customer-Level Elasticities of Substitution Regression Results**

Variable	Parm. Est.	t-Stat
Intercept	0.1976	0.88
Commercial/Retail	0.1640	1.34
Manufacturing	0.0155	0.17
Gov't/Education	0.1227	1.09
Health Care	0.0590	0.37
EDRP Participant	0.1794	2.53
SCR Participant	-0.0610	-0.63
Installed EMCS	-0.1489	-2.46
Monitor RTP Frequently	0.0579	0.52
Installed On-Site Gen	0.0262	0.45
Increased kWh	0.0811	1.34
Avg. Pk. Off-Pk. Load Ratio	-1.3114	-0.94
R-Squared	0.31	
White's Test Statistic	52	0.10

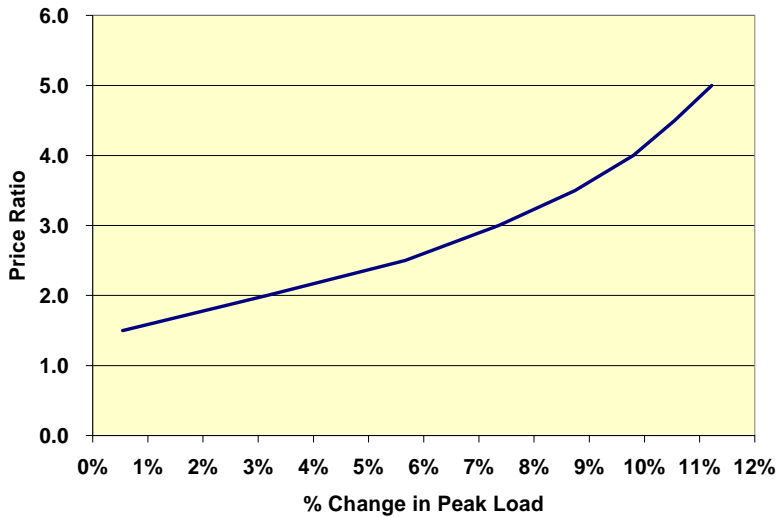
In general, the estimated parameters values do yield some insight into the factors that explain differences in customers' average ability to reduce peak load in response to price. While many of the coefficients are of the expected sign, only two effects are statistically different from zero ( $t > 2$ ). Rather than suggesting that these other factors have no effect, the low t-statistics could also be due to the relatively small sample size and the resulting limited variation in the level of the explanatory variables. In this case, the low t-ratio means that we are measuring the effect with very little precision, a situation that might well have been avoided if these variables had been available for all 119 customers.

The estimated coefficient for the EMCS variable is negative, indicating that firms with these systems are less able to shift load in response to higher relative peak prices, on average, than firms that do not have them. This result, has been consistent throughout this study, and comports with previous studies of price response among customers throughout New York participating in NYISO demand response programs.

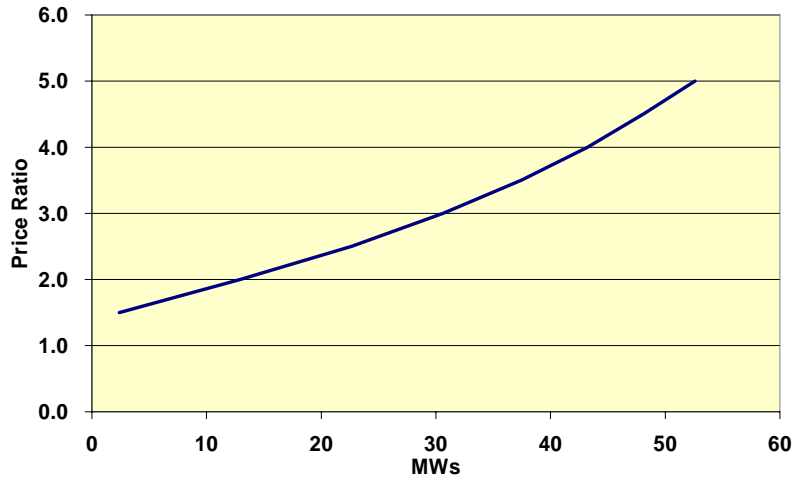
The coefficients on the variables for participation in NYISO's two demand response programs are of different signs. The coefficient for participation in the Emergency Demand Response Program is positive, and significant, indicating that such participation results in a higher average ability to be price responsive. This would seem to be an intuitively correct result. But the estimated coefficient on the ICAP/SCR participation variable is negative, and insignificant. In other words, the specification finds no (or at best a weak) relationship between the imposition of a high penalty prices and price response in the case of SCR, in contrast to the EDRP case. This counter-intuitive result is discussed in more detail in Chapter 3 of this report.

### C.3. Aggregate Load Response

To obtain some sense of the overall impact on the shift in peak to off-peak usage at various peak to off-peak electricity price ratios, the elasticities of substitution for individual firms were used to simulate the peak load reduction as the price ratio changes (see Appendix B for more details, particularly the discussion surrounding equation (35)). These results are illustrated in Figures 6 and 7.



**Figure 6. Percentage Reduction in 119 SC-3A Customers' Peak Demand**



**Figure 7. Reduction in 119 SC-3A Customers' Peak Demand**

At the highest peak to off-peak price ratio observed in the SC-3A price data – 5:1 – the 119 modeled customers are estimated to reduce their peak-period usage by about 50 MW, a 10% reduction from their typical usage. SC-3A customers' aggregate load response is non-linear – it increases as the price ratio increases but at a decreasing rate, especially at



ratios above 3:1. This occurs primarily because the relationship between price ratios and the elasticity of substitution is negative for ~57% of the customers (see Table 7). As the price ratio increases, the elasticity of substitution decreases modestly among manufacturing, healthcare, and public works customers. The overall level of load response therefore increases for higher price ratios, but the rate of change for higher and higher price ratios becomes smaller and smaller.