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MODELING PASSIVE SOLAR BUILDINGS WITH HAND CALCULATIONS

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ABSTRACT

Passive solar design can be encouraged by a better theoretical understanding of the performance of passive solar buildings and the ability to predict the thermal response of various designs. But existing public-domain computer programs do not yet handle solar gains precisely and are inaccurate in modeling buildings with large solar gains. Even when they are revised to properly describe solar effects, they may still fail to provide insight into the thermally important features of the building.

To address these problems, we derive an analytic model of passive solar building performance. We use heat balances on the surfaces of materials that absorb sunlight (e.g., the inside surface of a mass wall or concrete floor), along with solutions to the diffusion equation, to derive response functions for surface temperature as a function of solar flux and ambient temperature. These expressions are combined to form building response functions. These explicit building response functions allow one to write relatively simple, analytic expressions for room temperature as a function of time over the course of a design day in terms of ambient temperature, sunlight, and heater output (if any).

Parallels between our analytic model and computer codes can be exploited to provide a better intuitive understanding of the programs and to assist in the incorporation of accurate passive solar simulation into these codes.

1. INTRODUCTION

A successful passive solar building will maintain comfortable room temperatures without the use of heater output, by collecting and storing solar energy. The size of the collector glazing and the amount, location, and properties of thermal mass will determine the room temperature the house provides in any given climate; these house parameters can be adjusted to give a desired performance. But such adjustment or optimization is difficult at present because of the lack of a simple method to predict the response of a building in the design stage.

Existing passive solar buildings have a fairly wide range of responses to their climates: some remain at or above 70°F most of the winter (1), others frequently drop into the low 60's F (2) (while maintaining conditions described by their occupants as "comfortable), and some designs regularly heat up to 90°F or higher (3). Whatever the range of temperatures considered acceptable, the designer requires a method of predicting room temperature to be able to select the response by means of the building parameters at his disposal. If an initial design is unacceptable, he can then see the results of using various alternative designs.

In this paper, we describe a hand-calculation model which can predict building response from design parameters and climate (design-day) data. While relatively simple compared to computer codes like DOE-1 (4) or NBSLD (5), it may still be too complex for designer use at present. However, it can be written as a relatively simple hand-calculator program, and can be employed to develop simple rules of thumb.

The model is best used to calculate room temperature as a function of time of day for simple passive buildings with free-floating (nonthermostated) temperature. Direct-gain and unmanaged Trombe-wall or waterwall buildings can be modeled. Response to multi-day weather cycles can be easily calculated for unmanaged buildings. The model has been applied to the Los Alamos Scientific Laboratory (LASL) passive test cells; the results are described below and are summarized in Figures 3 through 7.

At present, the hand calculations are more accurate than the major public domain computer programs in their descriptions of passive solar buildings. This occurs because the computer models generally fail to look at the distribution of solar energy within the room. That is, they calculate the solar gain through the window correctly, but then spread this heat gain out over all the surfaces of materials on the inside of the room. In contrast, the hand calculations allow the user to specify the proportions of solar heat absorbed on each surface. The improvement in accuracy that this approach yields is illustrated in Figure 1, which shows the results of four different

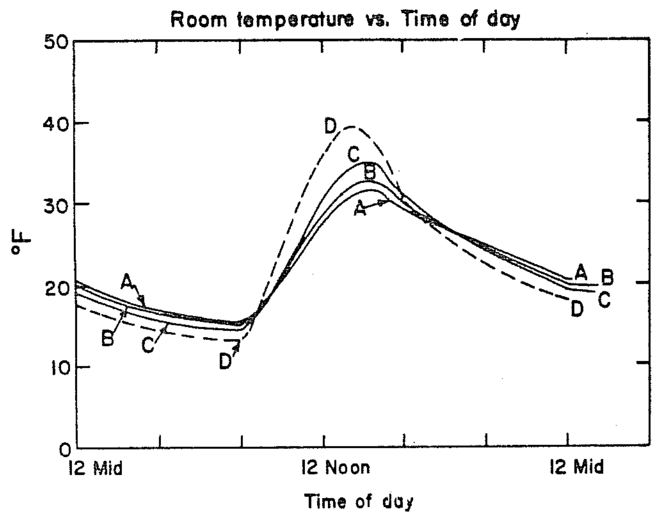

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Fig. 1. Room temperature vs. time of day for a passive solar house under four different cases of solar energy absorption. In case A, 6/7 of the solar energy is absorbed on the floor and 1/7 is absorbed on the walls. Cases B and C successively double the solar absorption on the walls and decrease the absorption on the floor. In case D all sunlight is absorbed in the air or on light furniture or carpets. All temperatures are measured with respect to average ambient temperature.

assumptions about where sunshine is absorbed in a direct-gain building. As seen in the figure, the different assumptions produce substantially different results. In contrast, the response is very insensitive to where sunshine is absorbed over a given surface.

This passive solar building model was developed as part of a project on analytic building calculations at Lawrence Berkeley Laboratory. The project was initiated by Sam Berman at LBL and Robert Richardson of New York University.

2. DERIVATION OF THE MODEL

Solar energy entering a direct-gain building passes through the room air until it reaches a surface (e.g., a floor). When it strikes the surface, some fraction is absorbed and the rest is reflected. The reflected component eventually is absorbed on some other surface. The fraction of sunlight which is absorbed on a given surface depends on the sun angles and room geometry in a very complex way. We assume that the internal solar radiation balance is already known, either from direct measurement or through simulation (6); and that the amount of sunlight absorbed on a surface "j" is given by α_j S, where S is the total amount of sunlight entering the building (in watts or Btu hr^{-1}).

For each surface "j", the surface Temperature T_{s_j} is determined by a heat balance at the surface, which can be expressed as:

$$h_j A_j (T_{s_j} - T_R) - A_j K_j \left. \frac{\partial T_j(x,t)}{\partial x} \right|_{x=0} = \alpha_j S \qquad (1)$$

where

h_j is the combined radiation/convection film heat transfer coefficient for the j^{th} surface (watts m^{-2} °C^{-1} or Btu hr^{-1} ft^{-2}°F^{-1}),

A_j is the area of the surface (m^2 or ft^2),

T_R is the room temperature

$T_j(x,t)$ is the temperature distribution within the j^{th} material,

K_j is the conductivity of the j^{th} material (watts m^{-1}°C^{-1} or Btu $hr^{-1}ft^{-1}$°F^{-1}),

α_j is the fraction of sunlight absorbed on the j^{th} surface,

x is the distance into the material.

This equation sets heat losses from the surface (left-hand side of (1)) equal to heat gains (right-hand side). It assumes that the surface transfers heat directly to the room air, rather than being in radiative contact with other surfaces, which results in a substantial simplification of the computational effort (7).

Heat-flows within a material satisfy the diffusion equation:

$$K_j \frac{\partial^2 T_j(x,t)}{\partial x^2} = (\rho c)_j \frac{\partial T_j(x,t)}{\partial t} \qquad (2)$$

where $(\rho c)_j$ is the heat capacity per unit volume of the j^{th} material (Joules °$C^{-1}m^{-3}$ or Btu °F^{-1} ft^{-3}). If the material consists of several layers, a separate diffusion equation is needed for each layer; but in practice, all layers beyond the first (inside) layer can usually be modeled as pure resistances.

Equations (1) and (2) describe heat-flows at the inside surface of a material and in its interior; at the outside surface we assume that the material is coupled to the ambient air (at temperature T_A) by a pure resistance which can be described by a heat transfer coefficient U_r. (For an uninsulated material this coefficient is just equal to an exterior surface film coefficient).

This description allows the solution for surface temperature in terms of the driving forces of solar gain and ambient temperature. This solution can be written in simple form if we look at the amplitudes of temperature (and solar) fluctuations at a steady harmonic frequency. The result can be expressed as:

$$T_{s_j} = (h_j T_R + \alpha_j\ S/A_j)\ R_{1j} + T_A R_{2j} \qquad (3)$$

where R_{1j} and R_{2j} are frequency-dependent response functions whose forms are given in Table 1. These response functions give all the information needed

Table 1: Equations for Response Functions

Materials Response Functions R_1 & R_2:

$$R_1(\omega) = \frac{\cosh kd + \frac{1}{RKk} \sinh kd}{(h + \frac{1}{R}) \cosh kd + (Kk + \frac{h}{RKk}) \sinh kd}$$

$$R_2(\omega) = \frac{\frac{1}{R}}{(h + \frac{1}{R}) \cosh kd + Kk + \frac{h}{RKk} \sinh kd}$$

where

K is the thermal conductivity of the material,

d is the thickness of the material,

R is the thermal resistance of the exterior insulating layer, ($R = U_r^{-1}$)

$k = \sqrt{\frac{i\omega\rho c}{K}}$ with ρc = the volumetric heat capacity of the material,

h is the film heat transfer coefficient for the surface.

Building Response Functions A, B, and C:

$$A(\omega) = \sum_{j=1}^{N} \hat{h}_j (1 - h_j R_{1j}) + \hat{U}_q$$

$$B(\omega) = \sum_{j=1}^{N} \alpha_j h_j R_{1j} + \alpha_R$$

$$C(\omega) = \sum_{j=1}^{N} \hat{h}_j R_{2j} + \hat{U}_q$$

where $\hat{U}_q$ is the sum of products of U-values times areas for all light materials, plus the infiltration loss rate.

to describe the behavior of a material. The R_{1j} function is generally more interesting because it describes the response to sunlight, which is most important in a passive solar building. R_1 is plotted as a function of frequency for a sample material in Figure 2; it is largest at low frequencies, and declines steadily with ω.

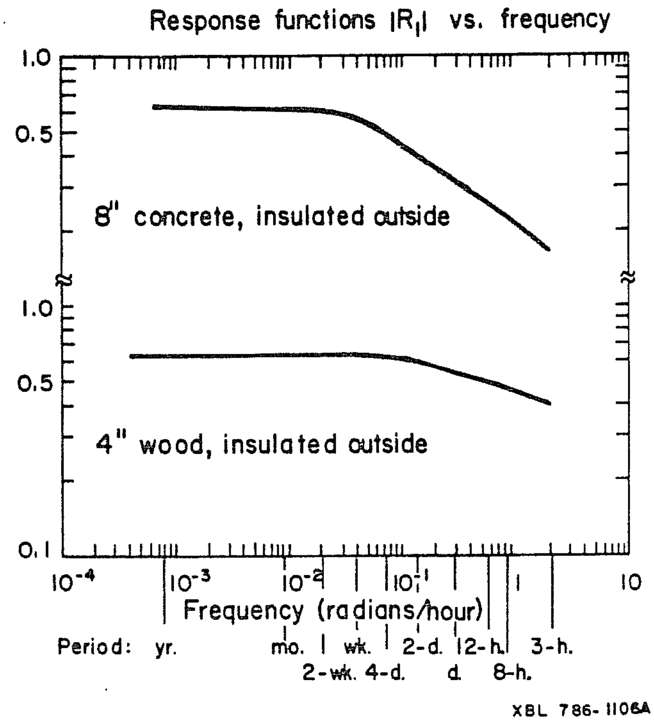

Fig. 2. Response functions as a function of frequency. Log $|R_1|$ is plotted vs. log ω for two materials with insulation of R-8 (8 ft^2-hr-°F/Btu) on the outside.

This shape means that the surface temperature has a large response to sunlight at low frequencies and a smaller response at higher frequencies. A good response function for a passive solar house would begin to decrease at frequencies comparable to weather changes (e.g., one cycle every week or two) and would be very small at diurnal frequency to damp out day-to-night changes in solar gain.

We combine these surface temperature results into an expression for room temperature using a heat balance for the room air. This is given by:

$$\sum_{j=1}^{N} \hat{h}_j (T_R - T_{s_j}) + \hat{U}_q (T_R - T_A) = H + \alpha_R\ S \qquad (4)$$

where

$\hat{h}_j = h_j\ A_j$,

H is the heater output,

α_R is the fraction of sunlight absorbed directly into the room air or on the surfaces of light objects (e.g., upholstery),

$\hat{U}_q$ is the quick heat transfer coefficient; the sum of U-values times areas for all pure conductances (e.g., windows) plus the loss rate due to infiltration.

This heat balance says that heat losses from the room air to material surfaces or losses directly to the outside air are equal to heat gains from the heater or from solar absorption on light material surfaces (which conduct immediately into the room air).

We can use equations (4) and (3) to derive the room temperature; at any frequency, the amplitude of room temperature is given by:

$$T_R \cdot A(\omega) = S \cdot B(\omega) + T_A \cdot C(\omega) + H \qquad (5)$$

where

$$A(\omega) = \sum_{j=1}^{N} \hat{h}_j \,(1-h_j R_{1j}) + \hat{U}_q \qquad (6a)$$

$$B(\omega) = \sum_{j=1}^{N} \alpha_j h_j R_{1j} + \alpha_R \qquad (6b)$$

$$C(\omega) = \sum_{j=1}^{N} \hat{h}_j R_{2j} + \hat{U}_q \qquad (6c)$$

The solution for room temperature as a function of time for a design day is given in Table 2.

The building response functions A, B, and C describe the reponse of the whole building to the driving forces of sunlight, ambient temperature, and heater output. The form of these coefficients can be understood fairly simply.

$A(\omega)$ and $C(\omega)$ are analogous to conventional design heat losses. In fact, for a frequency of $\omega=0$ (the steady-state term), $\hat{h}_j(1-h_jR_{1j})$ is equal to U_jA_j where U_j is the conventional U-value of the material. For $\omega=0$, $\hat{h}_jR_{2j}$ is also equal to U_jA_j. Thus, $A(\omega)$ and $C(\omega)$ are the conventional design heat losses for steady-state results, and are simply frequency-dependent heat transfer coefficients for

Table 2: Room Temperature as a Function of Time

Let $T_A(t) = \overline{T}_A + \Delta T_A \, e^{i\omega_0 t}$

where $\omega_0 = 2\pi$ radians day^{-1}

$$S(t) = \begin{cases} S_1 \sin\omega_1 t & \text{day} \\ 0 & \text{night} \end{cases}$$

Then $S(t) = \sum_n d_n e^{in\omega_0 t}$

$$T_R(t) = S_1 \sum_{n=0} \frac{B(n\omega_0)}{A(n\omega_0)} d_n e^{in\omega_0 t} + \overline{T}_A + \frac{C(\omega_0)}{A(\omega_0)} \Delta T_A e^{i\omega_0 t} + \frac{1}{A(0)} H(0)$$

$\omega \neq 0$. The solar coefficient is like a collector efficiency for the windows: it is a dimensionless number roughly equal to 1 for a direct gain house at zero frequency, and it decreases for higher frequencies. $B(\omega)$ completely describes the reduction in room temperature fluctuations due to locating thermal mass in the sun, since it is the only term which uses the radiation balance fractions α_j.

The response to sunlight is given by $B(\omega)/A(\omega)$. A good passive solar house will have B/A large for low frequencies but small for daily frequencies. B/A will decrease with frequency for any building, since B is composed of response functions which get smaller with frequency and A is made up of functions $(\hat{h}_j(1-h_jR_{1j}))$ which get larger with frequency. Typically, $A(\omega)$ begins to increase significantly before $B(\omega)$ decreases much, so the longest-term storage should be relatively insensitive to where the sunlight falls within the building.

The description above applies to a direct gain building, but adaptation to a Trombe wall is straightforward. The result is that (5) is still valid, but (6) changes form to reflect the changes in internal heat balance in a Trombe wall structure. The Trombe wall response functions are the same as those described here.

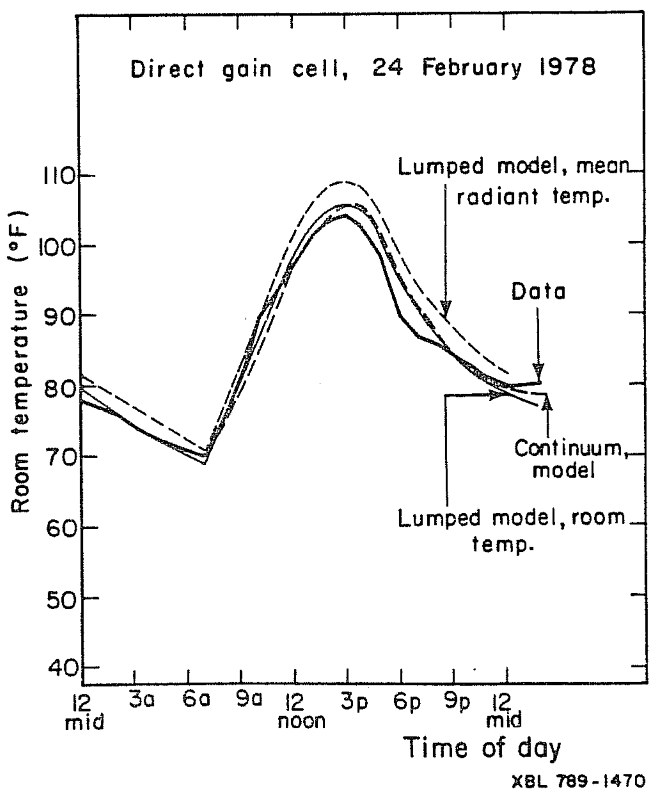

Fig. 3. Predicted room temperature and observed data as a function of time of day for the LASL direct gain cell for 24 February 1978.

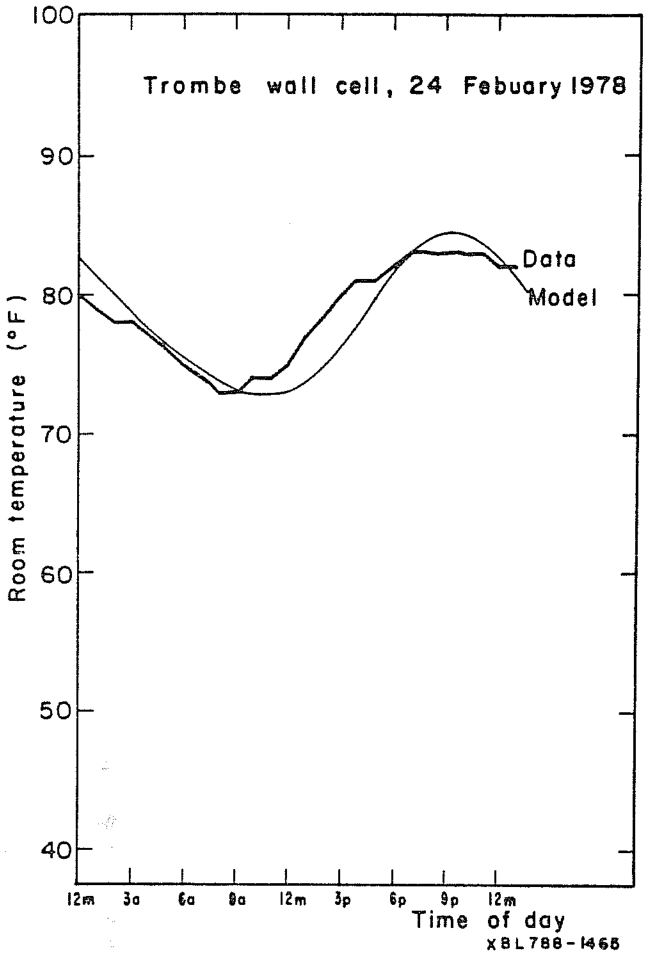

Fig. 4. Predicted room temperature and observed data as a function of time of day for the LASL Trombe wall cell, 24 February 1978.

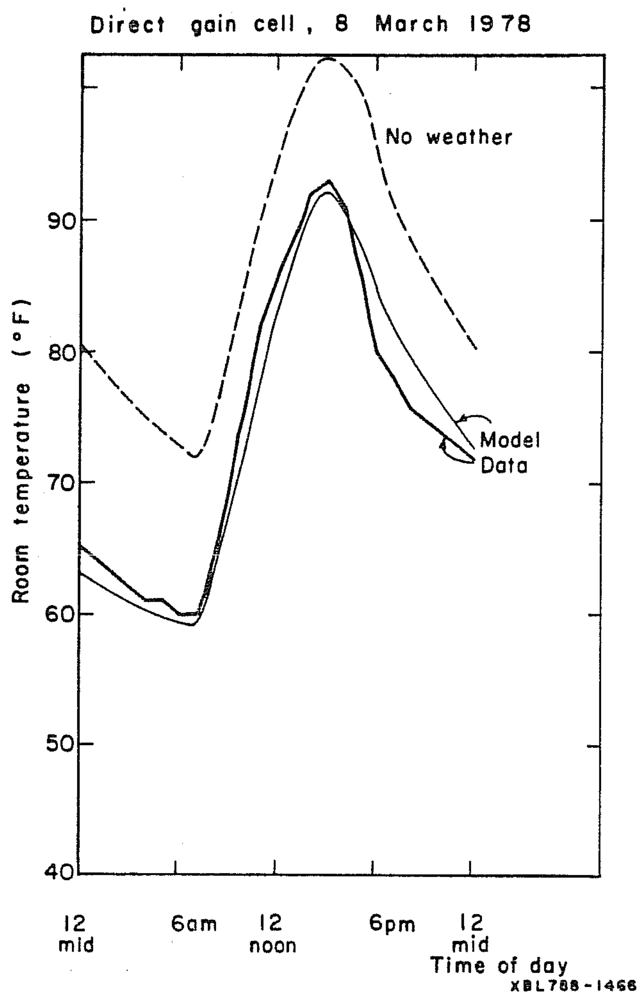

Fig. 5. Predicted room temperature and observed data for the direct gain cell as a function of time of day on March 8, 1978. The curve labelled "model" was calculated taking into account the weather variations for the previous two weeks. The curve labelled "no weather" shows the predictions of the model for the case where all days before March 8 were assumed to have the same weather.

The method above is applicable only to unmanaged buildings (that is, those without night insulation). A lumped parameter approximation can be made which allows the treatment of managed buildings (8).

3. EXPERIMENTAL RESULTS

These models have been applied to the Los Alamos passive solar test cells (9). We used handbook values for materials properties and measured the dimensions of the cells to derive most of the building parameters. For the concrete thermal properties, we used the measurement of conductivity described by Bob McFarland at LASL. Heat capacity was determined using data from thermocouples buried within the concrete. Weather data were recorded by LASL.

The results are shown in Figures 3-7. Figures 3-4 describe the response to a day in which weather patterns had been constant for a long time. Figures 5 and 6 describe the cells' performance on a day when the past two weeks' weather had been varying harmonically. The dashed lines on these figures show what the predicted response would have been if we had ignored previous days' weather.

As seen in the figures, agreement is excellent, with less than 10% error at all hours of the day. Beyond 5 or 10%, the accuracy is limited by data uncertainties, such as infiltration rates, net transmissivity of the collector windows, etc.

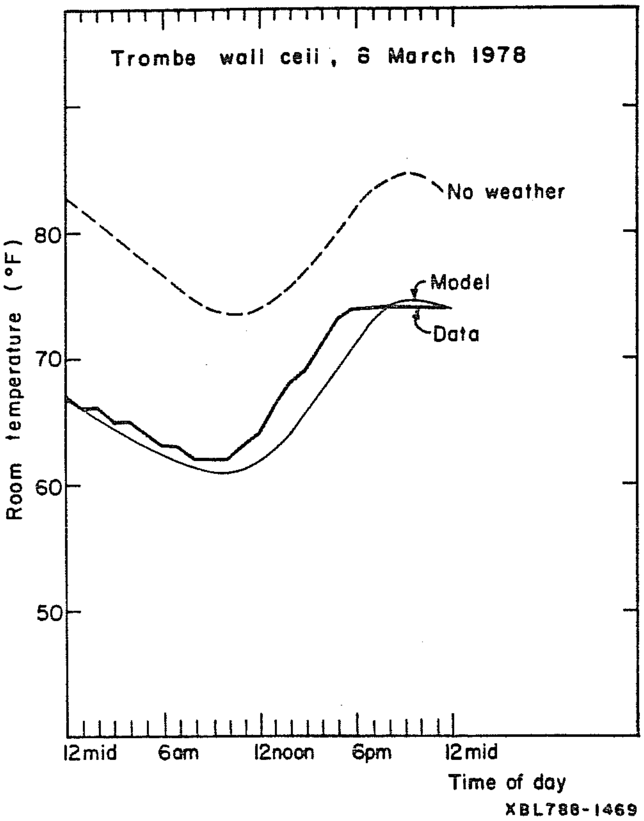

Fig. 6. Predicted room temperature and observed data as a function of time of day for the LASL Trombe wall cell, 8 March 1978. The curve labelled "no weather" was calculated by assuming that all days before March 8 had the same weather; the curve labelled "model" accounts for the previous two weeks' weather.

The models described in this paper are potentially usable as a design tool for new buildings. They can at present be evaluated in about 1/2 hour using a card-reading hand calculator: more efficient routines can reduce the calculational effort.

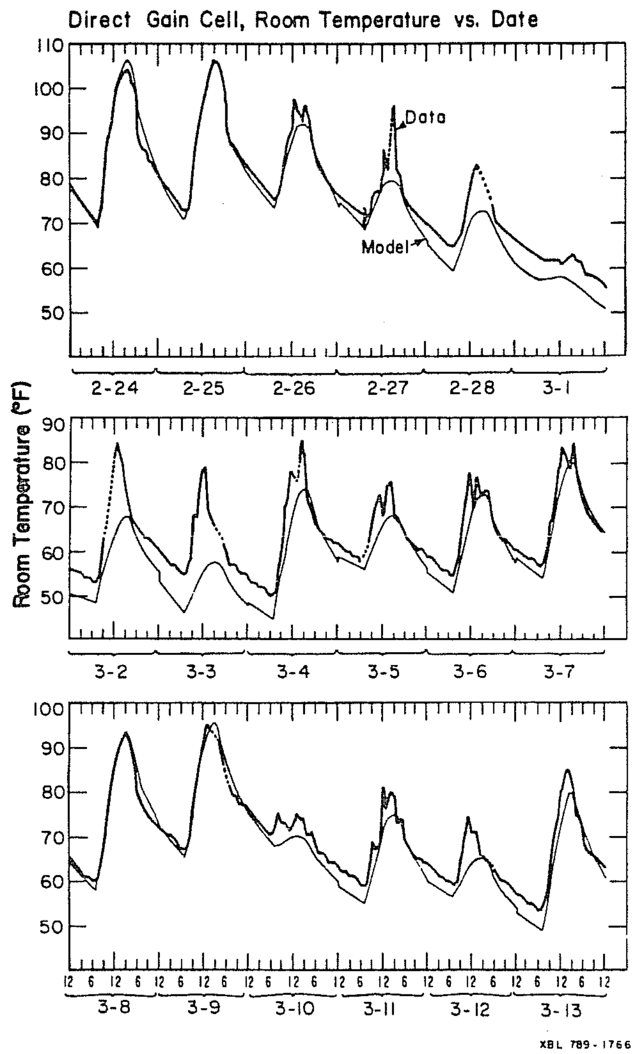

Fig. 7. Comparison between room temperature observations and predictions of the lumped parameter approximation for the direct gain cell. Dotted portions of the "data" curve indicate where data is missing. Model input data, such as solar flux, is also missing at these times; interpolations were used for the calculations.

4. REFERENCES

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8) D.B. Goldstein, op. cit., Section 2.3.

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