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Speed, Accuracy, and VL

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pending upon the task, cognition may proceed partially in parallel with vision and thus add little to the overall time.

Speed is the inverse of the total time, and for normal visual work the total time is the sum of t_{nv} , the time needed for any non-visual components of the task such as cognition, and t_v , the time needed for the strictly visual components of the task. The visual component of the task can be broken down into visibility for guidance and stability, so that eye motions are made to the correct locations and the subject knows where in the visual field they are, and visibility for detection, recognition, and identification of the visual target that is being fixated. The VL method was developed to fit accuracy of detection, or resolution of an isolated target of fixed size, fixed location in the visual field, and fixed exposure time. If the bulk of the visual work involves a well defined spatial frequency range or size, then the VL method should provide reasonable estimates for speed and accuracy.

In our VL approximation we replace t_v with t_{vi} , and t_{nv} with t_{nvi} . The t_{nvi} term includes saccade times as well as cognition times. It also includes any visual work involving a different size range than that fit by VL. Spatial organization and localization, for instance, can be expected to involve lower spatial frequencies (larger sizes) than resolution of task detail. If these factors are relatively unimportant, then t_{nvi} will be relatively independent of the task's visibility. Conversely, the VL approximation should not be expected to work well if there is no dominant size. When the visual work involves text or numbers, identification of the print should involve resolution of its line width, and this should provide a dominant spatial frequency to the overall task. Our best fits of Rea's data, which involve comparing number lists, seem to fit this approximation fairly well in that we find no more than about an 8 percent variation in t_{nvi} with luminance.

In the VL approximation, accuracy is a compressive function of VL alone. Blackwell has reported excellent fits using both normal and log-normal functions. We used the log-normal form in fitting Rea's data:

$$A = \int_{-\infty}^x e^{-(z^2)/2} dz \quad (1)$$

where $x = (\log_{10}(V) - \alpha)/\sigma$, V is a dynamic visibility level that takes into account time effects (see below), and α and σ are the fitted mean and standard deviation of the log-normal. At this stage in the development of the model, we are not attempting to predict the parameter values from first principles. The values are to be estimated by fitting to the data of interest.

The time needed to see a target is inversely related to its visibility, hence we expect t_{vi} to have the form $Q/g(VL)$ where Q is a time constant that is not dependent upon VL, and $g(VL)$ is some function of VL that can range from 0 to ∞ . The resulting expression for speed, S , at a fixed level of accuracy, A , is similar to an expression we derived in an earlier paper:¹⁰

$$S(VL,A) = (t_{nv} + t_v)^{-1} \sim (t_{nvi} + t_{vi})^{-1} = [t_{nvi} + Q/g(VL)]^{-1} \quad (2)$$

To get an expression for $Q/g(VL)$ it is helpful to recall that a given VL(t) is supposed to relate to a fixed level of accuracy of detection, or resolution. Thus we want to fix VL(t) to the value V that corresponds to a particular accuracy level, A , and then for a given nominal VL value find the value of the time, t , necessary to transform VL to V . To do this requires an expression for VL(t).

The dependence of the visibility of a task on time depends on the type of task. For simple detection, visibility appears to be dependent upon total flux (intensity times time) up to a critical time, and intensity alone above the critical time.¹¹ For a more complex task, such as acuity, the transition between the limiting types of behavior is smooth rather than abrupt.¹² Adrian has reviewed the data in this field and has proposed a rational fraction expression which applies to resolution tasks, and can be written in terms of VL(t):¹³

$$VL(t) = VL(\infty) * t / [t + a(L_b, \text{size})] \quad (3)$$

where t and a have units of seconds, and a is a function of the background luminance, L_b , and the angular size of the target. The parameter a is Q of Equation 2. Physically, a is the time needed to reach half the limiting sensitivity of the visual system. The function a varies from 0.12 to 0.35 s for L_b in the range from 10^{-6} to 10^4 cd/m², and size in the range from 0.3–50 minutes of arc. Adrian claims that his expression is a refinement of an earlier expression of Blondel's, where a was fit as a constant equal to 0.21 s.

Let $V \equiv VL(t)$ be the visibility level needed to reach the accuracy level A . Solving Equation 3 for the time yields the expression for t_{vi} :

$$t_{vi} = a(L_b, \text{size}) * V / [VL(\infty) - V] \quad (4)$$

CIE 19 and CIE 19/2 give VL as measured with an exposure time of 0.2 s. The value of VL in these two reports was scaled so that VL = 1 corresponds to the reference contrast level in a method produced by an adjustments type of detection experiment. This contrast level is about 2.5 times higher than what is

measured on forced-choice types of experiments. This point is discussed more fully in the validation section. Rewriting $VL(\infty)$ in terms of $VL = VL(0.2)$ yields an expression for speed in terms of the forced-choice definition of VL :

$$S(VL,A) = \left[t_{nv1} + a(L_b, \text{size}) \right. \\ \left. * V / (VL * [(a + 0.2)/0.2] - V) \right]^{-1} \quad (5)$$

and in the Blondel approximation that $a = 0.21$ s:

$$S(VL,A) \cong [t_{nv1} + 0.21 * V / (2.05 * VL - V)]^{-1} \quad (6)$$

In the approximate expression given by Equation 6 the parameter Q of Equation 2 is actually a constant. However, the more general expressions in Equations 4 and 5 show that speed, unlike accuracy, should depend upon luminance and size, independently of its dependence upon VL .

Equation 5 (or 6) has a minimum of three unknown (fittable) parameters: t_{nv1} , α and σ , the last two of which enter implicitly via Equation 1. As we noted earlier, t_{nv1} should be only approximately constant, so a fit may involve more than three unknowns.

To use Equation 5 (or 6) the value of V must be computed from accuracy via the inverse of Equation 1. A simple approximation for this operation is given in Abramowitz and Stegun.¹⁴ The values of luminance and contrast enter in Equation 5 in the computation of VL . This computation requires knowing, or being able to compute, reference contrast levels. Rea measured reference contrast values for the numerical verification experiment, and these values can be used directly. Alternatively, reference values can be computed from fits to Blackwell's data.^{13,15} We discuss fits with both procedures later.

Equations 5 (or 6) and 1 can also be used to compute accuracy as a function of speed. In this case V is determined by inverting Equation 5 and is then inserted into Equation 1 to compute the accuracy.

Equation 5 (or 6) suggests that the time taken to perform visual work should be roughly inversely proportional to VL . Figure 1 plots values from Rea's numerical verification experiment against $(VL - V/2.05)^{-1}$. VL was calculated from Rea's published contrasts and "threshold" contrasts. Rea's experiment involved comparing 20 numbers on a test sheet against those on a response sheet.² Both time and accuracy were dependent upon the visibility. The response sheet always had high contrast, so Rea computed what we have called *adjusted times* by subtracting an estimate of the time to read the response sheet from the total reading time.

Rea used nine parameters to fit his time data. The line in Figure 1 shows that in the approximation

of fixed accuracy (fixed V), a two-parameter fit is all that is needed to provide a qualitatively excellent fit. The intercept of the line is 13 s, corresponding to a non-visual component of 0.65 s per fixation. This suggests that the numerical verification task is moderately difficult from a cognitive viewpoint, as 0.65 s is substantially longer than the fixation times that have been demonstrated for simpler tasks.⁶ The parameter V determines the slope and zero performance point. Its value, 2.15, is less easily interpreted, as values of α and σ were not determined in this fit, and it cannot be directly related to a known accuracy. All that can be said is that its value is consistent with the VL values that give moderate accuracies on Landolt-C resolution experiments, and that moderate accuracies were measured for the Rea experiment. The simplicity of the fit shown in Figure 1, when coupled with its reasonableness, is a powerful argument for our model.

Comparison to Rea and CIE 19/2 models

In this section we discuss the Rea and CIE models and their differences. Rea's model is more flexible than the CIE model, and potentially can give better fits. We feel that our model is better yet, in that it is more physically based.

Performance has an ogival or compressive shape versus stimulus intensity. Past a certain point increases in the stimulus provide little gain in perfor-

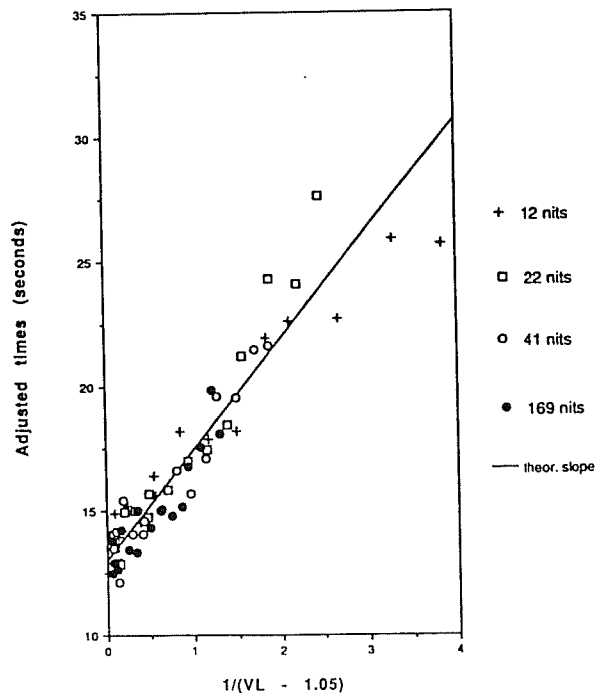


Figure 1—Adjusted times

mance. Rea uses a ratio of power law terms that he attributes to Naka and Rushton to fit this compressive shape:²

$$S = S_{\max} * \Delta L^n / [\Delta L^n + k^n] \quad (7)$$

where S_{\max} is the fitted maximum speed, $\Delta L = L_t - L_t(P=0)$, with L_t being the target luminance and $L_t(P=0)$ the maximum target luminance that gives zero performance, and n and k are free parameters. All three of the fitted parameters, S_{\max} , n , and k , were given as three parameter functions of the background luminance in Rea's paper.

The log-normal form used by Blackwell also has a compressive shape and, as Rea notes, fits the Landolt-C orientation data equally well. Rea's initial justification for using the ratio format instead of the log-normal form is that the "robustness" of the former has "led to its current preference in the visual sciences for modeling compressive suprathreshold visual response. . . ."²

The log-normal form fits the Landolt-C data with only two parameters, so there is no obvious advantage in using the ratio form for this relatively simple type of visual performance data. Blackwell claims that the Landolt-C data has the particularly simple form that for fixed target location and exposure time, different levels of performance (accuracy) result from a simple contrast or VL multiplication. The CIE 19/2 model retains this feature for any given task, in that performance, which is now given as some unspecified mix of speed and accuracy, is supposed to be fit as a function of VL alone. Rea's model can be rewritten with VL as a parameter by first rewriting ΔL in terms of VL:

$$\Delta L = (L_b * C_{50}) * (VL - C_0 / C_{50}) = (L_b * C_{50}) * (VL - X) \quad (8)$$

where L_b is the background luminance, C_{50} and C_0 are the contrast levels for 50 and 0 percent performance, and the ratio $X = C_0 / C_{50}$ is the ratio of these contrasts at the 0 and 50 percent criterion accuracy levels. Substituting from Equation 8 into Equation 7 gives:

$$S = S_{\max} * (VL - X)^n / [(VL - X)^n + [k / (L_b * C_{50})]^n] \quad (9)$$

This function will be dependent upon VL alone only if X is independent of size and luminance and there are restrictions on S_{\max} , k , and n . Rea claims that the numerical verification test data cannot be fit well simply as a function of VL alone. A significant advantage of the ratio model over the CIE 19/2 model is that it easily handles this situation.

Our derivation of how VL enters into "normal" visual work provides several reasons why performance

may be dependent solely upon VL in one type of experiment and not in another. In the Landolt-C experiment, resolution of the C's orientation requires detection of features of a well defined size. In the numerical verification experiment the eye makes a saccade from one set of digits to another. The set of digits as a whole is a fairly large target. After the saccade, the subject has to resolve each of the digits, a task of a fairly small size. Localization of the digits with respect to each other may require information from an intermediate size range. The shape and magnitude of the contrast sensitivity curve depend upon size, thus a single VL value should not be sufficient to completely characterize performance. A second consideration arises from the fact that performance on the numerical verification experiment is in terms of speed, not accuracy. Speed, size, and luminance enter independently through their effect on the parameter a in Equation 3. Finally, Blackwell has shown evidence that σ of Equation 1 is weakly dependent upon the exposure time.¹ We have not yet included this effect in our model. Rea's approach fits the data, but it is not linked to the macroscopic visual processes. Rea uses the ratio function because it fits the electrophysiological response curves, but this approach ignores eye motion and cognition. What is perhaps most important, however, is that Rea treated accuracy and speed separately. It should be obvious that taking longer at a task allows one to reach higher accuracies. Rea noted a clear

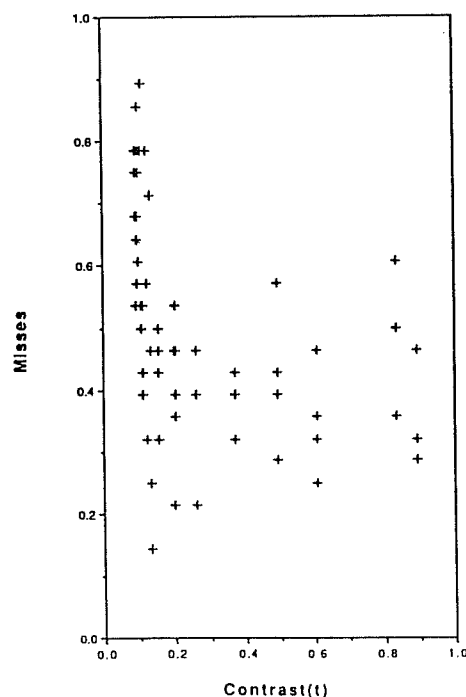


Figure 2—Misses versus contrast

decline in accuracy at the lower contrasts. **Figure 2** plots the miss rate against contrast. It shows an essentially constant level of performance at high contrasts and a sharp decline at lower contrasts. To maintain a fixed accuracy at the lower contrast levels would have required the subjects to take even more time than they did. This would have obviously resulted in different fitted values for Rea's model. In short, changing the subject's accuracy criteria changes the fit to the model. Our model, although primitive, explicitly links accuracy and speed and is therefore criterion free.

Criterion dependence is also a problem for the CIE 19/2 model. On the other hand, its attempt to include information about the structure of visual work, such as saccade speed, fixation accuracy, viewing eccentricity, and information requirements, is of great importance. Its attempt was logically flawed by the failure to distinguish between speed and accuracy, as well as some other errors, but the catalog of factors remains one that a complete model must consider.

Validation

In this section we describe our fit of **Equations 1** and **5** against Rea's numerical verification test time and accuracy data. We checked for the general validity of the fit in terms of standard statistical criteria, and we examined the relative goodness of the fit against Rea's fit of the data. We have already shown, via **Figure 1**, that under the assumption of fixed accuracy the time data follow an inverse VL type relationship fairly well. However, **Figure 2** showed that this fixed accuracy assumption is not correct. The normal fitting procedure would be to use either accuracy or time as an independent parameter, and then minimize the error in the other parameter. Unfortunately both time and accuracy had large errors. Instead of simply calculating V from accuracy or time and then minimizing the other variable, we chose the value of V that minimized the joint error in both time and accuracy. Two technical problems had to be dealt with before we did this constrained fit.

The first problem is that two types of accuracy were measured: the number of false positives (claiming that two numbers were different when in fact they were the same) and the number of misses (claiming that two numbers were the same when in fact they were different). Rea states that on average 17 of the 20 paired comparisons were in fact between identical numbers. The number of false positives, FP, averaged over subjects and runs ranged from 0 to 0.393 with an average of 0.032. The distribution was highly skewed, with the maximum number of false positives for a given condition (0.393) being over twice as large as the next largest value. On the other hand, a full 60 percent of the conditions had no recorded false positives. Converted

to accuracies via the formula $\text{accuracy} = (17\text{-FP})/17$, the false positive rates represent an accuracy range from 97.7–100 percent, with an average of 99.8 percent. The number of misses, M , ranged from 0.143 to 0.893 with an average of 0.483. This distribution was much more symmetric. Converted to accuracies via the formula $\text{accuracy} = (3\text{-}M)/3$, the miss rate ranges from 70 to 95 percent with an average of 84 percent.

The discrepancy in the two accuracy rates is similar to what is found when subjects are forced to state yes or no as to whether a target is present in a detection task.¹⁶ An alternative procedure, the forced-choice procedure, is to present the target in one of a number of spatial or temporal intervals and then require the subject to guess which had the stimulus. Subjects typically adopt biased criteria on the yes/no procedure in that they don't tend to say that they see the target until there is a considerably higher signal than that needed to get accuracy above chance on the forced-choice type procedure. Subjects can adopt fairly stable criteria, so the yes rate can be used as an accuracy measure. It is a measure that is subject to criterion shifts and may therefore be impossible to compare between experiments, and it is a noisier measure than the criterion-free value found in a forced choice procedure. In short, the numerical verification experiment has more non-visual complexities than had been thought.²

In the numerical verification experiment a difference in the numbers appears to be equivalent to the presence of a signal in the detection task. We therefore took the miss rate as our measure of accuracy. The extreme smallness and stability of the false-positive rate indicate a fairly stable criterion, and we ignored it in the rest of the analysis. The second problem we had was to get an objective measure of the "goodness" of fit. We fit our model to the data in Table A1 of Rea's paper. His data have already been averaged over subjects and runs, and standard error estimates were listed for each visibility condition. If each subject's data come from the same model, with different values of the parameters, then a fit to the averaged data can be spuriously good. Basically, what happens is that the data points have an inflated error estimate relative to the model predictions.

A check of Rea's model predictions versus the averaged time data gives a reduced χ^2 of 0.48, which is rejected as being too good at below the 0.05 percent significance level. We want to emphasize that this does not indicate a problem with Rea's analysis or fit. The problem we are addressing is how to use the averaged data from Table A1. A comparison of the sum of the squares of the time errors from Table A1, and from the analysis of variance Table A2.2 which averaged over repetitions before examining the deviations shows

that the between-subject variability is large relative to the within-subject variability. This confirms that our problem is data averaging and not simply over-parameterization of Rea's model. To get a more reasonable estimate of the time errors we used the property of the reduced χ^2 , that it is a ratio of the measured fit errors to an estimate of the intrinsic errors and should go to one if a fit is correct, or nearly so. We scaled the time error uncertainties by 0.689 so as to give Rea's fit a reduced χ^2 of 1.0.

A similar analysis of the miss rate error estimates indicated that between-subject variability was not large relative to overall variability. A fit of Rea's form to the accuracy data gave a reduced χ^2 of 1.16, which is perfectly reasonable. We therefore did not adjust the miss-rate error values. The net effect of our analysis is that we gave relatively more importance to the time data than would result from a direct application of the error rates of table A1.

The root-mean-square (rms) time error from Rea's model was 0.9 s. The error was calculated by applying the model to both the response and test luminances and contrasts. Rea's response time correction was applied to the data.

We fit Rea's accuracy data to his model by assuming that overall accuracy was the product of accuracies on the response and test sheets. The parameters were fit as functions of luminance using the same functional forms as Rea's time fit. The least-squares fit of the data gives an rms accuracy error of about 4.2 percent. The

parameters of the ratio fit had substantially different values for the accuracy and speed data. This is expected for the maximum performance level, since for speed it has units of inverse seconds and for accuracy it is a unitless percentage. The changes in n and k are evidence of the empirical nature of the fits.

At high visibility levels the value of V in Equation 5 is fairly unimportant. The visibility level on the response lists was always high, therefore to calculate times from Equation 5 we approximated by assuming that the same dynamic $VL(t)$ level, V , was reached on both lists. Reading time on the two lists was computed separately and summed. To calculate the accuracy of the comparison between the lists we used Equation 1 as a function of V . Our best fit was obtained by calculating VL from the reference contrast values given by Rea and using a separate value of t_{mvi} for each of the four luminance levels. Our use of four t_{mvi} values, the fact that there is both a response and test list, the fact that there are multiple comparisons in a run, and that each comparison requires two glimpses result in the following equation for the time calculation:

$$S(VL,A) = [40 * t_{mvi}(L_b) + 20 * a * V * \{1/(B * VL_r - V) + 1/(B * VL_t - V)\}]^{-1} \quad (10)$$

Here B is the collection of parameters $(a + 0.2)/0.2$; and VL_r and VL_t are the visibilities on the response and test lists respectively. The constants 20 and 40 represent the number of comparisons and the total number of fixations per run. The parameter a is a function of both size and luminance, as noted earlier. We have explicitly written t_{mvi} as a function of L_b in Equation 10 to emphasize the fact that we allowed different values of t_{mvi} for each of the different luminance levels in the experiment.

The four t_{mv} values ranged from 0.63 s per glimpse to 0.675 s per glimpse, with the slower values at the lower luminances. These differences are about equivalent to taking an extra saccade to settle properly on each number. The mean and standard deviation of the log-normal were 0.26 and 0.097 respectively. The rms time and accuracy errors were 0.76 s and 2.4 percent, respectively. Both errors are smaller than those found using Rea's method. The value of V ranged from 2.05 to 2.57. Positive and negative time estimation errors were almost evenly balanced, but there is a trend toward overestimation of time for the middle range of contrasts. Figure 3 shows a plot of Rea's adjusted times against the VL values on the test list. To illustrate the fit, we have plotted two lines. One line gives the calculated times with the value of t_{mvi} for the low luminance condition along with the maximum value of V calculated for that condition. The second

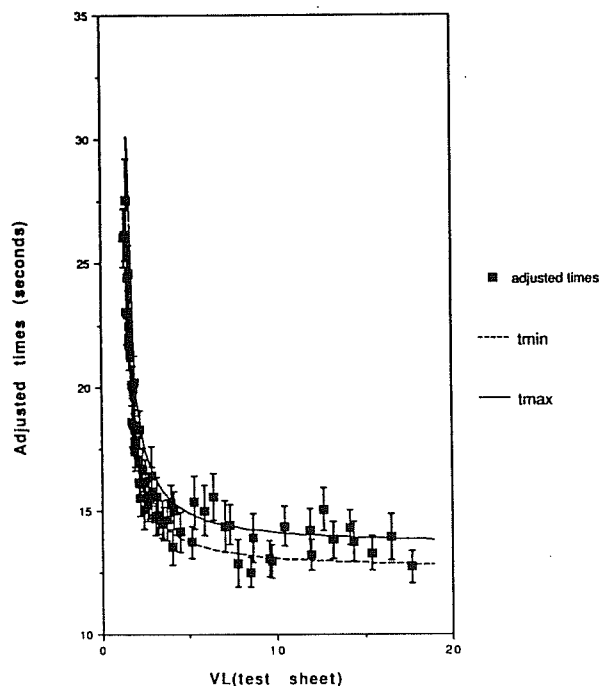


Figure 3—Rea's adjusted times versus approximate fitted bounds

line uses the value of t_{nvl} for the high luminance condition along with its minimum calculated V value. The V values calculated in this way are not the extreme values, but the lines do tend to bound the data.

We also did the fit, as above, with a reference contrast formula that fits Blackwell's contrast detection data in place of Rea's measured reference contrasts.¹⁵ The four t_{nvl} values for this fit range from 0.64 to 0.665 s and do not follow an obvious pattern. The mean and standard deviation of the log-normal are significantly different, although barely so, from the first fit, being 0.18 and 0.21 respectively. In conjunction with the fatter slope, the fitted values of V had a wider range, from 1.96 to 3.03. The fit is not as good as the fit with the measured reference contrasts and is barely acceptable. Rea has indicated concern over the applicability of Blackwell's contrast sensitivity data to more general situations. Our fits here suggest that Rea's concern may be a valid one.

Discussion

The joint time/accuracy fit gives a lower residual error rate than the separate ratio fits. Again what we are emphasizing is the joint nature of the fit. The accuracy and time-dependent portions of the fit are not sophisticated, and conceivably could be better fit with a ratio model. The disadvantage of the ratio model is that it is strictly empirical and fairly complex, as least as implemented with luminance-dependent parameters. The data are not precise enough to require such added complexity without also adding explanatory power in terms of physically measurable terms.

There is obviously more work to be done on the joint time/accuracy form. Although the derivation is consistent with small changes in t_{nvl} as a function of luminance, we have not explicitly modelled this variation and do not know if its magnitude is reasonable. Since the fits with Rea's reference contrast values gave a larger variation in t_{nvl} than the fits with reference contrasts derived from Blackwell's data, a better handle on expected variation would help in deciding whether the latter fits were reasonable and, by extension, whether the Blackwell reference contrast data are generally applicable. A similar lack of specificity exists for accuracy modelling. The log-normal form provides an empirical fit; we do not have an explicit way of determining the log-normal parameters. Our discussion of what accuracy is on the numerical verification experiment indicates that the problem is moderately complex and deserves more analysis.

Conclusion

We have presented a simple visibility/performance model that incorporates two critical ideas. The first is that speed and accuracy have to be analyzed as joint

independent/dependent variables. Analyzing an experiment in terms of one or the other variable without fixing the second variable makes the results criterion-dependent and non-generalizable. The second idea is that eye motions and cognition affect the structure of the performance relationship. This is why we separated the total time into non-visual and visual components. An advantage of this separation is that the parameters and predictions are in units of seconds, instead of being in units of relative performance. Our analysis is in fact very primitive, but it nonetheless captures the salient features of how these factors will influence performance. The model presented is capable of providing an excellent fit to joint time/accuracy data such as the numerical verification study data. Because the form of our model is tied to the structure of visual work it can be extended or modified in a logical manner as more is learned. This emphasis on structure provides some assurance of physical explanatory power and not just good correlations.

Acknowledgement

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Discussion

This is a paper to study not to read, and listening to an oral presentation leaves hardly anyone with full understanding. In order to comprehend the interesting approach made by the authors, one has to study the mathematical descriptions used by Blackwell given as Equation 1, and those named after Blondel-Rey that have been amalgamated to obtain, after some substitutions and careful consideration of visual and non-visual components hidden in a visual task, the relationship between performance and visibility level.

The existence of that plausible relationship has been often denied. The authors apply Rea's data on visual performance obtained by verifying a series of numbers with reference numbers with a time adjustment. Perhaps this time correction explains why the original data still showed visual performance at a task contrast below its threshold value. In contrast, the extrapolation of Weston's results obtained with Landolt-C orientation detection, as well as Muck's findings with a number search task, indicated zero performance when the contrast of the test reached the limit of visibility. This result makes sense. The great merit of the system described here lies in the showing of the relation between the suprathreshold factor, termed as

VL, and the visual performance if purified from non-visual components as much as possible.

The mathematics and explanation are too terse to be easily understandable. The paper resembles notes made for highly specialized experts who are familiar with all the functions used. I wish for greater publicity, which the paper definitely deserves, so that the authors will be more elaborate in the mathematical descriptions. Such elaboration could be well supported by graphs, especially displays of Equations 1,2,5,10.

Werner Adrian
University of Waterloo

To Werner Adrian

We apologize to Dr. Adrian for the terseness of the paper. Unfortunately, the space constraints of a conference paper did not permit the elaboration on equations that Dr. Adrian desires. However, despite the number of equations we used, the basic concept of the paper is simple. We have added explanatory material to the introduction in an attempt to make the ideas clearer.

We believe Dr. Adrian's comment about finite visual performance at contrasts below the threshold value refers to Rea's reaction time data¹ not the data from the 1986 paper that we analyzed here. In his 1988 paper Rea defined threshold as the contrast level that gave 50 percent accuracy of detection. Rea's model predicts zero speed for contrast values below the threshold value, while the data clearly show finite speeds. Defining threshold as the zero accuracy limit would result in better modelling of the low contrast data and poorer modelling of the high contrast data. Our model explicitly handles the threshold problem by making speed a function of the accuracy. Thus, if the contrast of a target is at the 25 percent detection limit, our model predicts that you cannot do the task in a finite time if you insist on 50 percent accuracy, but you can do it in a finite time if you are willing to settle for lower accuracy. This is in concordance with the data.

Our analysis breaks the visual task into a visual, or VL, dependent component and cognition and reaction components that are not related to VL. Further work needs to be done to better understand how these components depend upon the visual stimulus.

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