SOLAR HEAT GAIN THROUGH FENESTRATION SYSTEMS CONTAINING SHADING: PROCEDURES FOR ESTIMATING PERFORMANCE FROM MINIMAL DATA

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Abstract

The computational methods for calculating the properties of glazing systems containing shading from the properties of their components have been developed, but the measurement standards and property data bases necessary to apply them have not. It is shown that with a drastic simplifying assumption these methods can be used to calculate system solar-optical properties and solar heat gain coefficients for arbitrary glazing systems, while requiring limited data about the shading. Detailed formulas are presented, and performance multipliers are defined for the approximate treatment of simple glazings with shading. As higher accuracy is demanded, the formulas become very complicated.

INTRODUCTION

Several years ago ASHRAE sponsored a research project, 548-RP, to develop a method of determining the solar heat gain coefficient for complex fenestration systems, i.e., those containing non-specular (and presumably geometrically complex) sun-control or visibility elements, such as shades, venetian blinds, or translucent glazings. These systems had been characterized using the shading coefficient, a concept developed when only clear single glazing was common. The basic idea behind the shading coefficient was that for most systems the chief dependence of the solar transmission on wavelength and incident angle came from the glass layer, and the reflectance and absorptance of this layer were not large. It was therefore possible to treat shading systems as providing a modifying factor to the single glass transmittance, and this factor being assumed independent of incident angle, it could be determined by a single measurement in a calorimeter.

As multiple glazings and coated glasses were introduced, the basic assumption of the shading coefficient ceased to be valid, and it was gradually replaced with the solar heat gain coefficient (SHGC), which is a quantity with an explicit dependence on incident angle. For "simple" glazing systems (those consisting only of layers of unshaded glass), the SHGC can be calculated from a simple formula,

$$SHGC(\theta) = T(\theta) + \sum_{k=1}^{L} N_k \cdot \mathcal{A}_k(\theta), \qquad (1)$$

where T represents the overall solar-optical transmittance of the system (at the incident angle θ) and $\mathcal{A}_k$ represents the solar-optical absorptance for each of the glazing layers (assumed to be L in number). The *layer inward-flowing fractions*, N_k, represent the fraction of the energy absorbed in each layer that reaches the interior space. The system transmittance and layer absorptances can be calculated from optical measurements made on the individual glass layers. These are generally made with a spectrophotometer. Systematic collections of such properties have been compiled. (Windows and Daylighting Group, 2001) The layer inward-flowing fractions can be calculated from heat transfer theory, utilizing measured thermal transmittance coefficients for the gas spaces between the glazings. Standardized computer codes exist to derive the solar heat gain coefficient from the spectrophotometric data. (Windows and Daylighting Group 1994; Wright 1995; Wright 1998)

Equation 1 and the associated calculation methodologies represent a very significant simplification of the problem of characterizing glazing systems. This can be seen from the following argument. If the number of glazing products from which glazing systems may be composed is G, then the number of possible double glazed systems is G^2, the number of possible 3-layer systems is G^3, etc. If one needs to characterize each glazing system by measurement, there is a much larger number of characterizations to be made than if one needs only to characterize the individual glazing layers. This is known as the "combinatorial problem". And the number G is not small. At present there are more than 900 products listed in the WINDOW-4.1 Spectral Data Library. While the differences between many of these products are small, even the type of generic characterization that ASHRAE favors could scarcely be expected to use a number for G smaller than 20. So even though wavelength-by-wavelength measurements are needed to calculate the T and $\mathcal{A}$ in equation 1, there is still a great savings in labor from not needing to measure all possible combinations.

If one now considers the problem of characterizing glazing systems with shading layers, and if the number of possible shading products is S, then just the fact that double glazing systems are common means that one must consider three-layer systems, and the number of possible systems becomes $3G^2S$ (assuming only one shading layer, which can be in any of three locations). But any type of shading product is likely to come in a very large number of colors and patterns. The author is aware of a venetian blind manufacturer that offers nearly one hundred color options for a single line of venetian blinds. Even assuming the (unrealistically) modest values of G=20, S=10 yields over 12,000 possible combinations. One sees how far we have departed from the era when the shading coefficient was developed, in which G was one, the number of possible shaded systems was 2S (interior and exterior shading), and S itself was fairly small.

Equation 1 cannot be used for glazing systems that contain a shading element because the reflectance and transmittance of such an element is not specular: incoming radiation from a particular direction produces transmitted or reflected radiation in many directions. This means that the only method of determining the solar heat gain coefficient for a glazing with shading was the procedure originally used to determine shading coefficient, measurement of the entire system in a calorimeter. But now measurements would need to be made for a number of different incident directions on each system. The difficulty and expense of doing these measurements, coupled with the combinatorial problem described above made it highly desirable to find a method of characterizing glazings with shading elements that was more consonant with Equation 1. (These systems were termed "complex glazing systems", because the shading elements add optical and geometric complexity.) This was the task of 548-RP.

The research project began by defining an analog of Equation 1 applicable to complex glazing systems:

$$SHGC(\theta, \phi) \,=\, T(\theta, \phi) \,+\, \sum_{k=1}^{L} N_k \cdot \mathcal{A}_k(\theta, \phi) \tag{2}$$

where now ϕ, the azimuthal angle of the incident direction enters the expressions for the solar-optical quantities, since complex glazing systems are not necessarily symmetric about the normal to the glazing plane (e.g., glazings with venetian blinds). Here L is now the total number of layers, including shading layers. That such an expression exists follows from basic physical principles, energy conservation and superposition; the problem lay in finding a way to determine T, $\mathcal{A}_k$ and N_k. The research project was able to do this, and in its several publications (Klems 1994A; Klems 1994B; Klems and Warner 1995; Klems, Warner et al. 1995; Klems and Kelley 1996; Klems, Warner et al. 1996) it set forth these methods and also verified the resulting expression for Equation 2 by calorimetric measurements on a double-glazed system with a venetian blind.

Many shading layers have an inhomogeneous construction that repeats in one or two directions; for example, drapes, woven shades, sun screens, and venetian blinds. The scale of this inhomogeneity varies. In a woven shade the weave provides a repeating rectangular lattice with a scale of a fraction of a millimeter. A venetian blind is a repeating assembly of slats along one direction, with a repeat dimension on the order of a few centimeters. A sun screen sometimes consists of either (or both) horizontal or vertical slats with a dimension of a few millimeters, while a drape has the rectangular weave of the shade with a superimposed and much larger linear inhomogeneity caused by folds, which are not strictly regular. All of the systems have irregularities stemming from broad manufacturing tolerances; they are not precision optical components.

This led the researchers in 548-RP to characterize shading elements by their spatially averaged optical properties. The averaging is done over a spatial region that is large compared with the features of the shading element, e.g., the slat width for a venetian blind. This allowed them to replace the shading systems with spatially uniform layer averages, and to do calculations using only the bidirectional solar-optical properties of the average layers (and the specular glazing layers). While they measured only the spectrally averaged layer properties, using a large-sample gonioradiometer, it was clear

that the method could be extended to gonio-spectrometric measurements, and could also be extended to include polarization. (A consistent treatment of polarization has not yet even been included in the treatment of multilayer specular glazings.) They also developed a calorimetric method for measuring the layer-specific inward-flowing fractions, N_k. This was necessary because the heat transfer coefficients necessary to do a heat transfer analysis, as is done for Equation 1, have not been determined for the thermal and geometrical situations that occur for most combinations of shading and glazing layers.

In principle, then, the stage was set for treating the solar heat gain through complex glazings in the same way as unshaded glazings. Detailed compilations of shading layer properties (like the WINDOW 4.1 Spectral Data Library for specular glazings) could be used, together with specular glazing properties and tabulated measurements of layer inward-flowing fractions, as inputs to standard calculation programs that would produce the T and $\mathcal{A}_k$ functions needed in Equation 2. Unfortunately, since the conclusion of 548-RP the necessary standardized instrumentation for measuring the shading layer properties has not been developed, the library of bidirectional optical properties has not been compiled, and the small table of inward-flowing fractions produced by the project (which is currently in the ASHRAE Handbook) has not been much extended, although some work in the latter direction has occurred. (Collins and Harrison 1999) As a result, there has been no way to incorporate the outcome of this research project into the ASHRAE Handbook of Fundamentals in a way that would enable designers to predict the solar heat gain of shaded fenestrations.

This paper is intended to bridge the gap temporarily, by substituting simple assumptions for the missing data on shading layer bidirectional solar-optical properties. It is not argued that these assumptions provide accurate representations of shading layer properties, although for some shading elements this will be the case. Rather, the assumptions allow the use of an accurate characterization of the unshaded glazing layers occurring in the system. Often the angular dependence of the specular glazings is a much more important determinant of the SHGC of a glazing with shading than the specific properties of the shading layer. In these cases even a very poor characterization of the latter can yield a useful result.

CHARACTERIZING MULTILAYER GLAZING SYSTEMS WITH SHADING

Problem Description

In any multilayer optical system, one can visualize any collection of sequential layers as a sub-system and replace the collection by a "black-box" single layer with the same overall properties as the subsystem. This means that with complete generality one may consider any system as a two-layer system. In a system consisting of L layers, the sub-system of the first (L-1) layers can be considered to be the first "layer", while the other "layer" is the Lth layer. The properties (i.e., overall transmittance, front and back reflectance) of this system of two "layers" can then be derived as if one were dealing

with a two-layer system, and the result will be the properties of the L-layer system expressed in terms of those for the layer L and those for the subsystem consisting of layers 1 through (L-1).

We therefore consider the combination of a specular layer (an unshaded glazing system, G) and a shading layer, S. For definiteness we will consider a shading layer that is interior to G, although later we will expand this to include any position for the shading.

Let us first exclude a trivial case that would otherwise complicate the discussion. Most shading layers, being inhomogeneous, have regions for which radiation reaching the layer pass through without encountering the material of the shading at all, as illustrated in Figure 1. We shall term these regions "gaps" in the shading layer. Figure 1(a) depicts a venetian blind, for which some radiation may pass between the slats; Figure 1(b) depicts a woven material for which some of the radiation passes between the threads. (It is assumed that the distance between the threads is very large compared with the wavelength of the radiation.) In both cases, the radiation has essentially passed through an unshaded system. We divide the glazing area into a fraction, $u(\theta, \phi)$, for which the radiation may pass through the entire system without encountering any shading material, and the remainder, $1 - u(\theta, \phi)$, for which this is not the case. Then for a total glazed area A_0 the area $A_0 u(\theta, \phi)$ can be treated as an unshaded glazing and Equation 1 applied, and we can restrict our attention to the *reduced shading area* $A_G = A_0\big(1 - u(\theta, \phi)\big)$, for which the shading is involved in an essential way. (We note that interreflections between the shading layer and the glazing break down this neat separation, a point we will discuss below.)

Consider the system shown in Figure 2(a). Radiation is incident on this system from a direction $\vartheta_0 = (\theta_0, \phi_0)$, (a vector) and we consider radiation that passes through the glazing system G and strikes the shading layer at $\mathbf{x}_0$ (a position vector, $\mathbf{x}_0 = (x_0, y_0)$, assuming that the z-axis of a three-dimensional coordinate system is chosen perpendicular to the glazing plane). If the incident irradiance is E_0, then the irradiance at $\mathbf{x}_0$ will be $E_0 T_G(\theta_0)$. The total amount of radiation passing through the shading layer S and emerging in the direction $\vartheta_e = (\theta_e, \phi_e)$ will be given by

$$I_{SYS}^{(0)}(\vartheta_0; \vartheta_e) = \int E_0 T_G(\theta_0) T_S(\mathbf{x}_0, \vartheta_0; \vartheta_e) d^2\mathbf{x}_0 , \qquad (3)$$

where the superscript indicates that no interreflections between the layers have yet been included in the calculation. The quantity $T_S(\mathbf{x}_0, \vartheta_0; \vartheta_e)$ is the bidirectional front transmittance of the shading layer S at the point $\mathbf{x}_0$. [We have omitted any qualifying notation to distinguish between front and back transmittance here because in this section we will only use the front transmittance of the shading layer. Note, however, that unlike specular glazings, bidirectional transmittances are not symmetric; rather $T_S^f(\mathbf{x}, \vartheta_1; \vartheta_2) = T_S^b(\mathbf{x}, \vartheta_2; \vartheta_1)$, where the superscripts f and b denote front and back incidence.] This quantity can be converted to an expression for the system transmittance (neglecting interreflections) by dividing by the incident energy:

$$T_{SYS}^{(0)}(\vartheta_0; \vartheta_e) = \frac{1}{A_G} \int T_G(\theta_0) T_S(\mathbf{x_0}, \vartheta_0; \vartheta_e) d^2\mathbf{x_0} \qquad (3a)$$

In addition, radiation at point $\mathbf{x_0}$ may be reflected by the shading layer, re-reflected by the glazing, and re-incident on the shading layer at point $\mathbf{x_1}$, as indicated in the figure. The radiation that is then transmitted by the shading layer and emerges in the direction ϑ_e will be

$$T_{SYS}^{(1)}(\vartheta_0; \vartheta_e) = \frac{1}{A_G} \int T_G(\theta_0) \left[\int R_S^f(\mathbf{x_0}, \vartheta_0; \vartheta_1) R_G^b(\theta_1) T_S(\mathbf{x_1}, \vartheta_1; \vartheta_e) \cos(\theta_1) d\Omega_1 \right] d^2\mathbf{x_0} \qquad (4)$$

where $R_S^f(\mathbf{x_0}, \vartheta_0; \vartheta_1)$ is the front bidirectional reflectance of the shading layer at the point $\mathbf{x_0}$. Front and back reflectances may be different, since incidence from the back side in a given direction may present a completely different physical situation. Now the shading layer transmittance appearing in the expression, $T_S(\mathbf{x_1}, \vartheta_1; \vartheta_e)$, is for a different position and incident angle, and could be completely different from that in Equation 3, as Figure 2(b) illustrates. Note that $\mathbf{x_1}$ is not an independent quantity; for a given system geometry, it can be calculated from $\mathbf{x_0}$ and ϑ_1. The expression introduces yet another integral when one considers two interreflections,

$$T_{SYS}^{(2)}(\vartheta_0; \vartheta_e) =$$
$$\frac{1}{A_G} \int T_G(\theta_0) \left\{ \int \left[\int R_S^f(\mathbf{x_0}, \vartheta_0; \vartheta_1) R_G^b(\theta_1) R_S^f(\mathbf{x_1}, \vartheta_1; \vartheta_2) \cos(\theta_1) d\Omega_1 \right. \right. \qquad (5)$$
$$\left. \left. \times R_G^b(\theta_2) T_S(\mathbf{x_2}, \vartheta_2; \vartheta_e) \right] \cos(\theta_2) d\Omega_2 \right\} d^2\mathbf{x_0}$$

Continuing this process develops the familiar multiple reflectance series for the system transmittance. In the case of uniform specular glazings, the new terms introduced with each reflection are the same, and the series can be summed analytically. Here, in contrast, each new reflectance introduces a complicated new integral that is buried in the integral produced by the previous reflection. Not only do the once- and twice-reflected rays arrive at different points on the shading layer, the angles at which they reflect from the glazing system are also different.

Research project 548-RP dealt with this situation through its spatial averaging methodology and obtained a matrix series using the average bidirectional properties of the layers. An alternative approach might be to carry through the expansion explicitly, using measured bidirectional properties at each point of the shading system. In practice, since the *materials* making up the shading layer could be considered to have macroscopically uniform properties, this would require a detailed model of the system geometry together with *point* measurements with a gonio-radiometer. Of course, all of the above calculations should be done for each wavelength, with spectral averaging applied to the result, so more precisely measurements with a spectro-gonio-radiometer are necessary. Research into this method is also being pursued, but has not yet provided any data.

Simplifying Assumptions

Given the absence of data, one would like to derive whatever consequences are possible from the information normally obtainable for a shading system, and one would like to simplify the solar-optical calculations into something tractable. The data normally obtainable for a shading layer are the hemispherical transmittance and reflectance, either of the layer itself, or, in the case of venetian blinds, of the slats. In the absence of any other information, the most natural simplifying assumption is that the shading layer is a uniform diffuser. It turns out that this assumption also provides the necessary simplification of the calculation.

If the shading layer is a uniform diffuser, then R_S^f and T_S are constants independent of incident or outgoing angle. These constants can be related immediately to the hemispherical transmittances, since

$$R_S^{fH} = \int R_S^f \cos(\theta) d\Omega = \pi R_S^f$$

$$T_S^{fH} = \int T_S \cos(\theta) d\Omega = \pi T_S$$

(6)

When these quantities are put into Equation 4 we obtain

$$T_{SYS}^{(1)}(\vartheta_0; \vartheta_e) = T_G(\theta_0) R_S^{fH} \left[\frac{1}{\pi} \int R_G^b(\theta_1) \cos(\theta_1) d\Omega_1 \right] \frac{T_S^{fH}}{\pi}$$

(7)

The quantity in brackets is the average back reflectance of the (specular) glazing system for uniform diffuse incident irradiation:

$$\langle R_G^b \rangle_D = \frac{1}{\pi} \int R_G^b(\theta_1) \cos(\theta_1) d\Omega_1$$

$$= 2 \int_0^{\frac{\pi}{2}} R_G^b(\theta) \cos(\theta) \sin(\theta) d\theta$$

(8)

Integrating over the outgoing hemisphere in Equation 7 is equivalent to multiplying by π, and we can use this fact and Equation 8 to express the total system directional-hemispherical transmittance, combining Equations 3a, 4 and 5, in which we now recognize the standard multiple reflection series,

$$T_{SYS}^{fH}(\theta) = T_G(\theta) \left[1 + R_S^{fH} \langle R_G^b \rangle_D + \left(R_S^{fH} \langle R_G^b \rangle_D \right)^2 + ... \right] T_S^{fH}$$

$$= \frac{T_G(\theta) T_S^{fH}}{1 - R_S^{fH} \langle R_G^b \rangle_D}$$

(9)

(Since we are now dealing with a single angle, the subscript on the incident angle has been dropped.)

The key insight to be gained from this discussion is the observation that, if the shading layer is considered as a uniform diffuser, then the net effect of a reflection from the shading layer to the glazing system and back to the shading layer (i.e., one round-trip) is to produce a hemispherical average of the glazing system (back) reflectance, multiplied by the shading hemispherical (front) reflectance. Each subsequent interreflection will produce another factor of this product. Similarly, a (diffuse) transmission through a shading layer will cause all further interactions (whether transmission or reflection) with the downstream portion of the glazing system to involve the diffuse average properties of the glazing.

Conversely, if the shading layer is not a uniform diffuser, the radiation reflected from or transmitted through it will effectively average over the angular properties of the glazing system in a different way. This means that if the uniform diffuser assumption for the shading layer fails, the effect could be large or small, depending on the glazing system characteristics.

APPROXIMATE ANALYSIS OF SHADED GLAZING SYSTEMS

The conclusions of the previous section allow us to calculate the approximate properties for any type of shaded glazing, but before doing so it will be necessary to introduce some notation. We shall consider glazings consisting of one shading layer, which we denote by S, and some number L of transparent layers. So in general we shall be discussing a system of L+1 layers. We will number these layers sequentially from outside to inside, so that we can refer to a specific layer by its position, n, in the assembly. If we removed a particular layer, n, from the assembly and measured its solar-optical properties in isolation, the resulting transmission, reflectance and absorption would be denoted T_n, R_n, and $\mathcal{A}_n$.

These quantities depend on a number of physical variables. All depend on the incident direction, the wavelength of the radiation, and the polarization state of the radiation entering them. The wavelength and polarization dependence will not be noted explicitly here. In principle, all of the equations to follow should be calculated for each wavelength and polarization, and the final result averaged over the polarization and wavelength spectrum of the incident radiation. In practice, however, because of the limited availability of characterization data, polarizations are typically averaged for each layer (assuming unpolarized incident radiation). For shading layers, a similar lack of data leads to use of wavelength-averaged layer properties. Neither practice is strictly correct.

In the discussion below, in order to avoid rewriting equations for special cases, it will sometimes happen that impossible combinations of indices appear in the equations, i.e., indices that refer to a nonexistent layer. For example, if the first layer of the system is layer 1 (and we have not defined a layer 0, as will occasionally happen below), then reference to a layer k-1 becomes nonsensical for k=1. This possibility will particularly arise in the discussion of layer absorptances. We deal with these cases by defining all nonexistent layers to have T=1, R=0, and $\mathcal{A}$=0.

As noted above, for specular glazings the solar-optical properties will depend on the incident angle, e.g., $T_n(\theta)$, while for a shading layer it is a *bidirectional* quantity, e.g., $T_n(\vartheta_i; \vartheta_e)$ for the incident (i) and emerging (e) directions, each of which is specified by a pair of angles. In addition, the properties of the layer depend on the side of the layer on which the radiation is incident. We shall denote this by the superscripts f ("front", for radiation coming from the outside) and b ("back", for radiation coming from the inside), e.g., $T_n^f(\vartheta_i; \vartheta_e)$. (To avoid confusion with exponentiation, all quantities raised to a power will be enclosed in parentheses.) As noted above, for specular layers (or systems) it is always true that $T_n^f(\theta) = T_n^b(\theta) \equiv T_n(\theta)$, so that we can drop the superscript, but this is not the case for $R_n^f(\theta)$ and $\mathcal{A}_n^f(\theta)$. For nonspecular layers the analogous relation is $T_n^f(\vartheta_i; \vartheta_e) = T_n^b(\vartheta_e; \vartheta_i)$. We shall also use the superscript H to denote a directional-hemispherical quantity, i.e., a bidirectional one summed over the outgoing direction, e.g., $T_n^{fH}(\vartheta_i) = \iint\limits_{hemisphere} T_n^f(\vartheta_i; \vartheta_e)d^2\vartheta_e$. Note that for a purely specular layer this operation does nothing, since outgoing radiation only occurs in a single direction: $T_n^H(\theta) \equiv T_n(\theta)$.

As also noted above, adjacent layers can be collected together and considered as systems. The notation for this is summarized in Figure 3, and consists of using as subscripts the layer numbers of the first and last layers in the system to identify the overall system properties. It is important to distinguish that these are different from the isolated layer properties. For example, $R_{m,n}^f$ gives the fraction of the radiation incident on layer m that is reflected, and includes multiple interreflections among layers m through n, but none from layers lying to the inside of (i.e., with a number larger than) layer n. (Of course, $R_{n,n}^f \equiv R_n^f$.) This fact necessitates additional notation for specifying layer absorptions. Consider a (sub)system consisting of the layers m through n. This system has an absorptance, $\mathcal{A}_{m,n}^f = 1 - T_{m,n}^f - R_{m,n}^f$. However, the absorptance actually takes place in the individual layers of the system and we denote this *layer absorptance within a system* by $\mathcal{A}_{k;(m,n)}^f$ for the kth layer. We note that the quantity $\mathcal{A}_k$ appearing in Equations 1 and 2 should be denoted more correctly $\mathcal{A}_{k;(1,L)}^f$. They are not isolated-layer absorptances, but rather layer absorptances within the system of layers 1 through L. Absorptance in the layer k comes from radiation incident both on its front surface and on its back surface (due to multiple reflectance from downstream layers). The superscript f denotes that this absorption is all due to radiation incident on the front side of layer m.

Complete formulas for assemblies of arbitrary layers are given in (Klems 1994A; Klems 1994B). Here we reproduce the key composition formulas, specialized to the case of specular glazings, for combining two adjacent layers or subsystems, M and N:

$$T_{M,N}(\theta) = \frac{T_M(\theta)T_N(\theta)}{1 - R_N^b(\theta)R_M^f(\theta)} \tag{10a}$$

$$R_{M,N}^f(\theta) = R_M^f(\theta) + \frac{T_M(\theta)R_N^f(\theta)T_N(\theta)}{1 - R_M^b(\theta)R_N^f(\theta)} \tag{10b}$$

$$R_{M,N}^{b}(\theta) = R_{N}^{b}(\theta) + \frac{T_{M}(\theta)R_{M}^{b}(\theta)T_{N}(\theta)}{1 - R_{M}^{b}(\theta)R_{N}^{f}(\theta)} \tag{10c}$$

Since M and N may be either adjacent layers or subsystems, these equations can be used to generate all possible cases. For example, if M were a subsystem consisting of layers 1 through L-1 (M=(1,L-1)) and N were the single layer L, then Equations 10 would give the "recursion relations" that yield the properties of the system (1,L) from the properties of the subsystem (1,L-1) and those of the layer L. The situation is somewhat more complex for the layer absorptances. We consider the layer k, lying somewhere between layer m and layer n in the subsystem (m,n), and the layer absorptances are given by

$$\mathcal{A}_{k;(m,n)}^{f}(\theta) = \frac{T_{m,k-1}(\theta)\mathcal{A}_{k}^{f}(\theta)}{1 - R_{m,k-1}^{b}(\theta)R_{k,n}^{f}(\theta)} + \frac{T_{m,k}(\theta)R_{k+1,n}^{f}(\theta)\mathcal{A}_{k}^{b}(\theta)}{1 - R_{m,k}^{b}(\theta)R_{k+1,n}^{f}(\theta)} \tag{11a}$$

$$\mathcal{A}_{k;(m,n)}^{b}(\theta) = \frac{T_{k+1,n}(\theta)\mathcal{A}_{k}^{b}(\theta)}{1 - R_{m,k}^{b}(\theta)R_{k+1,n}^{f}(\theta)} + \frac{T_{k,m}(\theta)R_{m,k-1}^{b}(\theta)\mathcal{A}_{k}^{f}(\theta)}{1 - R_{m,k-1}^{b}(\theta)R_{k,n}^{f}(\theta)} \ . \tag{11b}$$

As can be seen from these equations, absorption in a layer can be due to both front- and back-incident radiation on the layer, due to multiple reflections within the subsystem (m,n). The total subsystem absorptances are of course given by

$$\mathcal{A}_{m,n}^{f}(\theta) = \sum_{k=m}^{n} \mathcal{A}_{k;(m,n)}^{f}(\theta) \tag{12a}$$

$$\mathcal{A}_{m,n}^{b}(\theta) = \sum_{k=m}^{n} \mathcal{A}_{k;(m,n)}^{b}(\theta) \ . \tag{12b}$$

With these preliminaries taken care of, we are now in a position to treat shading systems consisting of a single diffusing shading layer combined with an arbitrary specular glazing system of L layers.

Interior Shading

Transmission through a system with an interior shading layer has already been given in Equation 9, but we restate it here in the more detailed notation described above. The unshaded glazing system has L layers, so the interior shading layer is taken to be S=L+1 in what is now an L+1 layer system. Equation 9 for the total system transmittance now becomes

$$T_{1,L+1}^{fH}(\theta) = \frac{T_{1,L}(\theta)T_{S}^{fH}}{1 - R_{S}^{fH}\langle R_{1,L}^{b}\rangle_{D}} \ . \tag{13a}$$

For diffuse incident solar radiation one uses the hemispherical average of this quantity,

$$\left\langle T_{1,L+1}^{fH} \right\rangle_D = \frac{\left\langle T_{1,L} \right\rangle_D T_S^{fH}}{1 - R_S^{fH} \left\langle R_{1,L}^{b} \right\rangle_D} \tag{13b}$$

Absorptance in the shading layer is given by

$$\mathcal{A}_{S;(1,L+1)}^{f}(\theta) \equiv \mathcal{A}_{L+1;(1,L+1)}^{f}(\theta) = \frac{T_{1,L}(\theta)\mathcal{A}_S^{f}}{1 - R_S^{fH} \left\langle R_{1,L}^{b} \right\rangle_D} \tag{14a}$$

while for other layers ($1 \leq k \leq L$) the absorptance can be expressed in terms of the absorptance for the unshaded L-layer system, which may be computed from Equation 11a:

$$\mathcal{A}_{k;(1,L+1)}^{f}(\theta) = \mathcal{A}_{k;(1,L)}^{f}(\theta) + \left\langle \mathcal{A}_{k;(1,L)}^{b} \right\rangle_D \frac{T_{1,L}(\theta) R_S^{fH}}{1 - R_S^{fH} \left\langle R_{1,L}^{b} \right\rangle_D} \tag{14b}$$

The hemispherical averaging of these quantities is straightforward:

$$\left\langle \mathcal{A}_{S;(1,L+1)}^{f} \right\rangle_D = \frac{\left\langle T_{1,L} \right\rangle_D \mathcal{A}_S^{f}}{1 - R_S^{fH} \left\langle R_{1,L}^{b} \right\rangle_D} \tag{14c}$$

$$\left\langle \mathcal{A}_{k;(1,L+1)}^{f} \right\rangle_D = \left\langle \mathcal{A}_{k;(1,L)}^{f} \right\rangle_D + \left\langle \mathcal{A}_{k;(1,L)}^{b} \right\rangle_D \frac{\left\langle T_{1,L} \right\rangle_D R_S^{fH}}{1 - R_S^{fH} \left\langle R_{1,L}^{b} \right\rangle_D} \tag{14d}$$

These equations apply to the reduced shading area of the glazing, A_G. The corresponding unshaded glazing system properties should be applied to the unshaded area, $u(\theta, \phi)A_0$.

Putting Equation 2 into the present notation, the solar heat gain coefficient for the shaded area of the overall system is

$$SHGC_{1,L+1}^{shaded}(\theta) = T_{1,L+1}^{fH}(\theta) + \sum_{k=1}^{L+1} N_k \cdot \mathcal{A}_{k;(1,L+1)}^{f}(\theta) \tag{15a}$$

which becomes, after combining the expression with Equations 13 and 14,

$$SHGC_{1,L+1}^{shaded}(\theta) = \frac{T_{1,L}(\theta) \cdot \left[T_S^{fH} + N_{L+1} \cdot \mathcal{A}_S^f + R_S^{fH} \sum_{k=1}^{L} N_k \cdot \left\langle \mathcal{A}_{k;(1,L)}^b \right\rangle_D \right]}{1 - R_S^{fH} \left\langle R_{1,L}^b \right\rangle_D}$$

$$+ \sum_{k=1}^{L} N_k \cdot \mathcal{A}_{k;(1,L)}^f(\theta) \qquad\qquad (15b)$$

where Equations 12 have been used to express the overall absorption in the unshaded glazing. It can be seen that this expression mixes the optical properties of the glazing and shading systems in a complicated manner.

While the optical properties for the subsystem (1,L) will be the same as those of the unshaded glazing, the values of N_k in Equations 15a and 15b for layers in the glazing system ($1 \leq k \leq L$) are not the same as the values $N_k^{(0)}$ for the glazing system without shading. The presence of the shading layer will alter the thermal transfer through the system to some degree, with the result that there will be a change in the inward-flowing fractions of the glazing layers, $N_k = N_k^{(0)} + \Delta N_k$. Consequently, when the unshaded equivalents of Equations 14 are put into Equation 2, the result is not exactly the solar heat gain coefficient of the unshaded glazing. Instead we obtain

$$SHGC_{1,L+1}^{unshaded}(\theta) = SHGC_{1,L}(\theta) + \sum_{k=1}^{L} \Delta N_k \mathcal{A}_{k;(1,L)}^f(\theta) \qquad\qquad (15c)$$

The SHGC and layer absorptions appearing on the right-hand side of this equation are those of the glazing system without shading.

Exterior and Between-Glass Shading

For exterior and between-glass shading the calculations become much more complex. They are given in Appendix B, and in the following sections we list the resulting formulas for the solar-optical quantities entering Equation 1, front transmittance and layer absorptances. The resulting equations for SHGC are quite complicated and not very illuminating; they are contained in Appendix B.

A new geometric complication enters the problem when there is glazing to the inward side of the shading (i.e., the side away from the incident radiation source). Radiation passing through the gaps in the shading may now be specularly reflected by the glazing. Some of this reflected radiation may go back through the gaps; the remainder will strike the shading layer material from the back side, where it will be diffusely reflected. This situation is illustrated in Figure 4. The fraction that strikes the shading is denoted as $s^b(\theta, \phi)$. This is a geometric quantity, and it is discussed further in Appendix B.

The scheme for renumbering the layers and referring back to the optical properties of the glazing without the shading layer also becomes more complicated, particularly in the case of between-glass shading. For this case the numbering schemes are illustrated in Figure 5. Layer numbering is explained in more detail in Appendix B.

Exterior Shading

For exterior shading Equation B.2a gives the system transmittance (the shading layer number is S=0):

$$T_{S,L}^{fH}(\theta) = u(\theta, \phi)T_{1,L}(\theta) + \left(1 - u(\theta, \phi)\right)\frac{T_S^{fH}\left\langle T_{1,L}\right\rangle_D}{1 - R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}$$

$$+ \frac{u(\theta, \phi)R_{1,L}^f(\theta)s^b(\theta, \phi)R_S^{bH}\left\langle T_{1,L}\right\rangle_D}{1 - R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D} \tag{16a}$$

For incident diffuse solar radiation one uses the hemispherical average of this quantity, which is given in Appendix B. The layer absorptance in the shading layer (S=0) is given by Equation B.3a:

$$\mathcal{A}_{S;(S,L)}^f(\theta) = \left(1 - u(\theta, \phi)\right)\left[\mathcal{A}_S^f + \frac{\mathcal{A}_S^b R_{1,L}^f(\theta)}{1 - R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}\right]$$

$$+ \frac{u(\theta, \phi)R_{1,L}^f(\theta)s^b(\theta, \phi)\mathcal{A}_S^b}{1 - R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D} \tag{16b}$$

and for the other layers ($1 \le k \le L$) the layer absorptances are expressed in terms of the layer absorptances of the unshaded system in Equation B.3b:

$$\mathcal{A}_{k;(S,L)}^f(\theta) = u(\theta, \phi)\mathcal{A}_{k;(1,L)}^f(\theta) + \left(1 - u(\theta, \phi)\right)\frac{T_S^{fH}\left\langle \mathcal{A}_{k;(1,L)}^f\right\rangle_D}{1 - R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}$$

$$+ \frac{u(\theta, \phi)R_{1,L}^f(\theta)s^b(\theta, \phi)R_S^{bH}\left\langle \mathcal{A}_{k;(1,L)}^f\right\rangle_D}{1 - R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D} \quad . \tag{16c}$$

Averaging these quantities to produce the corresponding ones applicable for diffuse incident solar is straightforward, as explained in Appendix B.

Between-Glass Shading

For between-glass shading the system transmittance consists of three parts

$$T_{S,L+1}^{fH}(\theta) = T_{unshaded} + \left[T_{S,L+1}^{fH}(\theta)\right]_{shaded} + \left[T_{S,L+1}^{fH}(\theta)\right]_{erefl} \tag{17a}$$

which are given in Equations B.5a, B.5c and B.5d:

$$T_{unshaded} = u(\theta, \phi)T_{1,L}^{unshaded}(\theta)\left[1 - s^b(\theta, \phi)R_{1,S-1}^b(\theta)R_{S+1,L+1}^f(\theta)\right] \tag{17b}$$

$$\left[T_{1,L+1}^{fH}(\theta)\right]_{shaded} = \frac{(1 - u(\theta,\phi))T_{1,S-1}(\theta)T_S^{fH}\left\langle T_{S+1,L+1}\right\rangle_D}{1 - \left\langle R_{1,S-1}^{b}\right\rangle_D R_S^{fH} - \left[R_S^{bH} + \left\langle R_{1,S-1}^{b}\right\rangle_D\left(R_S^{fH}R_S^{bH} - T_S^{fH}T_S^{bH}\right)\right]\left\langle R_{S+1,L+1}^{f}\right\rangle_D} \quad (17c)$$

$$\left[T_{1,L+1}^{fH}(\theta)\right]_{rerefl} = \frac{u(\theta,\phi)s^b(\theta,\phi)R_{S+1,L+1}^{f}(\theta)\left\langle T_{S+1,L+1}\right\rangle_D\left(R_S^{bH} + \left(T_S^{bH}T_S^{fH} - R_S^{bH}R_S^{fH}\right)\left\langle R_{1,S-1}^{b}\right\rangle_D\right)}{1 - R_S^{fH}\left\langle R_{1,S-1}^{b}\right\rangle_D - \left(R_S^{bH} + \left(T_S^{bH}T_S^{fH} - R_S^{bH}R_S^{fH}\right)\left\langle R_{1,S-1}^{b}\right\rangle_D\right)\left\langle R_{S+1,L+1}^{f}\right\rangle_D} \quad (17d)$$

The shading layer absorptance is given in Equation B.7a:

$$\mathcal{A}_{S;(1,L+1)}^{f} = \frac{(1 - u(\theta,\phi))T_{1,S-1}(\theta)\mathcal{A}_S^{f}}{1 - \left\langle R_{1,S-1}^{bH}\right\rangle_D\left\langle R_{S,L+1}^{fH}\right\rangle_D}$$

$$+ \frac{\left[u(\theta,\phi)T_{1,S-1}(\theta)R_{S+1,L+1}^{f}(\theta)s^b(\theta,\phi) + (1 - u(\theta,\phi))T_{1,S}^{fH}(\theta)\left\langle R_{S+1,L+1}^{fH}\right\rangle_D\right]\mathcal{A}_{S;(1,S)}^{b}}{1 - \left\langle R_{1,S}^{bH}\right\rangle_D\left\langle R_{S+1,L+1}^{f}\right\rangle_D} \quad (17e)$$

For glazing layers outside the shading ($1 \leq k \leq S\text{-}1$) the layer absorptances are given in Equation B.7b:

$$\mathcal{A}_{k;(1,L+1)}^{f}(\theta) = u(\theta,\phi)\mathcal{A}_{k;(1,L)}^{f,unshaded}(\theta)$$

$$+ (1 - u(\theta,\phi))\left(\mathcal{A}_{k;(1,S-1)}^{f}(\theta) + \frac{T_{1,S-1}(\theta)R_S^{fH}\left\langle \mathcal{A}_{k;(1,S-1)}^{b}\right\rangle_D}{1 - \left\langle R_{S,L+1}^{f}\right\rangle_D\left\langle R_{1,S-1}^{b}\right\rangle_D}\right) \quad (17f)$$

and for glazing layers in inside the shading ($S\text{+}1 \leq k \leq L\text{+}1$), Equation B.7b gives

$$\mathcal{A}_{k;(1,L+1)}^{f}(\theta) = u(\theta,\phi)\mathcal{A}_{k;(1,L)}^{f,unshaded}(\theta)$$

$$+ (1 - u(\theta,\phi))\frac{T_{1,S}^{fH}(\theta)\left\langle \mathcal{A}_{k;(S+1,L+1)}^{f}\right\rangle_D}{1 - \left\langle R_{S+1,L+1}^{f}\right\rangle_D\left\langle R_{1,S}^{b}\right\rangle_D} \quad (17g)$$

SHADING MULTIPLIERS

It is clear from the above equations and those in Appendix B that the solar heat gain coefficient of a glazing with shading depends on the incident angle and the glazing layer properties in a complex way. In earlier ASHRAE treatments of shading, assuming simple unshaded glazing systems, a simple multiplier, the shading coefficient, was used to determine SHGC's for design purposes. Shading coefficients were determined largely by measurements. As an interim measure, until more accurate data is available, it is useful to cast the above discussion into a form that would allow these measurements to be used for the systems for which they are appropriate. In the following section we cast the solar heat gain coefficient for each shading type into an approximate multiplier form.

From the above equations we can then determine an expression (in general very complicated) for the multiplier. This expression can then be evaluated for a range of glazing classes to determine when the multiplier is reasonably constant, and therefore useful for design purposes.

Interior Shading

On physical grounds, interior shading should behave (in simple cases) like an attenuation of the energy flux admitted by the unshaded glazing SHGC. The latter determines the amount of energy arriving at the shading layer. If the glazing system has a high reflectance (e.g., at high incident angle), then little energy should reach the shading layer, but approximately the same fraction of this energy would be admitted. This picture would give a quantity we could call the Interior (solar) Acceptance Coefficient, IAC:

$$SHGC_{1,L+1}^{shaded}(\theta) \; = \; IAC \cdot SHGC_{1,L}(\theta) \tag{18a}$$

(with a similar relation holding between the SHGC's for diffuse incidence). If one recasts Equation 15b into this form, an expression for the quantity IAC is obtained:

$$
\begin{aligned}
IAC \; = \; & \frac{1}{1 - R_S^{fH}\langle R_{1,L}^b \rangle_D} \left[T_S^{fH} \; + \; N_S\mathcal{A}_S^f \; + \; R_S^{fH}\left(\langle SHGC_{1,L}\rangle_D \; - \; \langle T_{1,L}\rangle_D \right) \right] \\[2ex]
& + \frac{\left(SHGC_{1,L}(\theta) - T_{1,L}(\theta) \right)}{SHGC_{1,L}(\theta)} \left[1 - \frac{T_S^{fH} \; + \; N_S\mathcal{A}_S^f \; + \; R_S^{fH}\left(\langle SHGC_{1,L}\rangle_D \; - \; \langle T_{1,L}\rangle_D \right)}{1 - R_S^{fH}\langle R_{1,L}^b \rangle_D} \right] \\[2ex]
& + \frac{\displaystyle\sum_{k=1}^{L} \Delta N_k\mathcal{A}_{k;(1,L)}^f(\theta) + \sum_{k=1}^{L} \Delta N_k\langle \mathcal{A}_{k;(1,L)}^b \rangle_D \; \frac{T_{1,L}(\theta)R_S^{fH}}{1 - R_S^{fH}\langle R_{1,L}^b \rangle_D}}{SHGC_{1,L}(\theta)}
\end{aligned}
\tag{18b}
$$

A diffuse analog of IAC could be obtained by substituting the hemispherical average for all incident-angle dependent quantities in Equation 18b; however, to the extent that this calculation gives a different value, it indicates that the basic approximation of Equation 18a is failing. In the picture that the shading layer simply attenuates the energy admitted by the glazing, this attenuation should be insensitive to incident angle, since the shading layer is a uniform diffuser. The physical processes that create a difference between specular and diffuse radiation, namely multiple reflections and modifications of the inward-flowing fractions of the glazing layers, have been left out of the simple attenuation picture of Equation 18a. Accordingly, in using Equation 18a to estimate solar heat gains, the same value of IAC will be used for both direct and diffuse radiation.

Equation 18b has the correct behavior in extreme cases. Once energy has reached the shading layer (either as solar-optical radiation or as heat flow), there are only two mechanisms by which the shading layer can reject it: solar optical radiation can be reflected back out through the glazing, or it can be absorbed and conducted back outward, which requires that N_S be small. In addition, the shading layer can reduce the inward flow of energy absorbed in the glazing layers if its presence produces negative values of ΔN_k for the glazing layers. Both of the thermal effects are maximal for

shading applied to the interior of single glazing. As the number of glazing layers increases, N_S also increases, and the shading layer has progressively less effect on the values of N_k.

For the limiting case of a glazing with high thermal resistance, we can set $N_S = 1$ and $\Delta N_k = 0$. Then $T_S^{fH} + N_S \mathcal{A}_S^f = 1 - R_S^{fH}$ and Equation 18b becomes

$$IAC \approx \frac{1 - R_S^{fH}}{1 - R_S^{fH} \langle R_{1,L}^b \rangle_D} + \left(\text{Increases in absorption} \right)$$

exhibiting the fact that energy can be rejected only by reflection from the shading layer.

At the level of accuracy implicit in the use of the IAC (i.e., not very high) one should neglect the second term in Equation 15c. The equation for the heat flux through the fenestration area A_0 then becomes

$$
\begin{aligned}
q = \; & E_{DN} \cos(\theta) SHGC_{1,L}(\theta) \Big[u(\theta, \phi) + \left(1 - u(\theta, \phi) \right) \cdot IAC \Big] \\
& + \left(E_d + E_r \right) \langle SHGC_{1,L} \rangle_D \Big[\langle u(\theta, \phi) \rangle_D + \langle \left(1 - u(\theta, \phi) \right) \rangle_D \cdot IAC \Big]
\end{aligned}
\tag{19}
$$

where E_{DN} is the beam solar irradiance, E_d incident irradiance from the sky excluding the sun, and E_r is the incident irradiance from ground-reflected radiation. Both of the latter are assumed to be uniformly diffuse.

Exterior Shading

With the exception of radiation passing through gaps in the shading, an exterior shading layer reduces the amount of radiation incident on the glazing system and converts it all to diffuse. It is therefore reasonable to define an "Exterior (solar) Acceptance Coefficient", EAC:

$$SHGC_{(S,L)}(\theta, \phi) \approx u(\theta, \phi) SHGC_{(1,L)}(\theta) + \left(1 - u(\theta, \phi) \right) \cdot EAC \cdot \langle SHGC_{(1,L)} \rangle_D \tag{20a}$$

where, using Equation B.4,

$$EAC = \left[\frac{T_S^{fH}}{1 - R_S^{bH}\langle R_{1,L}^f \rangle_D} + \frac{u(\theta,\phi)R_{1,L}^f(\theta)s^b(\theta,\phi)R_S^{bH}}{(1 - u(\theta,\phi))(1 - R_S^{bH}\langle R_{1,L}^f \rangle_D)} \right]$$

$$+ \frac{1}{\langle SHGC_{(1,L)} \rangle} \times \left\{ \frac{u(\theta,\phi)}{(1 - u(\theta,\phi))} \sum_{k=1}^{L} \Delta N_k \mathcal{A}_{k;(1,L)}^f(\theta) \right.$$

$$+ \left[\frac{T_S^{fH}}{1 - R_S^{bH}\langle R_{1,L}^f \rangle_D} + \frac{u(\theta,\phi)R_{1,L}^f(\theta)s^b(\theta,\phi)R_S^{bH}}{(1 - u(\theta,\phi))(1 - R_S^{bH}\langle R_{1,L}^f \rangle_D)} \right] \sum_{k=1}^{L} \Delta N_k \langle \mathcal{A}_{k;(1,L)}^f \rangle_D$$

$$+ N_S \cdot \left[\mathcal{A}_S^f + \left(\frac{T_S^{fH}\langle R_{1,L}^f \rangle_D}{1 - R_S^{bH}\langle R_{1,L}^f \rangle_D} + \frac{u(\theta,\phi)R_{1,L}^f(\theta)s^b(\theta,\phi)}{(1 - u(\theta,\phi))(1 - R_S^{bH}\langle R_{1,L}^f \rangle_D)} \right) \mathcal{A}_S^b \right] \right\}$$

(20b)

The part of the expression within the curly brackets must be relatively small in order for Equation 20a to be of much value. This will occur when adding the exterior shading layer has little effect on the inward-flowing fractions of the glazing system layers (ΔN_k of small magniture) and little of the energy absorbed in the shading is transferred into the interior space ($N_S \mathcal{A}_S^f$ small). These assumptions are reasonable if there is good ventilation of the space between the exterior shading and the glazing, and if the heat flux due to thermal radiation from the shading is not too large.

Between-Glass Shading

The case of between glass shading is handled by conceptually dividing the fenestration system into the outer glazing, which is unshaded, and the combination of the shading layer and the inner glazing. From this point of view, the system is a glazing with interior shading, and accordingly the heat flux should have a form similar to Equation 19:

$$q = E_{DN}\cos(\theta)u(\theta,\phi)SHGC_{1,L}^{unshaded}(\theta) + (E_d + E_r)\langle u(\theta,\phi) \rangle_D \langle SHGC_{1,L}^{unshaded} \rangle_D$$

$$+ \left[E_{DN}\cos(\theta)(1 - u(\theta,\phi))SHGC_{1,S-1}(\theta) \right. \tag{21}$$

$$+ (E_d + E_r)\langle(1 - u(\theta,\phi))\rangle_D \langle SHGC_{1,S-1} \rangle_D \left. \right] \cdot \text{BAC}$$

This means that the SHGC for the complete system must have the form

$$SHGC_{1,L+1}(\theta) = \left[u(\theta,\phi))SHGC_{1,L}^{unshaded}(\theta) + (1 - u(\theta,\phi))SHGC_{1,S-1}(\theta) \cdot \text{BAC} \right] \tag{22}$$

with a similar equation holding for the hemispherical average quantities. Requiring the solar heat gain coefficient to have this form leads to an expression for the "Between-pane (solar) Acceptance Coefficient", BAC. The complete expression is given in Appendix C.

It contains a number of terms involving re-reflection of radiation that passes through gaps in the shading and modifications of the inward-fraction of absorbed energy due to the presence of the shading layer. These terms are of interest only in that, to the extent they are significant, it implies that equation 21 should not be used.

In the cases where these terms can be neglected, the expression for BAC is

$$
BAC \approx \frac{T_{S,L+1}^{fH}(\theta)}{1 - \left\langle R_{1,S-1}^{b} \right\rangle_{D} R_{S,L+1}^{fH}(\theta)} + \frac{R_{S}^{fH} \sum_{k=1}^{S-1} N_{k} \left\langle \mathcal{A}_{k:(1,S-1)}^{b} \right\rangle_{D}}{1 - \left\langle R_{S,L+1}^{f} \right\rangle_{D} \left\langle R_{1,S-1}^{b} \right\rangle_{D}}
$$

$$
+ \frac{T_{S}^{fH} \sum_{k=S+1}^{L+1} N_{k} \left\langle \mathcal{A}_{k:(S+1,L+1)}^{f} \right\rangle_{D}}{\left(1 - \left\langle R_{S+1,L+1}^{f} \right\rangle_{D} \left\langle R_{1,S}^{b} \right\rangle_{D}\right)\left(1 - \left\langle R_{1,S-1}^{b} \right\rangle R_{S}^{fH}\right)} \qquad (23)
$$

$$
+ N_{S} \frac{\mathcal{A}_{S}^{f}}{1 - \left\langle R_{1,S-1}^{bH} \right\rangle_{D} \left\langle R_{S,L+1}^{fH} \right\rangle_{D}} + N_{S} \frac{\left\langle R_{S+1,L+1}^{fH} \right\rangle_{D} T_{S}^{fH} \mathcal{A}_{S:(1,S)}^{b}}{\left(1 - R_{1,S}^{bH} \left\langle R_{S+1,L+1}^{f} \right\rangle_{D}\right)\left(1 - \left\langle R_{1,S-1}^{b} \right\rangle R_{S}^{fH}\right)}
$$

This expression can be considered as expressing the fraction of the energy incident on the "interior shading" system that reaches the building interior space. The first term in the expression gives the amount of radiation transmitted. The second and third terms represent energy absorbed in the glazing layers after either reflection or transmission by the shading layer, and the last two terms represent energy absorbed in the shading layer.

THE 2001 VERSION OF THE HANDBOOK OF FUNDAMENTALS

The calculations and approximations described in this paper formed the basis for updating the equations for solar heat gain in shaded fenestrations in the forthcoming version of the Handbook of Fundamentals. The equations given above were presented there in a somewhat simplified form. In that treatment it was assumed that $s^{b}(\theta,\phi) = 0$, and $\left\langle u(\theta,\phi) \right\rangle_{D} = 0$ in order to reduce the complexity of the material presented. These are likely to be good approximations in any case. They have been relaxed here in order to present a complete treatment that can be applied to unusual cases.

The existing tables of shading coefficients for simple systems were used to construct tables of IAC, EAC, and BAC. The reasoning is as follows: The shading coefficient tables were originally constructed from measurements, usually made at a single incident angle, and the measured SHGC for the system was converted to shading coefficient by dividing by the SHGC for single glazing at that angle. The shading coefficient was assumed to be angle-independent, and in the shading coefficient/solar heat gain factor treatment the SHGC for standard single glazing was contained in the solar heat gain factor, which depended on incident angle. Because the IAC, EAC and BAC characterize subsystems for which the outer layer is a uniform diffuser, they can be expected to be

independent of incident angle under the assumption $u(\theta, \phi) = 0$. This assumption was made for all of the tables except that of EAC for louvered sun screens. The table value of shading coefficient was assumed to apply at normal incidence, and it was multiplied by the SHGC for standard 3mm (1/8 inch) clear single glazing at normal incidence to give back the system SHGC. Equation 18a, 20a or 22 was then applied with the appropriate glazing system SHGC at normal incidence to calculate the multiplier listed in the table.

For louvered sun screens, the shading coefficient table, the profile angle, and the louver construction were used to calculate $u(\theta, \phi) \equiv u(\psi)$ (which is termed F_u in the Handbook). At the largest profile angle $u(\theta, \phi) = 0$, and the shading coefficient for this profile angle was used to calculate EAC.

The detailed formulas presented above for the multipliers will enable one to calculate when the assumption of a constant IAC, EAC or BAC is violated in particular systems.

CONCLUSIONS

The computational methods for calculating the properties of glazing systems containing shading from the properties of their components have been developed, but the measurement standards and property data bases necessary to apply them have not.

In order to apply the calculations when there is very limited data, and to simplify them to a level appropriate to a handbook treatment, a drastic simplifying assumption was necessary. This assumption is that all shading layers behave as uniform diffusers in both transmission and reflection for radiation that in any way encounters the material of the layer.

Many shading layers also contain gaps, through which radiation can penetrate without encountering the shading material, leading to the physical picture of a shaded fenestration as a mixture of a completely shaded and a completely unshaded one. This picture is valid only when certain multiple reflections are negligibly small.

Complete formulas for the solar-optical properties of such shaded fenestrations have been presented.

Performance multipliers are defined for simple systems and may be used with appropriate caution.

When both penetration through shading gaps and multiple reflections are significant, performance multipliers cannot be used, and the solar-optical property formulas become very complicated.

REFERENCES

Collins, M. R. and S. J. Harrison (1999). "Calorimetric Measurement of the Inward-Flowing Fraction of Absorbed Solar Radiation in Venetian Blinds." ASHRAE Trans. **105**(2): Paper SE-99-15-2.

Klems, J. H. (1994A). "A New Method for Predicting the Solar Heat Gain of Complex Fenestration Systems: I. Overview and Derivation of the Matrix Layer Calculation." ASHRAE Trans. **100**(pt. 1): 1065-1072.

Klems, J. H. (1994B). "A New Method for Predicting the Solar Heat Gain of Complex Fenestration Systems: II. Detailed Description of the Matrix Layer Calculation." ASHRAE Trans. **100**(pt.1): 1073-1086.

Klems, J. H. and G. O. Kelley (1996). "Calorimetric Measurements of Inward-Flowing Fraction for Complex Glazing and Shading Systems." ASHRAE Trans. **102**(Pt. 1; Symposium Paper AT-96-16-3): 947-954.

Klems, J. H. and J. L. Warner (1995). "Measurement of Bidirectional Optical Properties of Complex Shading Devices." ASHRAE Trans. **101**(pt 1; Symposium Paper CH-95-8-1(RP-548)): 791-801.

Klems, J. H., J. L. Warner, et al. (1995). ASHRAE Solar Heat Gain Project 548-RP Final Report: A New Method for Predicting the Solar Heat Gain of Complex Fenestration Systems. E. O. Lawrence Berkeley National Laboratory, Technical Report LBL-36995.

Klems, J. H., J. L. Warner, et al. (1996). "A Comparison between Calculated and Measured SHGC for Complex Glazing Systems." ASHRAE Trans. **102**(Pt. 1; Symposium Paper AT-96-16-1): 931-939.

NFRC (2000). NFRC Spectral Data Library. National Fenestration Ratings Council. Avail: www.nfrc.org/software.html.

Windows and Daylighting Group (1994). WINDOW 4.1: A PC PRogram for Analyzing Window Thermal Performance in Accordance with Standard NFRC Procedures. Lawrence Berkeley Laboratory, Berkeley, CA 94720, Technical Report LBL-35298.

Windows and Daylighting Group (2001), WINDOW-4.1 Spectal Data Library, Lawrence Berkeley National Laboratory. Avail: http://windows.lbl.gov/materials/optical_data/default.htm.

Wright, J. L. (1995). "Summary and Comparison of Methods to Calculate Solar Heat Gain." ASHRAE Trans. **101**(1): 802-818.

Wright, J. L. (1998). "Calculating the Centre-glass Performance Indices of Windows." ASHRAE Trans. **104**(Pt. 1): 1230-1241.

(a)

(b)

$(1-u)$
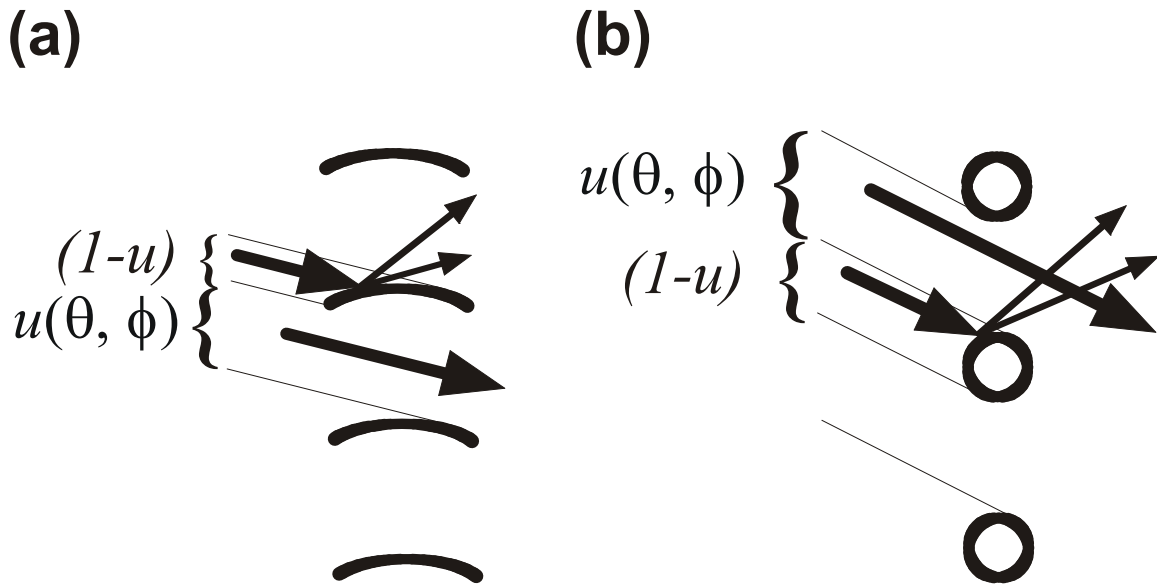
$u(\theta, \phi)$

$u(\theta, \phi)$

$(1-u)$

Figure 1. Definition of Unshaded and Shaded Parts of a Shading Layer. For either a louvred or slatted construction (a) or a woven shading layer (b) there is a trivial or geometric transmittance resulting from the fact that some incident rays encounter no material in the shading layer. Rays encountering material may contribute to the transmittance, but the angular distribution of the transmitted radiation will be modified. Note that since the constructions are not azimuthally symmetric, the unshaded fraction u will in general depend on both angles characterizing the incident direction.
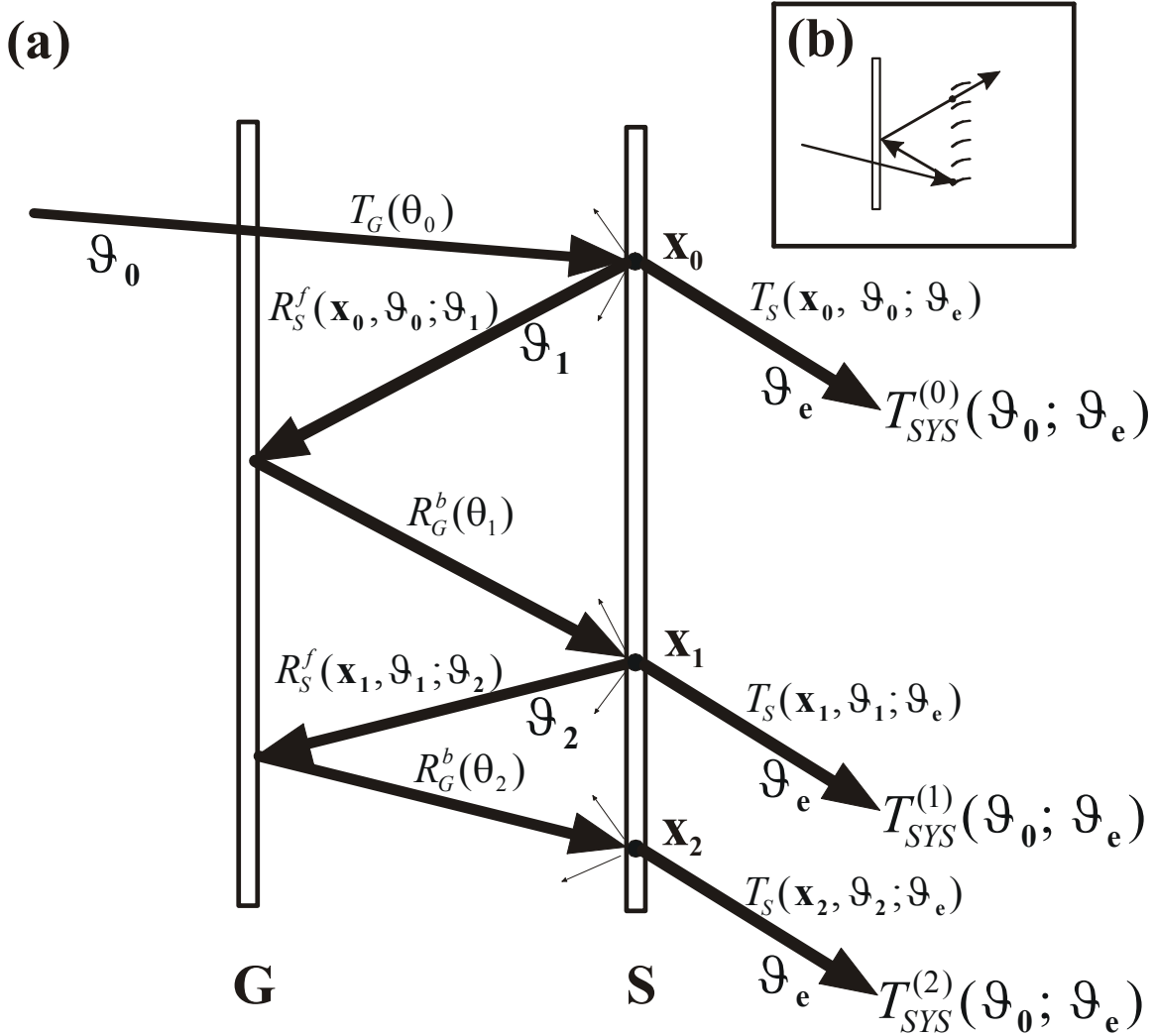
Figure 2. **Contribution of Multiple Interreflections to the Transmittance of a Glazing G with an Interior Shading Layer S. (a) Mathematical Structure of the Transmittance. The symbol below each ray indicates the direction (specified by two angles). At the shading layer, one direction is selected from the outgoing distribution. (Light arrows indicate the existence of other rays.) Above each ray is the term resulting from the last encounter with a solid material. The transmittance contribution indicated at the end of the arrow is the product of all terms along the preceeding path, integrated over intermediate directions. (b) A venetian blind example illustrates how a ray may encounter different shading layer transmittances on initial incidence and after interreflection.**
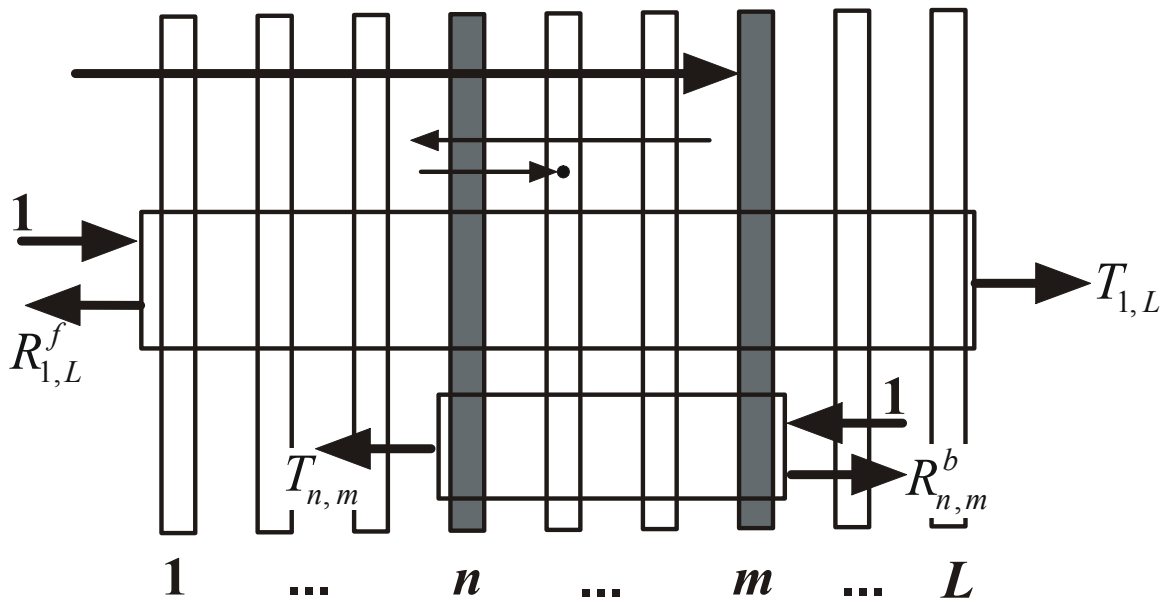
Figure 3. **Multilayer glazings considered as systems and subsystems. A total of L layers is assumed. Glazing layers are numbered sequentially, beginning with the side toward the incident radiation (upper dark arrow). Due to reflections among the layers, radiation may be incident on a given layer from either direction (light arrows) and interreflected radiation may contribute to the absorption in a given layer. The large dashed box indicates the notation for the entire system (transmittance and front reflectance are shown). The smaller dashed box indicates the notation for a subsystem consisting of layers *n* through *m* (unit incidence on the back side of the subsystem, transmittance and back reflection are shown).**
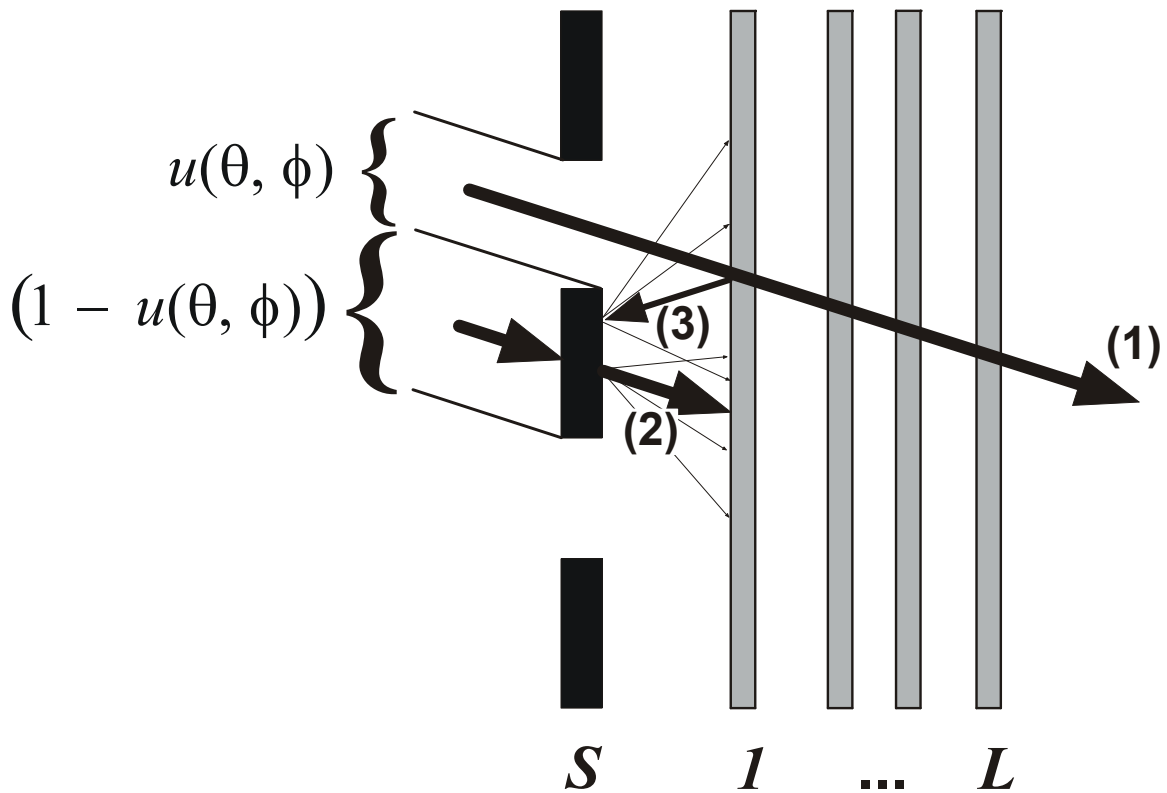
Figure 4. Exterior Shading. Layer numbering and definition of shaded and unshaded portions are indicated. The numbered rays are cases discussed in the text. Ray (1) enters through the unshaded portion and is transmitted through the glazing system. Ray (2) is incident on the shaded portion and is transmitted (diffusely) through the shading layer. Ray (3) is incident on the unshaded portion, reflected by the glazing system, and (diffusely, as indicated by the light arrows) re-reflected from the back of the shading layer.
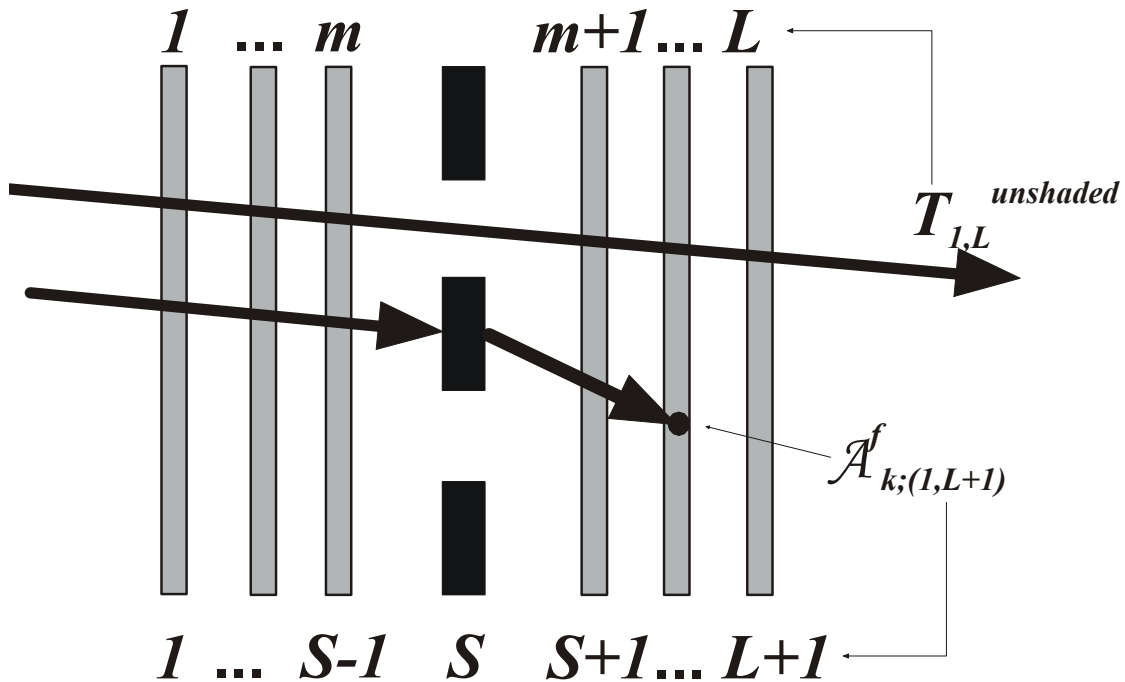
Figure 5. Layer numbering for between-pane shading. A shading layer S is assumed to be inserted within an L-layer glazing system, dividing the system into an outer glazing (layers 1 thru m) and an inner glazing (layers m+1 through L). The layers of the new system are renumbered to include the shading layer (S-1=m), producing an L+1-layer system with numbering shown at the bottom. For radiation passing through gaps in the shading, the optical properties are effectively those of the unshaded glazing, for example, transmission as indicated in the figure. These properties are given the superscript "unshaded", and use the layer numbering of the unshaded glazing system, shown at the top of the figure. For radiation that interacts with the shading layer, the relevant properties (e.g., layer absorptivity, as indicated) are those of the glazing with the shading layer, and the indices refer to the numbering at the bottom of the figure.

Appendix A. Nomenclature

Symbols

A	Area
$\mathcal{A}$	Absorptance
BAC	Between-pane (solar) Acceptance Coefficient
E	Solar Irradiance
EAC	Exterior (solar) Acceptance Coefficient
G	An integer; glazing system number, number of glazing systems, or the number specifying a particular glazing or glazing system in a list
I	Total radiant energy per unit time
IAC	Interior (solar) Acceptance Coefficient
L	Number of layers in a system or subsystem
N	Inward-flowing fraction of absorbed solar energy
R	Reflectance
S	An integer; shading system number, number of shading systems, or the number specifying a particular shading layer in a list
$SHGC$	Solar Heat Gain Coefficient
T	Transmittance
x	A position vector
x, y, z	Coordinates in a right-handed Cartesian coordinate system
θ	Incident angle or polar angle in a right-handed 3-D Cartesian coordinate system
ϕ	Azimuthal angle in a right-handed 3-D Cartesian coordinate system
ψ	Profile angle
ϑ	A direction vector, specified by a pair of angles

Subscripts

0	Denotes initial value, or before the application of some process or qualification; overall (as in area)
d	Diffuse
DN	Direct normal; beam
e	Emerging, or exit
G	Pertaining to a glazing or glazing system; for area, denotes reduced shading area

i	Integer denoting a particular layer in a list
j	Integer denoting a particular layer in a list
k	Integer denoting a particular layer in a list
m	Integer denoting a particular layer in a list
n	Integer denoting a particular layer in a list
M	A layer designation that may refer to either a single layer or a set of layers, e.g., (i, j)
N	A layer designation that may refer to either a single layer or a set of layers, e.g., (i, j)
i, j	Pertaining to the set of layers from i through j
(i, j)	Pertaining to the set of layers from i through j
$k; (l, m)$	Pertaining to the layer k, considered to be part of a (sub)system consisting of the layers l through m
r	Ground-reflected
S	Pertaining to a shading layer or system; also, an integer label identifying a shading layer in a list of layers
SYS	Denotes a quantity pertaining to an entire system

Superscripts

(0)	Initial, before the application of some change or action, or the zero-order approximation
(1)	First; after one application of some change or action; first in a succession of approximations
(2)	Second in a series of successive actions or approximations
f	Front incidence; i.e., on the side toward the ultimate source of radiation
b	Back incidence; i.e., on the side away from the ultimate source of radiation
H	Hemispherical total, i.e., summed over the outgoing hemisphere
shaded	A system containing a shading layer
unshaded	Denotes a glazing system without shading layers

Operations

$\langle X \rangle_D$	Average of the quantity X over incident directions uniformly distributed over a hemisphere
ΔX	Denotes a small change in the quantity X resulting from some action or change

Appendix B. Detailed Derivation of Solar-Optical Property Formulas For Exterior and Between-Glass Shading

Exterior Shading

   For exterior shading layers the separation into shaded and unshaded areas is not quite so clean as for interior shading. As illustrated in Figure 4, there are three cases that need consideration: (1) radiation incident on the openings in (unshaded portion of) the shading layer, that is transmitted by the remainder of the glazing system; (2) radiation incident on the material (reduced shading area) of the shading layer that is transmitted by the layer; and (3) radiation incident on the unshaded portion of the shading layer that is reflected by the remainder of the glazing system, re-reflected by the back of the shading layer, and re-incident on the remainder of the glazing system. Cases (1) and (2) both occur for interior shading and are treated in much the same way for exterior shading. Case (3), however, is new and requires further discussion.

As indicated in Figure 3 we consider the glazing with shading to form an L+1 layer system, but now we denote the shading layer by the index $S=0$, in order to continue to use the expressions involving $(1,L)$ above for the unshaded glazing. The usual ASHRAE treatment of exterior shading, utilizing the full arsenal of profile angles, etc., is essentially a calculation of $u(\theta, \phi)$. Once this has been determined cases (1) and (2) then result in, for example, a system transmittance of

$$\left[T_{S,L}^{fH}(\theta)\right]' = u(\theta, \phi)T_{1,L}(\theta) + \left(1 - u(\theta, \phi)\right)\frac{T_S^{fH}\left\langle T_{1,L}\right\rangle_D}{1 - R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D} \qquad (B.1a)$$

where the bracket and prime symbol on the left hand side indicate that case (3) has been excluded.

To treat case (3) we shall need to introduce a new quantity, denoted $s^b(\theta, \phi)$. It is an analog of the quantity $1 - u(\theta, \phi)$. A specular quantity, $s^b(\theta, \phi)$ is defined as the effective shaded fraction of the shading layer as seen by the rays labeled as (3) in Figure 4; that is, the shaded fraction of the shading layer *as seen from the back for rays that have already passed through an unshaded area of the layer*. The value of $s^b(\theta, \phi)$ introduced above may be determined by the same kind of geometric analysis used to determine $u(\theta, \phi)$.

Evaluation of case (3) then proceeds as follows: The fraction of incident radiation that passes through the unshaded portion of the shading layer and is reflected from the glazing system is $u(\theta, \phi)R_{1,L}^f(\theta)$, and a fraction $s^b(\theta, \phi)$ of this strikes the material of the shading layer from the back side. (The remainder passes through and is lost.) Of this, a fraction R_S^{bH} is reflected by the shading layer and is re-incident on the glazing system (as diffuse radiation). A fraction $\left\langle T_{1,L}\right\rangle_D$ of this is transmitted and $\left\langle R_{1,L}^f\right\rangle_D$ is reflected. The reflected

radiation now strikes the shading layer material and of this R_S^{bH} is reflected and re-incident on the glazing system. This is the first term of an infinite interreflection series, and collecting terms we can write the case (3) part of the transmittance as

$$\left[T_{S,L}^{fH}(\theta)\right]'' = \frac{u(\theta,\phi)R_{1,L}^{f}(\theta)s^{b}(\theta,\phi)R_{S}^{bH}\left\langle T_{1,L}\right\rangle_{D}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}} \tag{B.1b}$$

Combining Equations B.1a and B.1b then gives the expression for the system transmittance:

$$T_{S,L}^{fH}(\theta) = u(\theta,\phi)T_{1,L}(\theta) + \left(1 - u(\theta,\phi)\right)\frac{T_{S}^{fH}\left\langle T_{1,L}\right\rangle_{D}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}}$$
$$+ \frac{u(\theta,\phi)R_{1,L}^{f}(\theta)s^{b}(\theta,\phi)R_{S}^{bH}\left\langle T_{1,L}\right\rangle_{D}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}} \tag{B.2a}$$

For incident diffuse solar radiation the system transmittance is

$$\left\langle T_{S,L}^{fH}\right\rangle_{D} = \left\langle u(\theta,\phi)T_{1,L}(\theta)\right\rangle_{D} + \left(1 - \left\langle u(\theta,\phi)\right\rangle_{D}\right)\frac{T_{S}^{fH}\left\langle T_{1,L}\right\rangle_{D}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}}$$
$$+ \frac{\left\langle u(\theta,\phi)R_{1,L}^{f}(\theta)s^{b}(\theta,\phi)\right\rangle_{D}R_{S}^{bH}\left\langle T_{1,L}\right\rangle_{D}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}} \tag{B.2b}$$

Following a similar argument, absorptance in the shading layer is given by

$$\mathcal{A}_{S;(S,L)}^{f}(\theta) = \left(1 - u(\theta,\phi)\right)\left[\mathcal{A}_{S}^{f} + \frac{\mathcal{A}_{S}^{b}T_{S}^{fH}\left\langle R_{1,L}^{f}\right\rangle_{D}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}}\right]$$
$$+ \frac{u(\theta,\phi)R_{1,L}^{f}(\theta)s^{b}(\theta,\phi)\mathcal{A}_{S}^{b}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}} \tag{B.3a}$$

and for the other layers ($1 \le k \le L$) the layer absorptances are expressed in terms of the layer absorptances of the unshaded system:

$$\mathcal{A}_{k;(S,L)}^{f}(\theta) = u(\theta,\phi)\mathcal{A}_{k;(1,L)}^{f}(\theta) + \left(1 - u(\theta,\phi)\right)\frac{T_{S}^{fH}\left\langle \mathcal{A}_{k;(1,L)}^{f}\right\rangle_{D}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}}$$
$$+ \frac{u(\theta,\phi)R_{1,L}^{f}(\theta)s^{b}(\theta,\phi)R_{S}^{bH}\left\langle \mathcal{A}_{k;(1,L)}^{f}\right\rangle_{D}}{1 - R_{S}^{bH}\left\langle R_{1,L}^{f}\right\rangle_{D}}. \tag{B.3b}$$

Averaging these quantities to produce the corresponding ones applicable for diffuse incident solar is straightforward, and is done in the same manner as the transition between Equations B.2a and B.2b.

Using Equation 2 to combine these quantities into a solar heat gain coefficient yields

$$
\begin{aligned}
SHGC_{(S,L)}(\theta,\phi) = \ & u(\theta,\phi)SHGC_{(1,L)}(\theta) \\
& +\left[\left(1-u(\theta,\phi)\right)\frac{T_S^{fH}}{1-R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}+\frac{u(\theta,\phi)R_{1,L}^f(\theta)s^b(\theta,\phi)R_S^{bH}}{1-R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}\right]\left\langle SHGC_{(1,L)}\right\rangle_D \\
& +u(\theta,\phi)\sum_{k=1}^{L}\Delta N_k \mathcal{A}_{k;(1,L)}^f(\theta) \\
& +\left[\left(1-u(\theta,\phi)\right)\frac{T_S^{fH}}{1-R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}+\frac{u(\theta,\phi)R_{1,L}^f(\theta)s^b(\theta,\phi)R_S^{bH}}{1-R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}\right]\sum_{k=1}^{L}\Delta N_k\left\langle \mathcal{A}_{k;(1,L)}^f\right\rangle_D \\
& +N_S\cdot\left[\left(1-u(\theta,\phi)\right)\mathcal{A}_S^f+\left(\frac{\left(1-u(\theta,\phi)\right)T_S^{fH}\left\langle R_{1,L}^f\right\rangle_D}{1-R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}+\frac{u(\theta,\phi)R_{1,L}^f(\theta)s^b(\theta,\phi)}{1-R_S^{bH}\left\langle R_{1,L}^f\right\rangle_D}\right)\mathcal{A}_S^b\right]
\end{aligned}
\tag{B.4}
$$

The presence of the shading layer may modify the inward-flowing fractions for the glazing by amounts ΔN_k from their values without the shading layer, leading to the corresponding terms in the equation.

While this expression is dependent on both θ and ϕ, the ϕ dependence comes from the geometric transmittance through the shading. This dependence is most significant for a venetian blind or louver system, where it is essentially a dependence on the profile angle. [The profile angle, ψ, is defined by the equation

$$\tan\psi = \tan\theta\sin\phi$$

where θ (incident angle) is the polar and ϕ the azimuthal angle of a ray pointing to the radiation source in a right-handed coordinate system for which the z-axis is perpendicular to the fenestration (pointing outward), the x- and y-axes lie in the plane of the fenestration, and the x-axis points along the one of the venetian blind or louver slats.]

Between-Glass Shading

We next consider a shading layer, S, placed somewhere within an L-layer glazing system. This case will require a renumbering of the glazing system layers. If we include the shading layer, S, in the system, then we have an L+1 layer system with the layers 1,…,S-1 outside the shading layer and the layers S+1,…,L+1 to the inside of the layer.

In determining the properties of this system we shall consider it to be composed of three subsystems, an outer glazing subsystem consisting of the layers 1,..,S-1, the shading layer, S, and an inner glazing subsystem consisting of the layers S+1,…,L+1. In addition, we shall need to consider two alternative groupings. First, we shall consider the

combination of an outer subsystem of the layers 1,…,S and an inner subsystem containing the layers S+1,…,L+1. Second, we shall consider the combination of an outer subsystem of layers 1,…,S-1 and an inner subsystem of layers S,…,L+1. In the first case the outer subsystem is a glazing with interior shading, while in the second case the inner subsystem is a glazing with exterior shading.

As in the case of exterior shading, we must first consider the geometric fraction $u(\theta, \phi)$ of the incident radiation that passes through the shading layer S without encountering any of the shading material. This situation is similar to the cases discussed above, except that (primarily near normal incidence) special provisions must be taken to account for specular reflections through the gaps in the shading:

$$T_{unshaded} \; = \; u(\theta, \phi)T_{1,L}^{unshaded}(\theta)\Big[1 \; - \; s^b(\theta, \phi)R_{S+1,L+1}^f(\theta)\Big] \tag{B.5a}$$

where, as in the treatment of exterior shading, the quantity $s^b(\theta, \phi)$ represents the fraction of the incident radiation which, having passed through gaps in the shading layer and been reflected by the inner glazing system, strikes the shading material from the back side. This quantity will be zero at normal incidence. The above expression does not attempt to account for off-normal-incidence cases where radiation passing through a particular gap in the shading passes through different gaps on reflection. Treatment of this situation is extremely complicated and lies beyond the scope of this paper.

Transmission through the shaded glazing system can be calculated from Equation 10a, taking the subsystem M to be the outer glazing layers (1,S-1), and N to be the layers (S,L+1):

$$\Big[T_{1,L+1}^{fH}(\theta)\Big]_{shaded} \; = \; \frac{\big(1 \, - \, u(\theta, \phi)\big)T_{1,S-1}(\theta)T_{S,L+1}^{fH}(\theta)}{1 \, - \, \big\langle R_{1,S-1}^b\big\rangle_D R_{S,L+1}^{fH}(\theta)} \tag{B.5b}$$

In this expression, following the discussion of Equation 9, the fact that (S,L+1) is a glazing with an exterior shading layer leads to the replacement of the specular quantities by either the hemispherical total or diffuse average quantities. We can obtain the transmittance for this subsystem from Equation B.1a:

$$\Big[T_{S,L+1}^{fH}(\theta)\Big]_{shaded} \; = \; \frac{T_S^{fH}\big\langle T_{S+1,L+1}\big\rangle_D}{1 \, - \, R_S^{bH}\big\langle R_{S+1,L+1}^f\big\rangle_D}$$

The front reflectance for this subsystem (excluding the radiation passing through gaps in the shading) is

$$\Big[R_{S,L+1}^{fH}(\theta)\Big]_{shaded} \; = \; R_S^{fH} \; + \; \frac{T_S^{fH}T_S^{bH}\big\langle R_{S+1,L+1}^f\big\rangle_D}{1 \, - \, R_S^{bH}\big\langle R_{S+1,L+1}^f\big\rangle_D}$$

When these are substituted into Equation B.5b we obtain

$$\left[T^{fH}_{1,L+1}(\theta)\right]_{shaded} = \frac{(1 - u(\theta,\phi))T_{1,S-1}(\theta)T^{fH}_S\langle T_{S+1,L+1}\rangle_D}{1 - \langle R^b_{1,S-1}\rangle_D R^{fH}_S - \left[R^{bH}_S + \langle R^b_{1,S-1}\rangle_D\left(R^{fH}_S R^{bH}_S - T^{fH}_S T^{bH}_S\right)\right]\langle R^f_{S+1,L+1}\rangle_D} \quad \text{(B.5c)}$$

In addition, there is the transmittance due to radiation that is transmitted through the outer glazing, passes through the gaps in the shading layer, is (specularly) reflected by the inner glazing and is (diffusely) re-reflected from the back of the shading layer. Since the portion of the specularly reflected radiation that does not strike the shading layer material has already been included in Equation B.5a, we can calculate this case by considering that a fraction $u(\theta,\phi)s^b(\theta,\phi)R^f_{S+1,L+1}(\theta)$ of the incident radiation passes through the gaps in the shading, is specularly reflected by the inner glazing system and is incident on the back side of the shading layer. One can consider this to be the back side of the system (1,S) consisting of the shading layer and the outer glazing system. All radiation reflected by this system will be diffuse, because it will be either diffusely reflected by the shading layer or diffusely transmitted by the shading layer, reflected (any number of times) by the outer glazing layer, and again (diffusely) transmitted by the shading layer. We can therefore write

$$\left[T^{fH}_{1,L+1}(\theta)\right]_{rerefl} = \frac{u(\theta,\phi)s^b(\theta,\phi)R^f_{S+1,L+1}(\theta)R^{bH}_{1,S}\langle T_{S+1,L+1}\rangle_D}{1 - R^{bH}_{1,S}\langle R^f_{S+1,L+1}\rangle_D}$$

where it is unnecessary to indicate a hemispherical average over the incidence angle for $R^{bH}_{1,S}$, since this quantity is by assumption independent of incident angle. In fact,

$$R^{bH}_{1,S} = R^{bH}_S + \frac{T^{bH}_S\langle R^b_{1,S-1}\rangle_D T^{fH}_S}{1 - R^{fH}_S\langle R^b_{1,S-1}\rangle_D} ,$$

we have

$$\left[T^{fH}_{1,L+1}(\theta)\right]_{rerefl} = \frac{u(\theta,\phi)s^b(\theta,\phi)R^f_{S+1,L+1}(\theta)\langle T_{S+1,L+1}\rangle_D\left(R^{bH}_S + \left(T^{bH}_S T^{fH}_S - R^{bH}_S R^{fH}_S\right)\langle R^b_{1,S-1}\rangle_D\right)}{1 - R^{fH}_S\langle R^b_{1,S-1}\rangle_D - \left(R^{bH}_S + \left(T^{bH}_S T^{fH}_S - R^{bH}_S R^{fH}_S\right)\langle R^b_{1,S-1}\rangle_D\right)\langle R^f_{S+1,L+1}\rangle_D} \quad \text{(B.5d)}$$

The resulting transmittance is the sum of Equations B.5a, B.5c, and B.5d,

$$T^{fH}_{S,L+1}(\theta) = T_{unshaded} + \left[T^{fH}_{S,L+1}(\theta)\right]_{shaded} + \left[T^{fH}_{S,L+1}(\theta)\right]_{rerefl} \quad \text{(B.6)}$$

Absorption in the shading system may be due to specular radiation that strikes the shading layer on either the front or back surface, since radiation may pass through the gaps in the shading and be specularly reflected by the inside glazing system back onto the shading layer. We consider first radiation incident on the front side the shading layer. Prior to any multiple reflections, this incident radiation will be a fraction $(1 - u(\theta,\phi))T_{1,S-1}(\theta)$ of the radiation incident on the whole system, taking into account

that some of the radiation has passed through the gaps in the shading layer. This radiation incident on the shading layer may be absorbed, reflected or transmitted. The reflected radiation, which is diffuse, may be rereflected by the outer glazing system, and will give rise to the usual multiple-reflection series enhancement of the radiation incident on the shading layer. However, the radiation transmitted by the shading layer (which is also diffuse after transmission) may be reflected by the inner glazing system, retransmitted by the shading layer, reflected by the outer glazing system, and again incident on the shading layer. This effect may be included by considering the shading layer and the inner glazing system as a unit and using its subsystem reflectance in the multiple reflectance series, so that the front side absorption of the shading layer is

$$
\mathcal{A}_{Front} = \frac{\left(1 - u(\theta, \phi)\right)T_{1,S-1}(\theta)A_S^f}{1 - \left\langle R_{1,S-1}^{bH} \right\rangle_D \left\langle R_{S,L+1}^{fH} \right\rangle_D}
$$

In writing this expression we have multiple-pass reflection of specular radiation through the shading gaps, which eventually strikes the front side of the shading material, assuming that the amount of this radiation is small. Prior to any consideration of multiple diffuse reflections between the shading layer and the inner glazing system, radiation can be incident on the back side of the shading layer by two mechanisms after first being transmitted by the outer glazing: (1) passing through the gaps in the shading and being (specularly) reflected by the inner glazing system, and (2) being (diffusely) transmitted by the shading layer and subsequently reflected by the inner glazing. We can write this back-incident radiation as the following fraction of the radiation originally incident on the system:

$$
u(\theta, \phi)T_{1,S-1}(\theta)R_{S+1,L+1}^{b}(\theta)s^b(\theta, \phi) + \left(1 - u(\theta, \phi)\right)T_{1,S}^{fH}(\theta)\left\langle R_{S+1,L+1}^{fH} \right\rangle_D
$$

In this expression use of the subsystem transmittance $T_{1,S}^{fH}(\theta)$ includes cases where there are multiple reflections between the shading layer and the outer glazing system prior to transmittance by the shading system. This radiation may be absorbed by the shading system, but it may also be reflected by the shading system and again by the inner glazing system prior to absorption. This may happen any number of times. Moreover, it might also be transmitted by the shading layer, reflected by the outer glazing system, transmitted by the shading layer, and again reflected by the inner glazing system prior to absorption. Any number of such double transmissions may occur in combination with multiple reflections. All of these effects are included by considering the outer glazing and the shading layer as a unit (1,S) in the multple reflection series, yielding

$$
\mathcal{A}_{back} = \frac{u(\theta, \phi)T_{1,S-1}(\theta)R_{S+1,L+1}^{f}(\theta)s^b(\theta, \phi) + \left(1 - u(\theta, \phi)\right)T_{1,S}^{fH}(\theta)\left\langle R_{S+1,L+1}^{fH} \right\rangle_D}{1 - \left\langle R_{1,S}^{bH} \right\rangle_D \left\langle R_{S+1,L+1}^{f} \right\rangle_D} \mathcal{A}_{S:(1,S)}^b
$$

where

$$\mathcal{A}^b_{S;(1,S)} = \mathcal{A}^b_S + \frac{T^{bH}_S \left\langle R^b_{1,S-1} \right\rangle_D \mathcal{A}^f_S}{1 - R^{fH}_S \left\langle R^b_{1,S-1} \right\rangle_D} \,. \tag{B.6a}$$

The sum of the front and back absorptions gives the shading layer absorption:

$$\mathcal{A}^f_{S;(1,L+1)} = \frac{\left(1 - u(\theta,\phi)\right)T_{1,S-1}(\theta)\mathcal{A}^f_S}{1 - \left\langle R^{bH}_{1,S-1} \right\rangle_D \left\langle R^{fH}_{S,L+1} \right\rangle_D}$$
$$+ \frac{\left[u(\theta,\phi)T_{1,S-1}(\theta)R^f_{S+1,L+1}(\theta)s^b(\theta,\phi) + \left(1 - u(\theta,\phi)\right)T^{fH}_{1,S}(\theta)\left\langle R^{fH}_{S+1,L+1} \right\rangle_D\right]\mathcal{A}^b_{S:(1,S)}}{1 - \left\langle R^{bH}_{1,S} \right\rangle_D \left\langle R^f_{S+1,L+1} \right\rangle_D} \tag{B.7a}$$

which can be further expanded by substituting in Equation B.6a.

In the following we will denote the layer absorption of layer k in the unshaded glazing system, with the shading layer left out of the numbering, by $\mathcal{A}^{f,\,unshaded}_{k;(1,L)}(\theta)$. The shaded and unshaded system layer numbering systems are shown in Figure 5. For layers in the outer glazing system (i.e., $1 \le k \le S\text{-}1$)

$$\mathcal{A}^f_{k;(1,L+1)}(\theta) = u(\theta,\phi)\mathcal{A}^{f,\,unshaded}_{k;(1,L)}(\theta)$$
$$+\left(1 - u(\theta,\phi)\right)\left(\mathcal{A}^f_{k:(1,S-1)}(\theta) + \frac{T_{1,S-1}(\theta)R^{fH}_S \left\langle \mathcal{A}^b_{k:(1,S-1)} \right\rangle_D}{1 - \left\langle R^f_{S,L+1} \right\rangle_D \left\langle R^b_{1,S-1} \right\rangle_D} \right) \tag{B.7b}$$

and for layers in the inner glazing system ($S+1 \le k \le L+1$)

$$\mathcal{A}^f_{k;(1,L+1)}(\theta) = u(\theta,\phi)\mathcal{A}^{f,\,unshaded}_{k;(1,L)}(\theta)$$
$$+\left(1 - u(\theta,\phi)\right)\frac{T^{fH}_{1,S}(\theta)\left\langle \mathcal{A}^f_{k:(S+1,L+1)} \right\rangle_D}{1 - \left\langle R^f_{S+1,L+1} \right\rangle_D \left\langle R^b_{1,S} \right\rangle_D} \tag{B.7c}$$

When these equations are substituted into Equation 2 we obtain

$$SHGC_{1,L+1}(\theta) = u(\theta,\phi)\left\{ SHGC_{1,L}^{unshaded}(\theta) + \sum_{k=1}^{L} \Delta N_k \mathcal{A}_{k;(1,L)}^{f,unshaded}(\theta) \right\}$$

$$+(1-u(\theta,\phi)) \times \left\{ \frac{T_{1,S-1}(\theta)T_{S,L+1}^{fH}(\theta)}{1-\left\langle R_{1,S-1}^{b}\right\rangle_D R_{S,L+1}^{fH}(\theta)} \right.$$

$$+\sum_{k=1}^{S-1} N_k \mathcal{A}_{k;(1,S-1)}^{f}(\theta) + \frac{T_{1,S-1}(\theta)R_S^{fH}\sum_{k=1}^{S-1} N_k\left\langle \mathcal{A}_{k;(1,S-1)}^{b}\right\rangle_D}{1-\left\langle R_{S,L+1}^{f}\right\rangle_D\left\langle R_{1,S-1}^{b}\right\rangle_D}$$

$$+\frac{T_{1,S-1}(\theta)N_S\mathcal{A}_S^{f}}{1-\left\langle R_{1,S-1}^{bH}\right\rangle_D\left\langle R_{S,L+1}^{fH}\right\rangle_D} + \frac{T_{1,S}^{fH}(\theta)\left\langle R_{S+1,L+1}^{fH}\right\rangle_D N_S\mathcal{A}_{S:(1,S)}^{b}}{1-\left\langle R_{1,S}^{bH}\right\rangle_D\left\langle R_{S+1,L+1}^{f}\right\rangle_D} \tag{B.8}$$

$$+\frac{T_{1,S}^{fH}(\theta)\sum_{k=S+1}^{L+1} N_k\left\langle \mathcal{A}_{k:(S+1,L+1)}^{f}\right\rangle_D}{1-\left\langle R_{S+1,L+1}^{f}\right\rangle_D\left\langle R_{1,S}^{b}\right\rangle_D} \left.\right\}$$

$$+u(\theta,\phi)s^{b}(\theta,\phi) \times \left\{ \frac{R_{S+1,L+1}^{f}(\theta)R_{1,S}^{bH}\left\langle T_{S+1,L+1}\right\rangle_D}{1-R_{1,S}^{bH}\left\langle R_{S+1,L+1}^{f}\right\rangle_D} \right.$$

$$-T_{1,L}^{unshaded}(\theta)R_{1,S-1}^{b}(\theta)R_{S+1,L+1}^{f}(\theta) + \frac{T_{1,S-1}(\theta)R_{S+1,L+1}^{f}(\theta)\mathcal{A}_{S:(1,S)}^{b}}{1-\left\langle R_{1,S}^{bH}\right\rangle_D\left\langle R_{S+1,L+1}^{f}\right\rangle_D} \left.\right\}$$

The second term in the first set of curly brackets in this equation occurs because the presence of the shading layer modifies the inward-flowing fraction of layer k by an amount ΔN_k.

Appendix C. Complete Expression for BAC

The equation for BAC is obtained by manipulating Equation B.8 to put it into the form of Equation 22, after which the terms corresponding to the quantity BAC in Equation 22 can be identified. When this is done one obtains the expression

$$
BAC = \left\{ \frac{T^{fH}_{S,L+1}(\theta)}{1-\langle R^b_{1,S-1}\rangle_D R^{fH}_{S,L+1}(\theta)} + N_S \frac{\mathcal{A}^f_S}{1-\langle R^{bH}_{1,S-1}\rangle_D \langle R^{fH}_{S,L+1}\rangle_D} + \frac{R^{fH}_S \sum_{k=1}^{S-1} N_k\langle\mathcal{A}^b_{k:(1,S-1)}\rangle_D}{1-\langle R^f_{S,L+1}\rangle_D\langle R^b_{1,S-1}\rangle_D} \right.
$$

$$
+ \frac{T^{fH}_S \sum_{k=S+1}^{L+1} N_k\langle\mathcal{A}^f_{k:(S+1,L+1)}\rangle_D}{\left(1-\langle R^f_{S+1,L+1}\rangle_D\langle R^b_{1,S}\rangle_D\right)\left(1-\langle R^b_{1,S-1}\rangle R^{fH}_S\right)} + N_S \frac{\langle R^{fH}_{S+1,L+1}\rangle_D T^{fH}_S \mathcal{A}^b_{S:(1,S)}}{\left(1-R^{bH}_{1,S}\langle R^f_{S+1,L+1}\rangle_D\right)\left(1-\langle R^b_{1,S-1}\rangle R^{fH}_S\right)}
$$

$$
+ \frac{u(\theta,\phi)s^b(\theta,\phi)}{(1-u(\theta,\phi))}\left[N_S \frac{R^f_{S+1,L+1}(\theta)\mathcal{A}^b_{S:(1,S)}}{\left(1-R^{bH}_{1,S}\langle R^f_{S+1,L+1}\rangle_D\right)} \right.
$$

$$
+ \frac{R^f_{S+1,L+1}(\theta)R^{bH}_{1,S}\langle T_{S+1,L+1}\rangle_D}{\left(1-R^{bH}_{1,S}\langle R^f_{S+1,L+1}\rangle_D\right)} - \frac{R^f_{S+1,L+1}(\theta)T_{S+1,L+1}(\theta)}{\left(1-R^f_{S+1,L+1}(\theta)R^b_{1,S-1}(\theta)\right)} \left.\left. \right] \right\}
$$

$$
- \frac{\sum_{k=1}^{L} N^0_k A^{f,unshaded}_{k;(1,L)}}{SHGC_{1,S-1}(\theta)} \times \left\{ N_S \frac{\mathcal{A}^f_S}{1-\langle R^{bH}_{1,S-1}\rangle_D\langle R^{fH}_{S,L+1}\rangle_D} + \frac{R^{fH}_S \sum_{k=1}^{S-1} N_k\langle\mathcal{A}^b_{k:(1,S-1)}\rangle_D}{1-\langle R^f_{S,L+1}\rangle_D\langle R^b_{1,S-1}\rangle_D} \right.
$$

$$
+ \frac{T^{fH}_S \sum_{k=S+1}^{L+1} N_k\langle\mathcal{A}^f_{k:(S+1,L+1)}\rangle_D}{\left(1-\langle R^f_{S+1,L+1}\rangle_D\langle R^b_{1,S}\rangle_D\right)\left(1-\langle R^b_{1,S-1}\rangle R^{fH}_S\right)} + N_S \frac{\langle R^{fH}_{S+1,L+1}\rangle_D T^{fH}_S \mathcal{A}^b_{S:(1,S)}}{\left(1-R^{bH}_{1,S}\langle R^f_{S+1,L+1}\rangle_D\right)\left(1-\langle R^b_{1,S-1}\rangle R^{fH}_S\right)}
$$

$$
+ \frac{u(\theta,\phi)s^b(\theta,\phi)}{(1-u(\theta,\phi))}\left[N_S \frac{R^f_{S+1,L+1}(\theta)\mathcal{A}^b_{S:(1,S)}}{\left(1-R^{bH}_{1,S}\langle R^f_{S+1,L+1}\rangle_D\right)} + \frac{R^f_{S+1,L+1}(\theta)R^{bH}_{1,S}\langle T_{S+1,L+1}\rangle_D}{\left(1-R^{bH}_{1,S}\langle R^f_{S+1,L+1}\rangle_D\right)} \right.
$$

$$
- \frac{R^f_{S+1,L+1}(\theta)T_{S+1,L+1}(\theta)}{\left(1-R^f_{S+1,L+1}(\theta)R^b_{1,S-1}(\theta)\right)} \left.\left. \right] \right\}
$$

$$
+ \frac{1}{SHGC_{1,S-1}(\theta)}\left\{ \left[\frac{1-\langle R^b_{1,S-1}\rangle_D R^{fH}_{S,L+1}(\theta) - T^{fH}_{S,L+1}(\theta)}{1-\langle R^b_{1,S-1}\rangle_D R^{fH}_{S,L+1}(\theta)}\right]\sum_{k=1}^{S-1} N_k\mathcal{A}^f_{k:(1,S-1)}(\theta) \right.
$$

$$
+ \frac{T^{fH}_{S,L+1}(\theta)\sum_{k=1}^{S-1}\Delta N_k\mathcal{A}^f_{k:(1,S-1)}(\theta)}{1-\langle R^b_{1,S-1}\rangle_D R^{fH}_{S,L+1}(\theta)} + \frac{u(\theta,\phi)}{(1-u(\theta,\phi))}\sum_{k=1}^{L}\Delta N_k\mathcal{A}^{f,unshaded}_{k;(1,L)}(\theta) \left. \right\}
$$