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DETERMINING OPTIMUM SECTOR SIZE FOR AUTOMATIC LIGHTING CONTROLS

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1.0 INTRODUCTION

Automatic lighting controls can significantly reduce operating costs in commercial buildings.[1,2,3] However, the design of these dynamically operated lighting systems has not kept pace with advancements in equipment. This paper focuses on determining how many lights should be grouped into sectors and controlled as a unit in order to minimize the life-cycle cost of an automatically controlled lighting system. The analysis is based on occupant behavior patterns, the cost of the lighting controls, the energy costs, and a decision criterion based on life-cycle cost.

Occupant switching patterns in the morning and afternoon were experimentally determined at a lighting control demonstration at the World Trade Center (WTC) building in New York City[3]. The experimental procedure used to estimate the occupancy probability distributions at the WTC site is described in section 2. Section 3 develops a general analytical model for determining how large to make independently controlled lighting sectors in order to most cost-effectively use the scheduling control strategy. In Section 4, the model is applied to the occupant behavior patterns measured at the WTC site. Section 5 discusses the significance of the results; the final section summarizes the study.

2.0 OCCUPANT BEHAVIOR PATTERNS

2.1 Automated Lighting Control

The lighting control demonstration at the World Trade Center's 58^{th} floor was designed to analyze the energy- and cost-savings potential of various scheduling and daylighting techniques in a realistic office environment.[1,3] The 58^{th} floor consists of 29,000 ft^2 usable office space lit by 450 six-lamp fluorescent fixtures. The average lighting power density was 2.7 watts per square foot (W/ft^2). Measured illumination levels were between 60 and 90 footcandles. The lighting control system consisted of a central microprocessor and remotely located relays for switching lighting loads. By installing relays on every ballast, each fixture could be independently controlled to provide four levels of illumination (0, 33%, 66%, and 100% of full light output). The microprocessor could also control groups of relays (sectors), simulating the operation of a relay-based control system in a typical commercial installation (in which each sector of lights is split-wired into two lighting circuits and each circuit switched by one relay). The microprocessor automatically switched sectors on or off at scheduled times. The schedule could be overridden by occupants keying in a code on their telephones. The computer's built-in monitoring capability was used to record the time at which each sector's light level changed.

2.2 Experimental Design

During one 10-day period, occupants' use of the telephone override system was recorded to determine the probability of an occupant switching on lights as a function of time, i.e., to study the occupant behavior patterns. The lighting system was divided into 150 zones with a mean size of 192 ft^2. Occupants used their telephones to switch on their sector lights when arriving in the morning or returning from lunch.

In this experiment, the control system was programmed to automatically switch off 2/3 of all "on" relays (lights) at 12:15 p.m. for the lunch period. At 5:30 p.m., it imposed a uniform 1/3 lighting pattern for the cleaning crew, and at 9:00 p.m., the control system turned off all lights. The control system did not automatically switch on any lights during normal working hours. (At 6:45 a.m., it activated a low-level lighting pattern for ingress using one fixture per 1000 ft^2). The lights in each sector had to be switched on in the morning and afternoon by the sector's occupant. Occupant behavior patterns were estimated by analyzing the switching records.

To estimate the number of active, single-occupant sectors, we considered only those sectors smaller than 150 ft^2 and activated at least twice during the test. Sectors not meeting these criteria were excluded for the following reasons. Sectors larger than 150 ft^2 might have contained more than one individual thus the switching activity in such sectors could represent the behavior of more than a single individual. Some of these multi-occupant zones could also be identified by examining the floorplans. Secondly, sectors that were never activated throughout the entire test period were considered atypical and were not included. For the 85 active sectors, the number of switch-ons per 5-minute interval was continuously recorded for 10 working days in May.

2.3 Behavior Patterns

Tables 1A and 1B list the data for mornings and afternoons, respectively. For the morning data (Table 1A), the number of switching actions would total 850 if all occupants always switched on their lights in the morning. From the measured total of 635 we infer that, on the average, 25% of the 85 active sectors were not switched on in the morning. From the afternoon switching data, we conclude that, on the average, 58% of the 85 active sectors were not switched back on again in the afternoon. We assume that absence of switching is due to a combination of absenteeism, occupant accommodation to low light levels, and under-utilization of space. The relatively low number of recorded actions in the afternoon may reflect the acceptance of the 1/3 illumination levels at some stations or the impact of available daylight. While the total number of switching actions is only 810, the number of events measured is statistically sufficient to describe the occupants' general behavior. Figure 1 shows the probability of a switching action occurring in a given time interval, as well as the cumulative probability of an occupant switching on lights prior to time t.

3.0 THE MODEL

This section develops a model for analyzing the life-cycle cost (net present cost method) of a control system that schedules operation of the lighting in a large commercial building. The model expresses the present worth of the control system as a function of the number of lighting sectors in the installation; thus it can be used directly to compute optimum sector size (the sector size that minimizes the present cost of the system over the time horizon of the investment).

3.1 Basic Assumptions

The model is based on a lighting control schedule that:

- switches off 2/3 of all "on" lights at noon,
- supplies a uniform 1/3 light level at 5:30 p.m.,
- reduces any overridden sectors to 1/3 at 7:00 p.m., and,
- switches off all lights at 9:00 p.m.

It is assumed that the probability of an individual switching on the lights is independent of the switching behavior of other occupants and that occupants only turn on their lights to full output, not a reduced level. We assume that each occupant stays for 9 hours; thus the departure probability distribution is the same as the morning switching probability distribution but 9 hours later. Since occupants stay 9 hours, some fraction may be present after the lights are reduced to 1/3 at 5:30 p.m.. These occupants restore their lights to full level. As described above, the control system again switches these sectors down to 1/3 level at 7:00 p.m..

3.2 Considerations

The model is designed to analyze the cost/benefits of installing centrally controllable relays on the lighting circuits in a new commercial building. The initial cost of the control system is expressed as the cost per control point and includes both equipment and installation costs. The cost of a central microprocessor does not depend on the number of control points and, for large buildings, the cost per ft^2 is negligible. Each sector consists of two control points permitting four levels of illumination (0, 33-1/3, 66-2/3, and 100% full light output) in each sector.

3.3 Formulation

The cost of lighting a space and employing one or more control points to schedule the system operation may be considered the sum of two costs: the initial component and installation cost of the control points, and the operating cost of the system.

The initial installed cost in dollars per floor for two control points per sector is:

Installed cost = 2 b N,

where

b = cost per control point in dollars, and
N = number of sectors per floor.

The present value of the operating cost of the lighting system, with 2 N control points installed per floor, is:

$$\text{Operating Cost} = \beta\, N\ \phi(\frac{K}{N}),$$

where

β = present cost of energy ($-day/sector-hr)
ϕ = lighting energy use (hr/day), and
K = number of occupants per floor.

The present cost of electricity, β, is:

$$\beta = c\ w\ d\ \frac{F}{N}\ g$$

where

c = cost of electricity ($/kWh),
w = lighting power density (W/ft^2),
d = working days per year (days/yr),
F = total area of floor space (ft^2/floor), and
g = present value factor (years).

The present value factor[4], g, is:

$$g = \frac{\delta}{r} \frac{(1-\frac{\delta}{r})^{Y}}{1-\frac{\delta}{r}},$$

where

δ = 1 + the fractional escalation rate of energy,
r = 1 + the fractional discount rate, and
Y = time horizon (yr).

The lumped parameter β is the present value (cost) of the energy associated with running a sector's lights for one hour each day for all (working) days in the investment.

The total cost in dollars for installing and operating two control points per sector is thus:

$$Total\ cost = 2\ b\ N + \beta\ N\ \phi([\frac{K}{N}]). \tag{1}$$

We wish to find the number of sectors, for $1 \leq N \leq K$, that minimizes the total cost expressed in Eq. 1.

The ϕ function is an expression for the number of hours the lights are on each day as a function of the number of occupants per sector. Its value depends on the behavior of occupants and the specifics of the lighting control technique.

To derive an expression for ϕ, we must introduce some new nomenclature, which is listed in Table 2 along with nominal values for the variables. Let $p_1(t)$ and $p_2(t)$ be the cumulative probabilities, for the morning and afternoon, respectively, of a given individual switching on the lights after time t. If e_m is the number of hours that the lights are expected to be off in a sector that is eventually occupied in the morning then:

$$e_m = \int_{t_1}^{t_2} p_1(x_1 > t)^{[\frac{K}{N}]}\ dt\ , \tag{2a}$$

An equivalent term, e_v , for the afternoon is:

$$e_v = \int_{t_{n_1}}^{t_{n_2}} p_2(x_2 > t)^{[\frac{K}{N}]}\ dt. \tag{2b}$$

Eqs. (2a) and (2b) express the notion that the expected number of hours that the lights are off with K/N people per sector is the product of the individual probabilities for each occupants in a sector integrated over the appropriate hours of use.

To account for the fact that individuals do not always switch on their lights, we define the term γ_n. If p_m is the probability that a given individual will never switch on the lights in the morning then the probability of the lights being on by time t_{n_1} is:

$$\gamma_n = 1 - p_m^{[\frac{K}{N}]} \tag{3}$$

Equation (3) states that the probability that the lights are on at t_{n_1} (the time of the light level reduction at lunch) is equal to 1 minus the the probability that no one in this sector has turned on their lights in the morning (i.e., $p_m^{[\frac{K}{N}]}$).

Now we are ready to determine ϕ:

$$\begin{aligned}\phi([\frac{K}{N}]) = {} & \gamma_n (t_{n_1} - t_{m_1} - e_m) \\ & + \gamma_n D_n (t_{e_1} - t_{n_1}) - (1 - p_n^{[\frac{K}{N}]})(t_{e_1} - t_{n_1} - e_v)(1 - D_n) \\ & + (1 - p_n^{[\frac{K}{N}]})(1 - p_{e_1}^{[\frac{K}{N}]})(t_{e_2} - t_{e_1})(1 - D_e) + D_e (t_{e_2} - t_{e_1}) \\ & + (1 - p_n^{[\frac{K}{N}]})(1 - p_{e_1}^{[\frac{K}{N}]})(1 - p_{e_2}^{[\frac{K}{N}]})(t_{e_3} - t_{e_2})(1 - D_e) + D_e (t_{e_3} - t_{e_2})\end{aligned} \tag{4}$$

As shown above, ϕ is the sum of four major terms that describe the expected lighting usage in the morning, afternoon, early evening (5:30 to 7:00 p.m.), and late evening (7:00 to 9:00 p.m.), respectively.

The function $\phi([\frac{K}{N}])$ is a monotonically increasing function of the sector size (or the number of occupants per sector). On the other hand, installation costs are inversely proportional to the sector size. The optimal sector size is the one that minimizes the sum of these two functions (Eq. 1).

4.0 RESULTS

4.1 Application to World Trade Center Data

Using the notation developed in the previous section, Figs. 1a and b show the interval and the cumulative switching probability densities, respectively, measured at the WTC site. The times of the automatic lighting reductions assumed for this example are indicated. The model accounts for any lighting use after t_{e_1} (time of evening light reduction) by assuming that the probability of a sector being vacant at time t_{e_1} is the same as the probability of a sector being switched on 9 hours earlier.

Figure 2 gives the value of the function ϕ computed from the WTC data as a function of K/N. The dashed curve shows the ϕ function if the automatically programmed lighting reduction at 12:15 p.m. is not implemented. For both curves, the function ϕ increases rapidly as the number of occupants per sector increases from unity, and slower as K/N becomes large. An important feature of these curves is that some energy savings can be achieved even if there are relatively few sectors (i.e., N is small). The graph also shows that the energy savings associated with the noon-time lighting reduction is negligible once the number of occupants in a sector exceeds 20.

The economic analysis is applied to an area equivalent to that occupied by the 85 active workstations: 9350 ft^2. Each person therefore occupies 110 ft^2. We will take the time horizon to be 10 years, and the discount rate 10%; the cost of energy will be assumed to increase 5% annually. Table 2 gives the values for the various other variables in the analysis.

Figure 3 shows the initial cost of the control points as a function of the number of occupants in a sector (K/N). As the number of occupants per sector increases, the sector size must increase proportionally (since the area per person is held fixed). Thus control point installation costs decrease in inverse proportion.

Figures 4a-d and 5a-d plot the total net present value (initial plus operating costs) of the system per square foot as a function of number of people in a sector using the experimentally determined probability distributions. Figure 4 was calculated assuming the noontime 2/3 light level reduction. Figure 5 shows the analogous results for the situation in which the lights are not reduced at noon. For both control schedules, four graphs are shown, each representing a different value of the lumped parameter, *wc* , the product of the lighting power density and cost of energy. The values of *wc* selected, 0.05, 0.1, 0.2, and 0.3 x 10^{-3} \$/hr-ft^2, represent the range of values typically encountered in commercial buildings. Plotting each curve for different values of *wc* increases the generality of the results because a *wc* value of, for example, 0.10 x 10^{-3} \$/hr-ft^2, represents a power density of 2.5 W/ft^2 at an energy cost of \$0.04 per kWh or a power density of 2.0 W/ft^2 at \$0.05 per kWh. The different curves on each graph represent different initial costs for the control points.

For a *wc* of 0.05 x 10^{-3} \$/hr-ft^2 and a cost per control point of \$100 or higher, the net present cost of the system tends to be lowest when all occupants occupy one large sector, Figs. 4a and 5a, if the cost per control point is \$100 or higher. (The cost of installing presently-available equipment in a new commercial building project is estimated to be \$100 to \$150 per control point.) For the switching scheme that incorporates the noon light level reduction (Fig. 4a), the net present cost is essentially unaffected by sector size if a control point costs just \$50.

Figures 4b and 5b show the results for *wc* = 0.1 x 10^{-3} \$/hr-ft^2, the approximate value of (*wc*) for the World Trade Center site at the time of the demonstration (2.7 W/ft^2 at \$0.037/kWh). In Fig. 4b, the cost curve reaches a broad minimum for K/N somewhere between 1 and 85 depending on the assumed cost of the control point. For example, at \$100/control point, the net present cost minimizes at 7 to 8 occupants per sector. It minimizes at 15 to 20 occupants per sector if a control point costs \$150.

At a moderate value of *wc* , 0.2 x 10^{-3} \$/hr-ft^2, the net present cost of the lighting system always decreases as the number of sectors per floor increases from 1 even if the control points are expensive (e.g., \$250). As the number of occupants per sector (K/N) decreases (i.e., increasing N), the cost curves for more expensive control points reach a broad minimum and then increase. If the control points are relatively inexpensive, the net present value continues to decrease, minimizing at 1 to 3 occupants per sector. Note that there are significant differences in the results for the two control techniques. For example, without the lunchtime reduction (Fig. 5c), the optimum solution is 10 people per sector (at \$150/point). However, with the lunch-time lighting reduction, the equivalent optimum is 2 to 3 occupants per sector.

At high values of *wc* , 0.3 x 10^{-3} \$/hr-ft^2, the cost curve minima are better defined (i.e., less broad) than at lower energy costs, and the cost/benefits of adding control points are more obvious. At this value of *wc* , the net present cost curves always minimize at 10 people per sector or fewer, depending on the various parameters. In Fig. 5d, one sees that 3 to 4 people per sector is the most economical solution even at \$250/control point.

In Figs. 4 and 5, the net present value at 85 people per sector (N = 1) is simply the cost of operating the lighting system with minimal controls. As the lighting system is divided into increasingly smaller independent sectors (more control points), the change in net present value from the limiting case (one aggregate point) represents the difference between the increased cost of the additional control points and the savings in energy costs.

4.2 Effect of Occupant Density

If other parameters are fixed, changing the density of people in the space should affect the calculated optimum sector size. As an example of this effect, we have used the same control scheme and *wc* value as shown in Fig. 4c and fixed the control point cost at \$100. Figure 6 shows present value curves calculated for these assumptions for three occupant densities: 110, 165, and 220 ft^2/person. For this example, one person per sector is the most economical solution regardless of occupant density. However, at this optimum value of one person per sector, the savings relative to the base case of one large sector is higher for lower densities of people. This is logical because at 220 ft^2/person, each person uses potentially twice as much lighting energy than at 110 ft^2/person. Thus the economic advantage associated with small sectors increases as the density of people in the space decreases.

4.3 Time Horizon

Generally, the time horizon for an investment reflects the expected life of the equipment. In the previous sections we used a time horizon of 10 years because equipment of the type used at the World Trade Center should have a life of 10 to 20 years. However, financial decisions are often based on a shorter time horizon than the expected equipment life. Figure 7 plots the net present value for the controls investment for *wc* = 0.2 x 10^{-3} \$/hr-ft^2, control point costs of \$100 and \$150, and for time horizons of 10 and 5 years. For a 5-year time horizon, the present cost curve minimizes at roughly 3 people per sector for \$100/control point; it minimizes at 85 people per sector if the control point costs \$150. With a 10-year time horizon, 1 to 3 people per sector would be optimum even for control points costing \$150. Thus, if other factors are held constant, the optimum sector size tends to increase as the time horizon considered shortens.

4.4 Occupants' Behavior Pattern

In the previous sections, we examined how different parameters affect the calculated optimum sector size for the behavior patterns measured at the WTC demonstration site. Different types of buildings will have different occupancy behavior patterns. For example, in a factory, where all workers usually arrive and leave at essentially the same times, there would be probably be little advantage to using small sectors. To investigate how occupancy arrival and departure distributions affect the calculation of sector size, several analytically generated functions are used to represent different occupancy behaviors. We use four truncated normal Gaussian distributions with standard deviations of 5, 15, 30 and 60 minutes (min.) to represent increasing spread in arrival and departure times (Fig. 8). To compare these results to measured probability results, we assume the same control schedule as that represented in Fig. 4, i.e., 2/3 light reduction at noon, a 1/3 lighting pattern after 5:30 p.m., and all lights off at 9:00 p.m.

Also, the median time of the Gaussian distributions is taken to be at 8:15 a.m., corresponding to the median for the WTC morning switching distribution. Figure 9 shows the equivalent full lighting power hours per day as a function of number of occupants per sector for the four Gaussian switching distribution functions and the experimentally obtained curve.

For a narrower distribution ($\sigma = 5$ min.), the lighting hours remain essentially unchanged if there are more than 10 occupants in a sector. Without the noon light reduction measure, there is negligible energy savings unless a sector holds only one person. For broader distributions, even for relatively large sectors (~ 10-20 occupants/sector or 1,000 - 2,000 ft^2/sector), there is a significant opportunity to reduce lighting hours relative to the hours used in the limiting case of one sector/floor. For example, with $\sigma = 30$, the lights were on nearly an hour less per day at 10 than at 85 people/sector (Fig. 9a). Also, the potential for reducing energy use at relatively large sector size increases as the spread in occupant arrival and departure times increases.

One notable feature of Fig. 9a is the close correspondence of the experimentally measured curve and the curve for the broad hypothetical distribution ($\sigma = 60$ *min.*). This indicates that analytical functions generated to characterize the occupancy behavior patterns in different building types (e.g., offices, factories, schools) can be used in lieu of experimentally measured switching distributions for determining optimum sector size.

The net present cost of the control point investment for the four analytically generated switching distributions and the experimental data are given in Fig. 10 for $wc = 0.2 \times 10^{-3}$ \$/hr-ft^2 and \$100/control point. For $\sigma = 5$ min, one large sector is clearly most economical, because as N increases (fewer occupants in a sector), the net present cost steadily increases. In other words, with this narrow switching distribution, as we split the space into increasingly smaller sectors, the burden of paying for controls increases faster than the energy savings can pay out. As the uncertainty associated with occupant arrival times increases, the optimum sector size decreases. For the values of wc and control point cost assumed in the graph, the optimum sector size is sensitive to the control technique. With a moderately broad switching distribution, $\sigma = 30$ min., the cost curve shows a broad minimum at ≈ 20 people/sector if the lights are not reduced at noon, but with the noon-time measure, 2 to 3 people per sector is the optimum size. If there is considerable uncertainty in occupant arrival times, $\sigma = 60$ min., then small sectors (≈ 1 person/sector) are strongly indicated.

5.0 DISCUSSION

The optimum sector size (or the number of occupants per sector) for an automatic control system has been shown to be sensitive to the cost of the control points, energy costs, and occupant behavior patterns. In some situations, the size of the sector having the lowest present cost doubled if the control point costs \$150 instead of \$100. The sensitivity of the results to changes in occupant behavior, lighting energy cost, and control point cost argues the importance of estimating these quantities during the design process.

An important result of this study is that at typical values of energy cost and power density ($wc = 0.2 \times 10^{-3}$ \$/hr-ft^2), the life-cycle cost of the lighting system with 10 to 20 people per sector is consistently lower than with only minimal controls (one sector/floor). This reduction in lighting energy use with a relatively small number of control points is almost entirely attributable to reduced lighting hours in the evening. This savings is possible because even with 10 to 20 people in a sector, the probability becomes high that a sector will be unoccupied for at least a few hours in the evening. To obtain a savings during the evening hours, the scheduling program must be properly designed. For example, a control schedule that did not re-assert the reduced level at

7:00 p.m. (data not shown) showed little change in lighting use between sectors holding 10 and 85 people. Because the model assumes that any occupant still present at 5:30 p.m. when the 1/3 light level is imposed will override the sector lights to full level, the absence of an automatically re-asserted 1/3 light level at 7:00 p.m. increases evening lighting usage significantly for large sectors. To ensure that lighting energy is saved with large sectors, the control system should be programmed to minimize evening usage by periodically switching any overridden sectors to the reduced level.

Results show that the occupant switching distributions measured at the WTC can be approximated by simple Gaussian distributions with standard deviations between 45 and 60 min. This indicates that occupant behavior patterns may be able to be modeled using analytical mathematical functions to determine the cost-effectiveness of the scheduling strategy. Thus various work activities (e.g. office work, factory work) would be categorized with simple Gaussian distributions and sector size guidelines would be developed for different building types using these distributions.

Closer inspection of the probability distributions show that the experimentally measured distribution has a smaller standard deviation than the Gaussian distribution that can replace it in the calculation. The reason for this lies in the shape of the experimental curve; the small number of people who turn on their lights significantly earlier than the others have a disproportionately large influence on energy use. To mimic the effect of the experimental curve that has a standard deviation of 20 to 25 min., we need to use a Gaussian with a σ of roughly 60 min. Also, the experimental distributions are asymmetrical with respect to the median value, while the Gaussian distributions are symmetrical.

The results indicate that the specific control schedule can be important. The simulations that assumed the lights were reduced to 1/3 level at noon showed significant shifts in the net present cost curves relative to a control schedule without the noon switching. Generally, noon switching increases the possibility that the cost of the lighting system can minimize at 2 to 4 people per sector. Based on the WTC data, we have assumed that there is a 58% chance of an individual never switching on lights in the afternoon. This means that there is a 20% chance that 3-person sectors will not be switched on in the afternoon. This probability is a consequence of many factors that may vary widely between buildings. A better understanding of how factors such as absenteeism, preference of low light levels, and available daylight affect occupant switching behavior in different building types would contribute to the development of design guidelines for dynamic lighting systems.

6.0 SUMMARY

We developed a general analytical model for determining the optimally cost-effective size of independently controlled sectors in buildings where the lighting system is automatically scheduled. We found that the optimum sector size was sensitive to many factors, including the cost of the control points, the cost of energy and the occupant behavior patterns. Applying the model to the occupancy switching behavior measured at the World Trade Center, using typical values for the cost of energy, lighting power density, and control point cost ($0.10/kWh, 2 W/ft^2, and $150/point), we determined that sectors containing approximately 10 people (i.e. 1100 ft^2/sector) had the lowest life cycle-cost when a simple control schedule (i.e. lights reduced to 1/3 in the evening with no noon-time lighting reduction) was used. A control schedule that reduced light levels at noon was shown to significantly reduce overall lighting use, reducing the optimum sector size to 2 to 3 occupants per sector (220 - 330 ft^2) assuming the same conditions as above.

Applying the analysis to the switching behavior found at the World Trade Center showed that, except at low energy costs and power densities, light level scheduling with even a modest number of control points (i.e. 1000 - 2000 ft^2 sectors) generally had a lower life-cycle cost than a system with only minimal controls (i.e. 1 sector/floor).

The behavior patterns of the occupants had a significant impact on calculating optimum sector size. Using analytically derived functions to approximate various occupant behavior patterns, we found that sectors in the 200 to 2000 ft^2 range were cost-effective unless the vast majority of the occupants all arrived and left within a narrow 10-minute window. Small sectors were generally indicated if there was a medium to large uncertainty in occupant arrival and departure times.

In the calculation of optimum sector size, we found that experimentally measured occupancy patterns could be approximated by simple Gaussian distributions. This indicates that by characterizing various occupant work schedules using various analytical distributions, it may be possible to develop generalized design techniques for calculating how large the sectors should be to most cost-effectively use the scheduling control strategy in any building type.

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Table 1A. Number of Sectors[a] Switched on in Morning During 10-Day Period

Time[b]	Sectors Activated in Interval	Sectors Activated (cumulative)	Probability of[c] Activation (cumulative)
7: 0 a.m.	9	9	0.014
7: 5	5	14	0.022
7:10	2	16	0.025
7:15	10	26	0.041
7:20	11	37	0.058
7:25	8	45	0.071
7:30	3	48	0.076
7:35	6	54	0.085
7:40	12	66	0.104
7:45	18	84	0.132
7:50	31	115	0.181
7:55	39	154	0.243
8: 0	73	227	0.357
8: 5	51	278	0.438
8:10	62	340	0.535
8:15	45	385	0.606
8:20	57	442	0.696
8:25	36	478	0.753
8:30	20	498	0.784
8:35	20	518	0.816
8:40	17	535	0.843
8:45	14	549	0.865
8:50	9	558	0.879
8:55	8	566	0.891
9: 0	8	574	0.904
9: 5	5	579	0.912
9:10	7	586	0.923
9:15	9	595	0.937
9:20	1	596	0.939
9:25	3	599	0.943
9:30	4	603	0.950
9:35	3	606	0.954
9:40	2	608	0.957
9:45	1	609	0.959
9:50	1	610	0.961
9:55	2	612	0.964
10: 0	2	614	0.967
10: 5	1	615	0.969
10:10	3	618	0.973
10:15	2	620	0.976
10:20	2	622	0.980
10:25	0	622	0.980
10:30	0	622	0.980
10:35	1	623	0.981
10:40	2	625	0.984
10:45	0	625	0.984
10:50	0	625	0.984
10:55	1	626	0.986
11: 0	9	635	1.000
TOTAL:		635	

[a]Sample size: 85 sectors

[b]Times shown indicate mid-point at 5-min interval

[c]Normalized relative to total number of measured switch-ons (635)

Table 1B. Number of Sectors[a] Switched on in Afternoon During 5-Day Period

Time[b]	Sectors Activated in Interval	Sectors Activated (cumulative)	Probability of[c] Activation (cumulative)
12:15 p.m.	18	18	0.102
12:20	1	19	0.108
12:25	1	20	0.114
12:30	0	20	0.114
12:35	1	21	0.119
12:40	3	24	0.136
12:45	1	25	0.142
12:50	2	27	0.153
12:55	11	38	0.216
1: 0	21	59	0.335
1: 5	25	84	0.477
1:10	23	107	0.608
1:15	16	123	0.699
1:20	9	132	0.750
1:25	13	145	0.824
1:30	3	148	0.841
1:35	3	151	0.858
1:40	3	154	0.875
1:45	2	156	0.886
1:50	4	160	0.909
1:55	1	161	0.915
2: 0	2	163	0.926
2: 5	0	163	0.926
2:10	1	164	0.932
2:15	1	165	0.938
2:20	2	167	0.949
2:25	0	167	0.949
2:30	1	168	0.955
2:35	1	169	0.960
2:40	0	169	0.960
2:45	2	171	0.972
2:50	0	171	0.972
2:55	1	172	0.977
3: 0	1	173	0.983
3: 5	0	173	0.983
3:10	0	173	0.983
3:15	0	173	0.983
3:20	2	175	0.994
3:25	0	175	0.994
3:30	0	175	0.994
3:35	0	175	0.994
3:40	0	175	0.994
3:45	0	175	0.994
3:50	0	175	0.994
3:55	1	176	1.000
4: 0	0	176	1.000
4: 5	0	176	1.000
4:10	0	176	1.000
4:15	0	176	1.000
TOTAL:		176	

[a]Sample size: 85 sectors

[b]Times shown indicate mid-point at 5-min interval

[c]Normalized relative to total number of measured switch-ons (176)

Table 2. Definitions of Variables and Nominal Values

Variable	Description	Value[a]
N	Number of Sectors Per Floor	(Decision Variable)
K	Number of Occupants Per Floor	(85 people)
F	Floor Size (9350 ft^2)	
$[\frac{K}{N}]$	Closest Integer Approximation to Number of Occupants in a Sector	(Decision Variable)
t_1	Earliest Time an Individual Switches Lights On in A.M.	(7 AM)
t_2	Latest Time an Individual Switches Lights On in A.M.	(11 AM)
t_{n_1}	Noon Light Reduction Time	(12:15 PM)
t_{n_2}	Latest Time an Individual Switches Lights On in P.M.	(4:15 PM)
t_{e_1}	Evening Light Reduction Time	(5:30 PM)
t_{e_2}	Re-Asserted Light Level Reduction Time	(7:00 PM)
t_{e_3}	Time When All Lights are Switched Off	(9 PM)
$p_1(x_i > t)$	Probab. an Individual Switches Lights On After Time t (morning) (normalized to 1)	-
$p_2(x_i > t)$	Probab. an Individual Switches Lights On After Time t (afternoon) (normalized to 1)	-
p_m	Probab. an Individual Never Switches Lights On in A.M.	(0.25)
p_n	Probab. an Individual Never Switches Lights On in P.M.	(0.58)
p_{e_1}	Probab. an Individual Leaves by Time t_{e_1}	(0.75)
p_{e_2}	Probab. an Individual Leaves by Time t_{e_2}	(0.97)
c	Cost of Energy [dollars/Kwh]	-
w	lighting density [watts/ft^2]	-
Δ	1 + Energy Escalation Rate in Percent	(1.05)
r	1 + Discount Rate in Percent	(1.10)
d	Number of Working Days in a Year	(260 days/year)
Y	Time Horizon	(10 years)
b	Cost of Single Control Point [dollars]	(50-250)
D_e	Fractional Light Level in Evening	(1/3)
D_n	Fractional Light Level at Noon	(1/3)

[a]Values listed in this column used for analysis unless otherwise indicated.

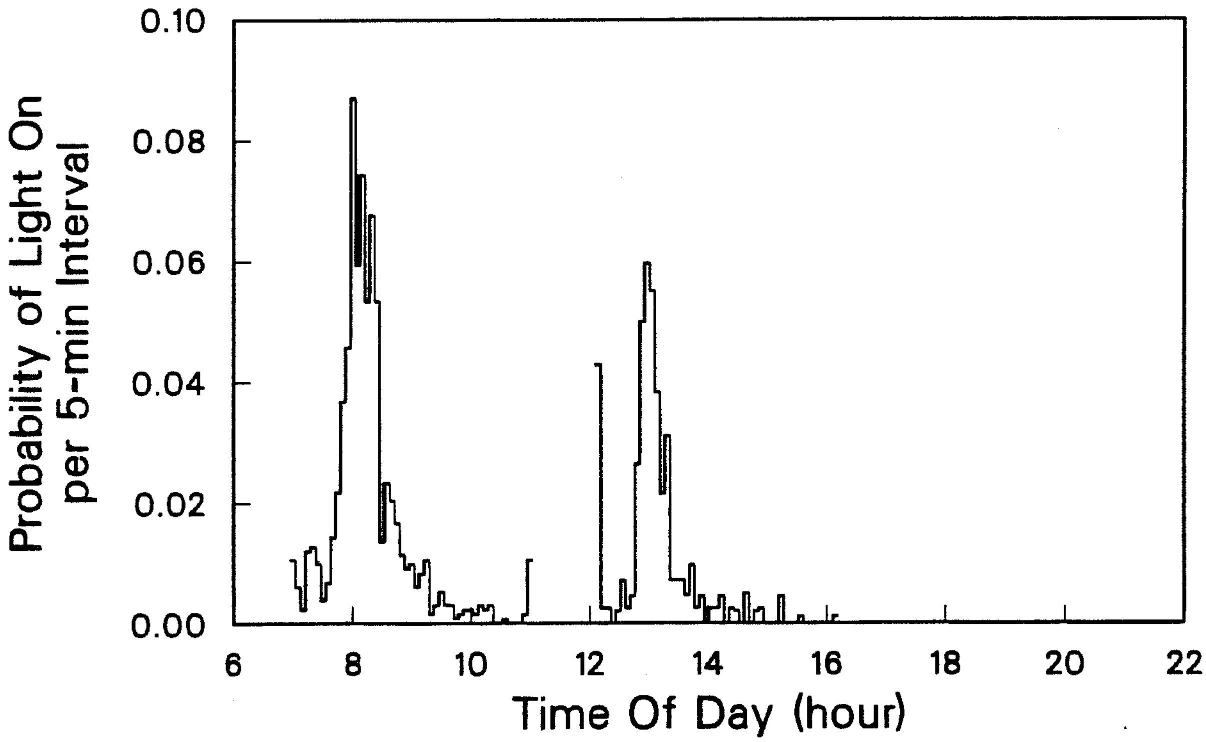

Figure 1a. Probability of occupants switching on lights per 5-minute interval in morning and afternoon as measured at World Trade Center. Curve in morning normalized to 0.75 (see Table 1a). Curve in afternoon normalized to 0.42 (see Table 1b).

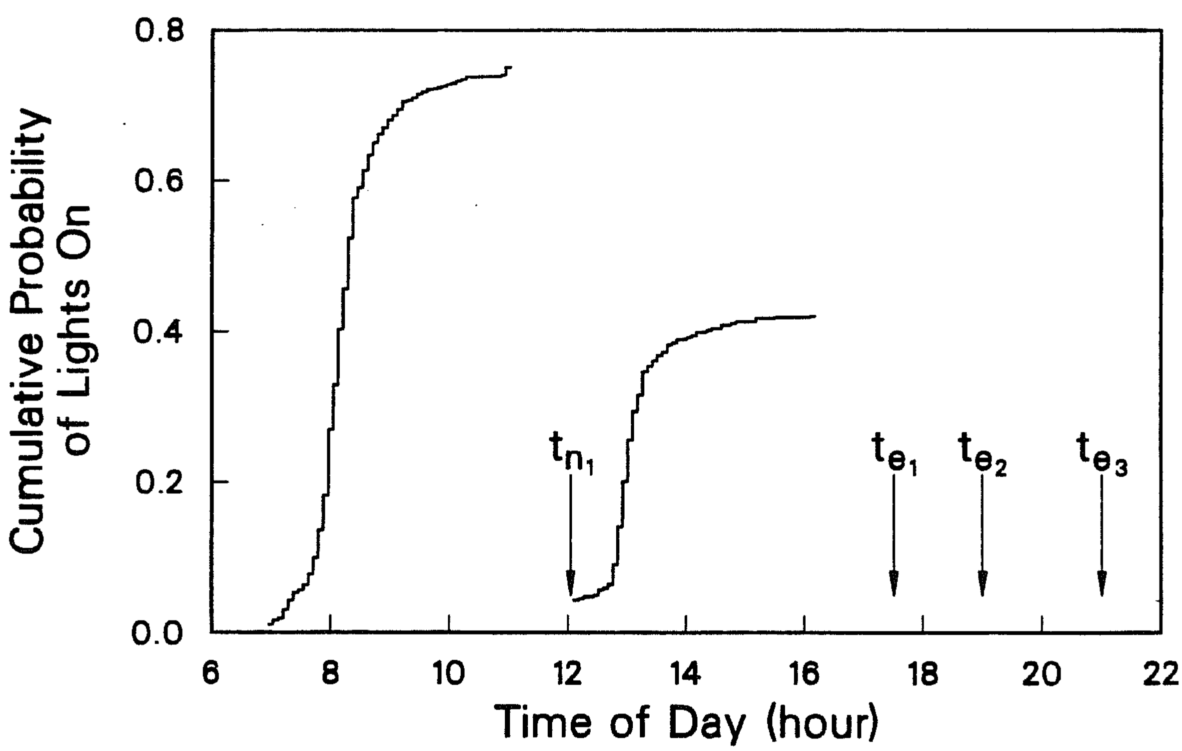

Figure 1b. Cumulative probability of occupants switching on lights in morning and afternoon as measured at World Trade Center. Curve in morning normalized to 0.75 (see Table 1a). Curve in afternoon normalized to 0.42 (see Table 1b). Control system switching times assumed in analysis are superimposed.

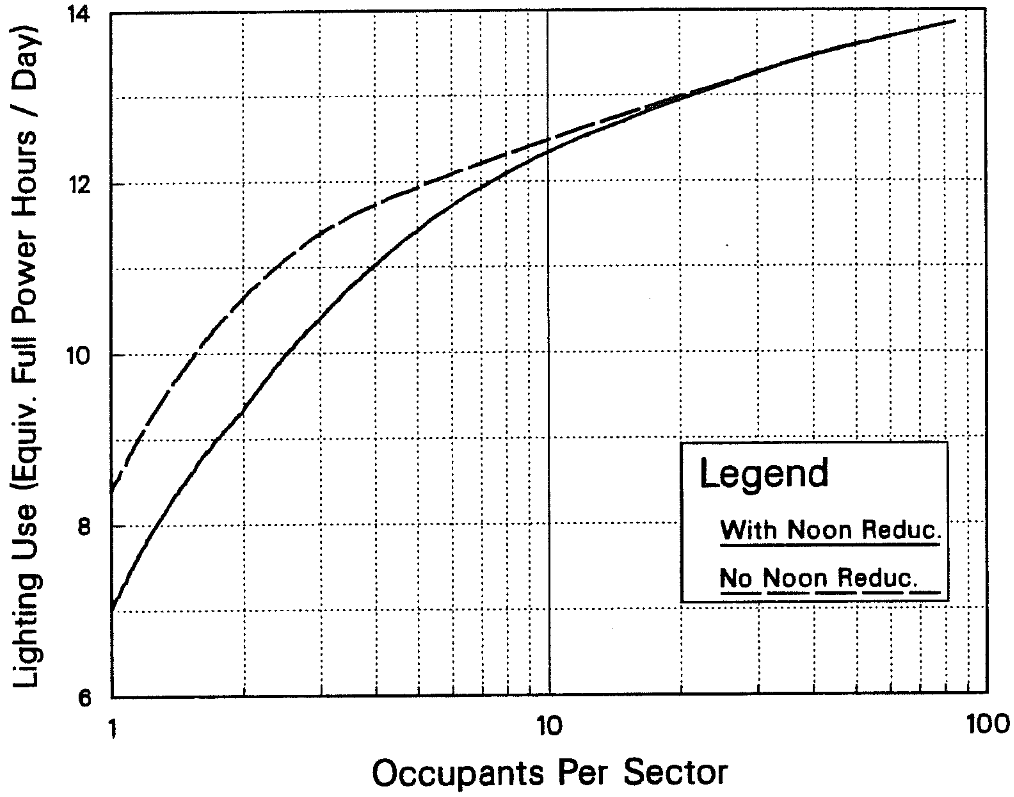

Figure 2. Lighting energy use per day as a function of number of occupants in a sector. Lighting use plotted as equivalent full power lighting hours per day for control schedule with noon-time lighting reduction (solid line) and without (dashed).

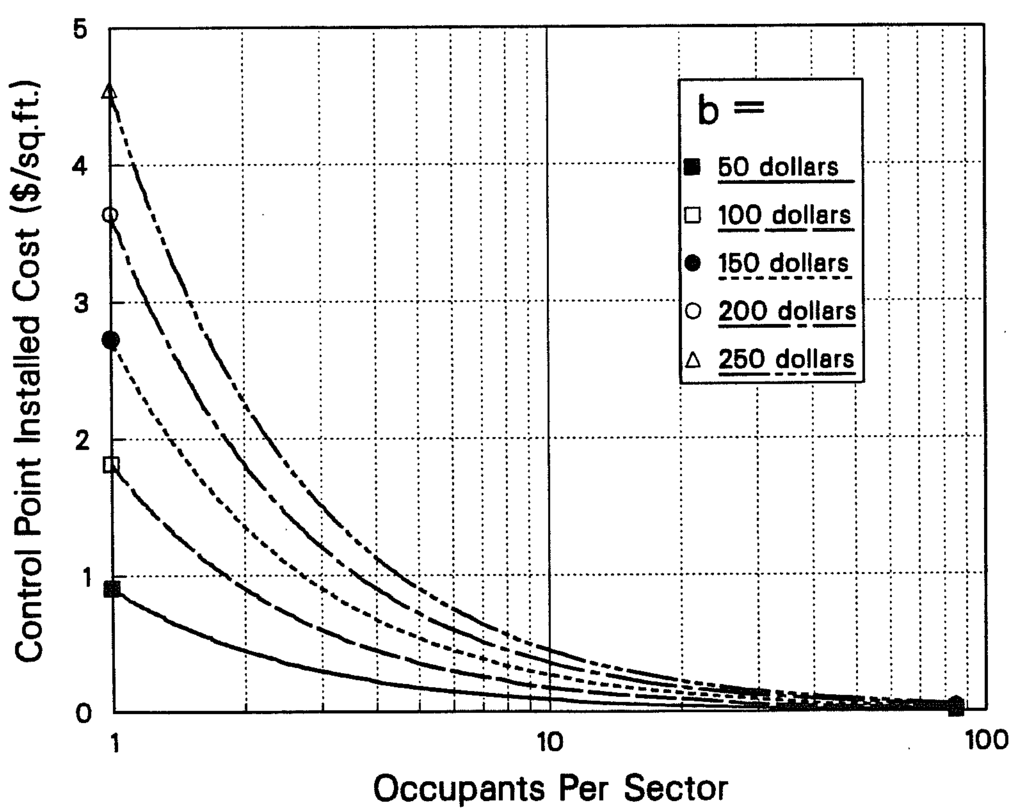

Figure 3. Cost of installing control points as a function of number of occupants per sector. Cost given in $\$/ft^2$ assuming 2 control points per sector with occupant density of 110 ft^2/occupant. Curves shown for installed control point costs of \$50, \$100, \$150, \$200, and \$250 per point.

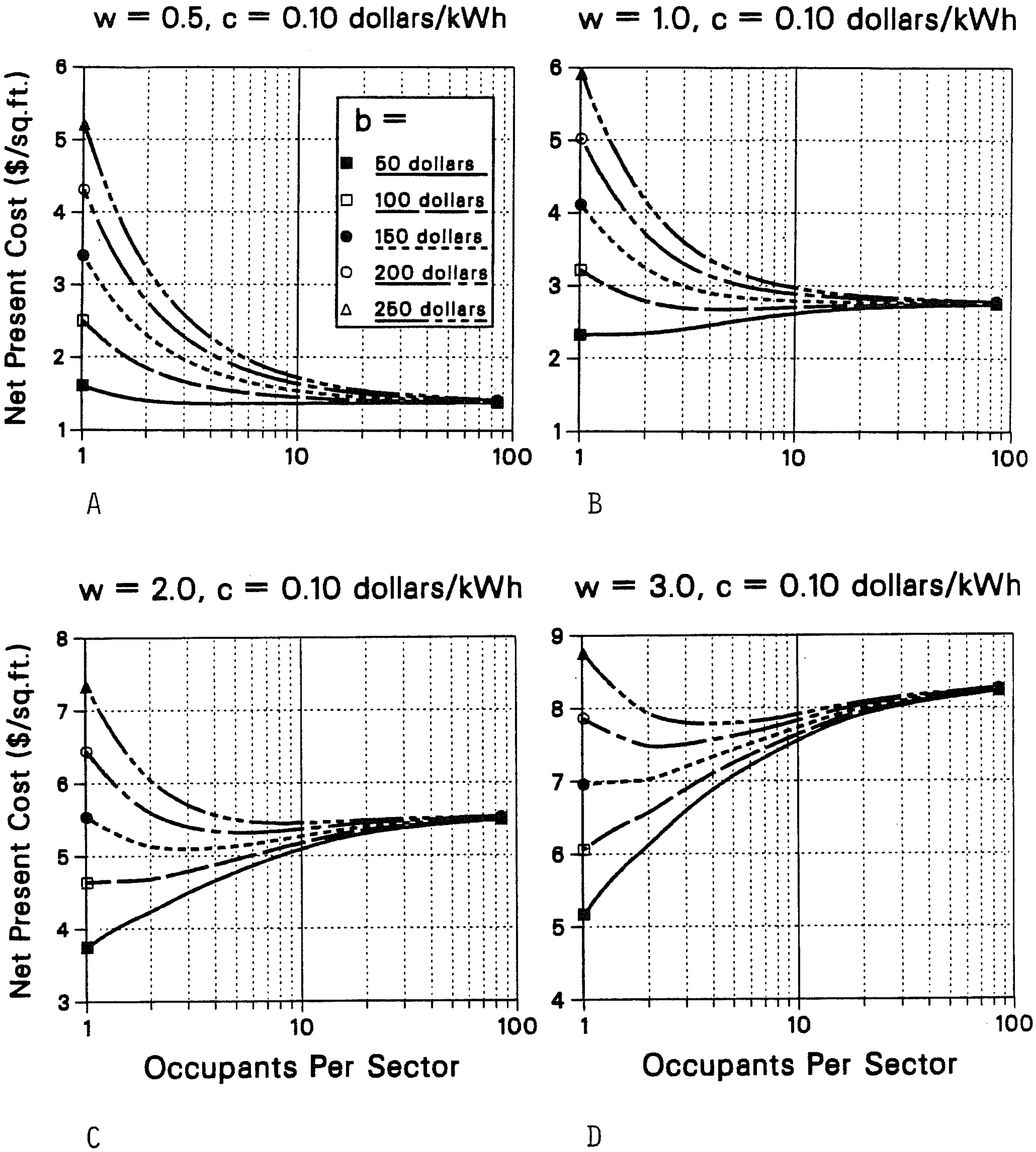

Figure 4. Net present cost of lighting system per square foot as a function of number of occupants per sector using measured switching probability distributions and switching times given in Fig. 1 for control schedule with noon light level reduction. Graphs are presented for four values of wc (energy cost x power density): (a) wc = 0.05×10^{-3} \$/hr-ft^2, (b) wc = 0.1×10^{-3} \$/hr-ft^2, (c) wc = 0.2×10^{-3} \$/hr-ft^2, and (d) wc = 0.3×10^{-3} \$/hr-ft^2. Occupant density is 110 ft^2/person.

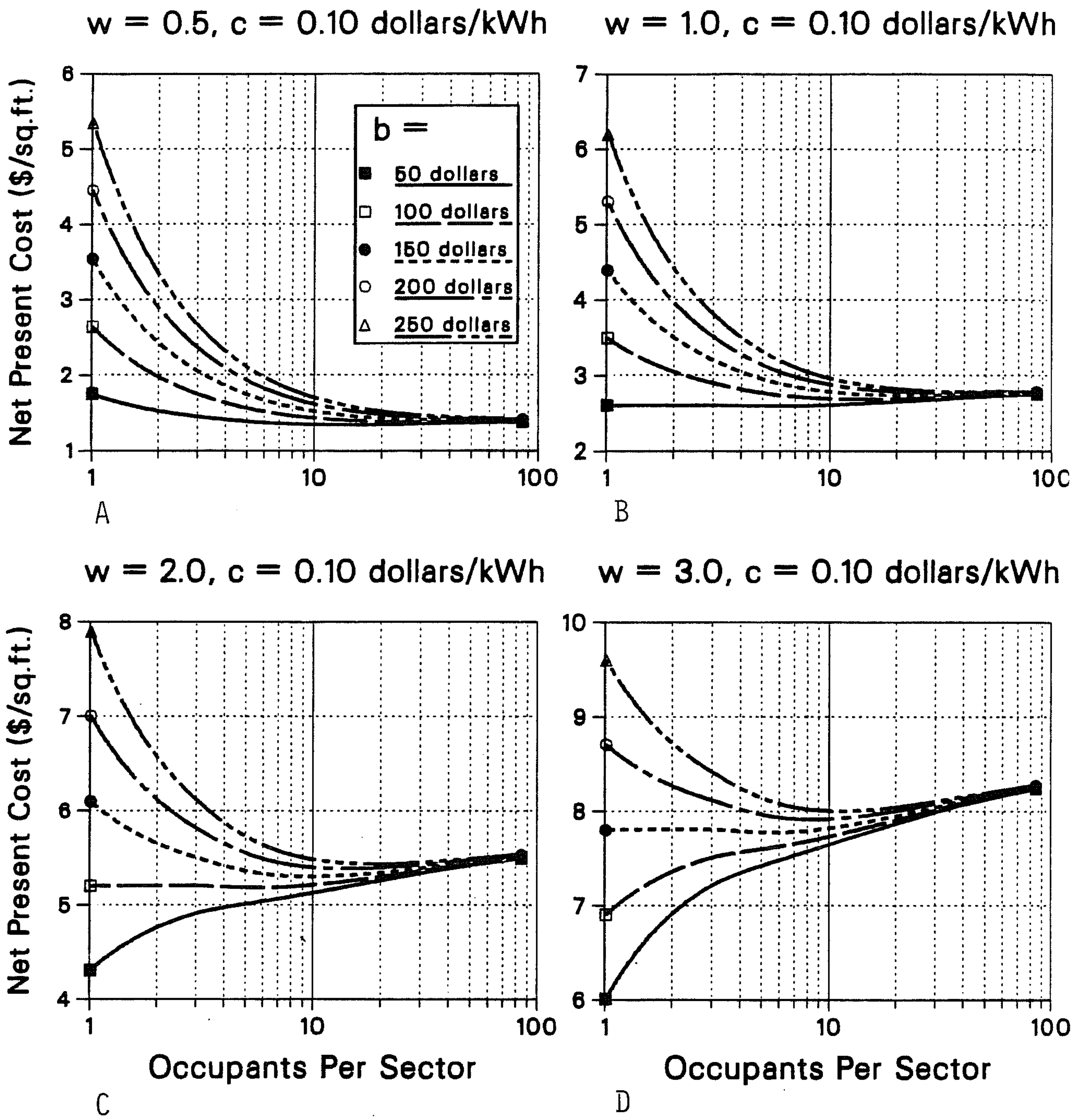

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Figure 5. Net present cost of lighting system per square foot as a function of number of occupants per sector using measured switching probability distributions and switching times given in Fig. 1 for control schedule without noon light level reduction. Graphs are presented for four values of wc (energy cost x power density): (a) wc = 0.05×10^{-3} \$/hr-ft^2, (b) wc = 0.1×10^{-3} \$/hr-ft^2, (c) wc = 0.2×10^{-3} \$/hr-ft^2, and (d) wc = 0.3×10^{-3} \$/hr-ft^2. Occupant density is 110 ft^2/person.

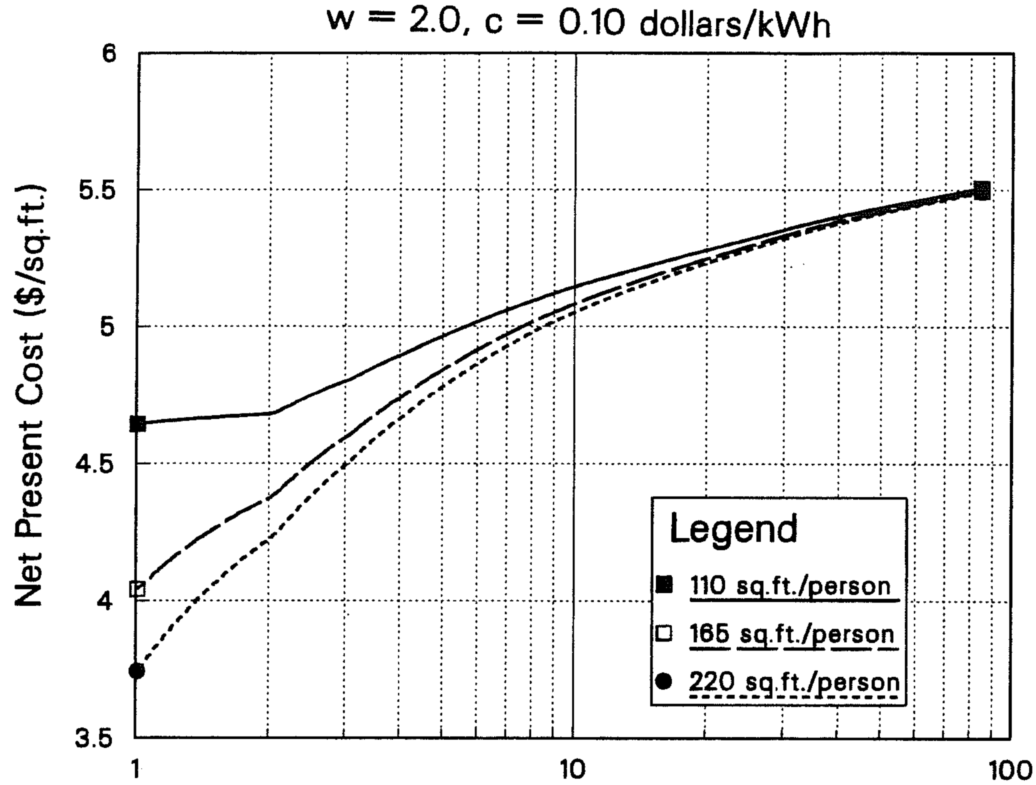


Figure 6. Net present cost of lighting system per square foot for 3 different occupant densities (110, 165, 220 ft^2/person) using measured switching probability distributions and switching times given in Fig. 1 for control schedule with noon light level reduction. Results are for wc = 0.2 x 10^{-3} \$/hr-ft^2, and \$100 installed cost per control point.

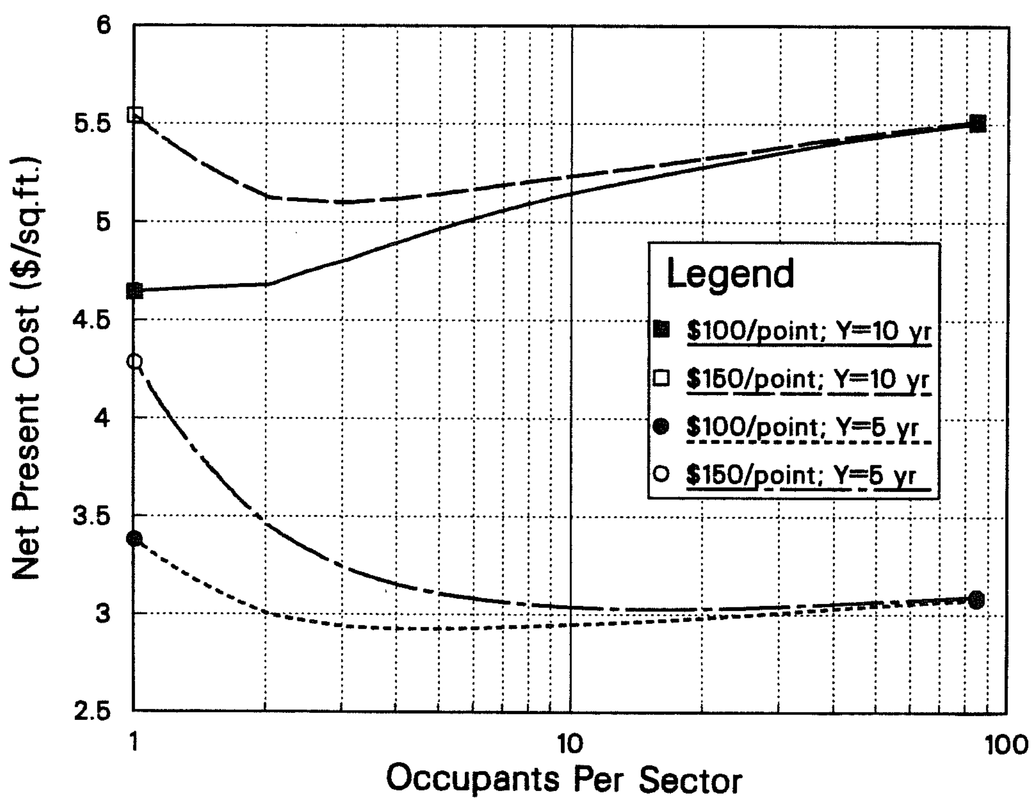


Figure 7. Net present cost of lighting system per square foot assuming time horizons of 5 and 10 years and installed control point costs of \$100 and \$150 per point using measured switching probability and switching times given in Fig. 1 for control schedule with noon light level reduction. Results are for wc = 0.2 x 10^{-3} \$/hr-ft^2, and occupant density of 110 ft^2/person.

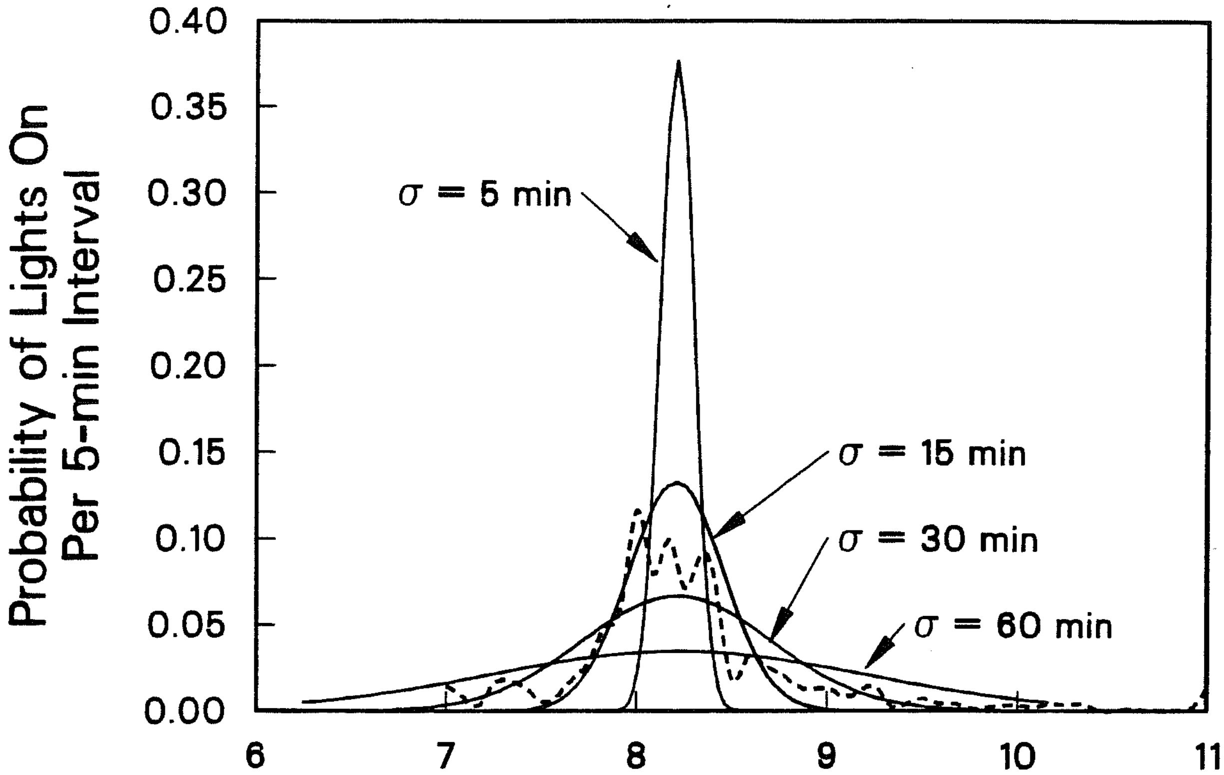


Figure 8a. Probability of occupants switching on lights per 5-minute interval in morning assuming Gaussian distributions with standard deviations of 5, 15, 30 and 60 min. Dashed line is experimentally measured distribution. The area under all curves normalized to 1.

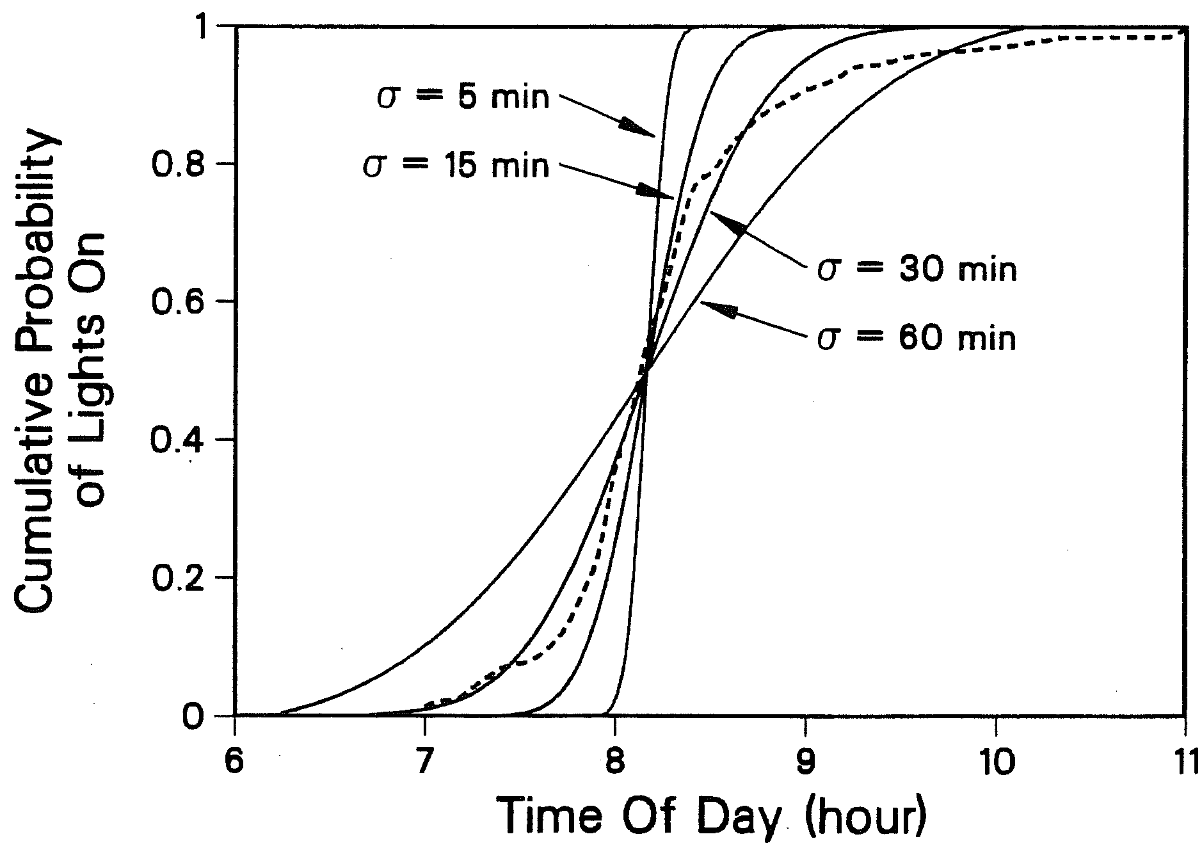
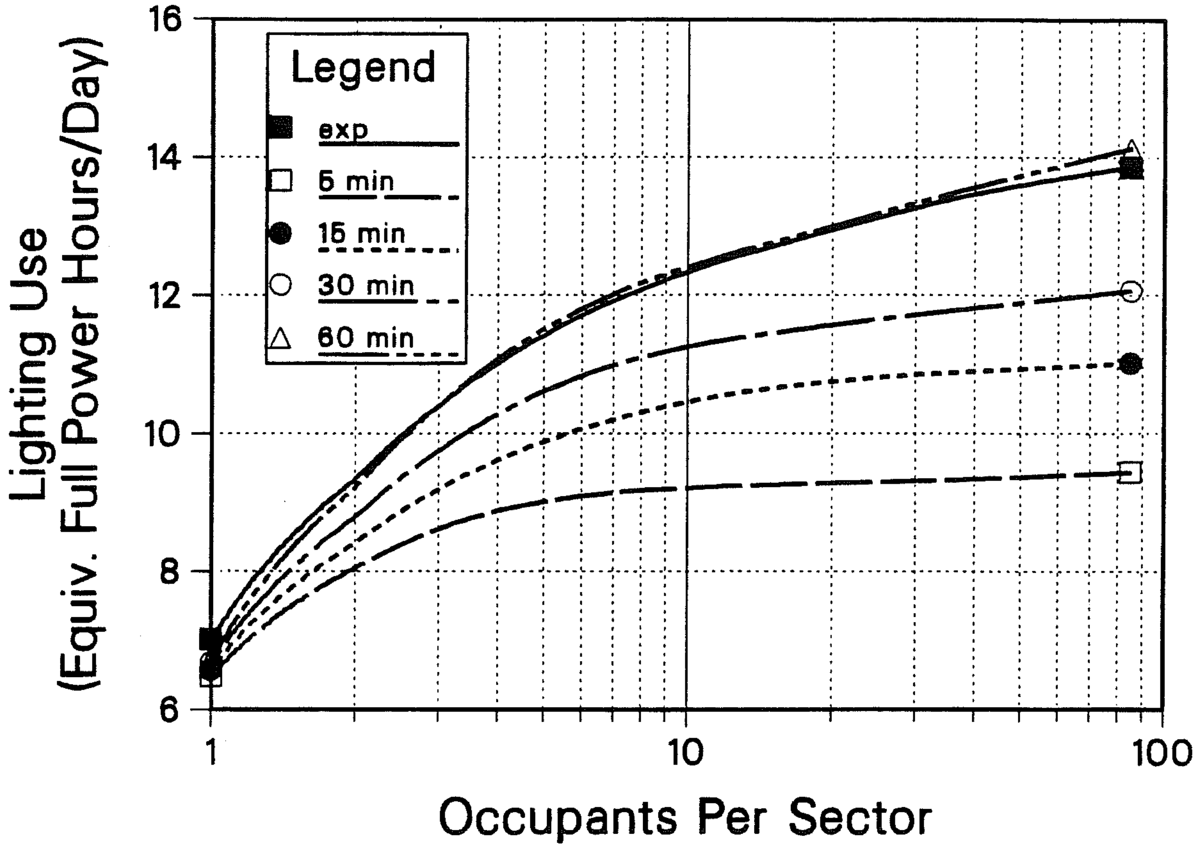


Figure 8b. Cumulative probability of occupants switching on lights in morning assuming Gaussian distributions with standard deviations of 5, 15, 30 and 60 min. Dashed line is experimentally measured distribution.

A



B

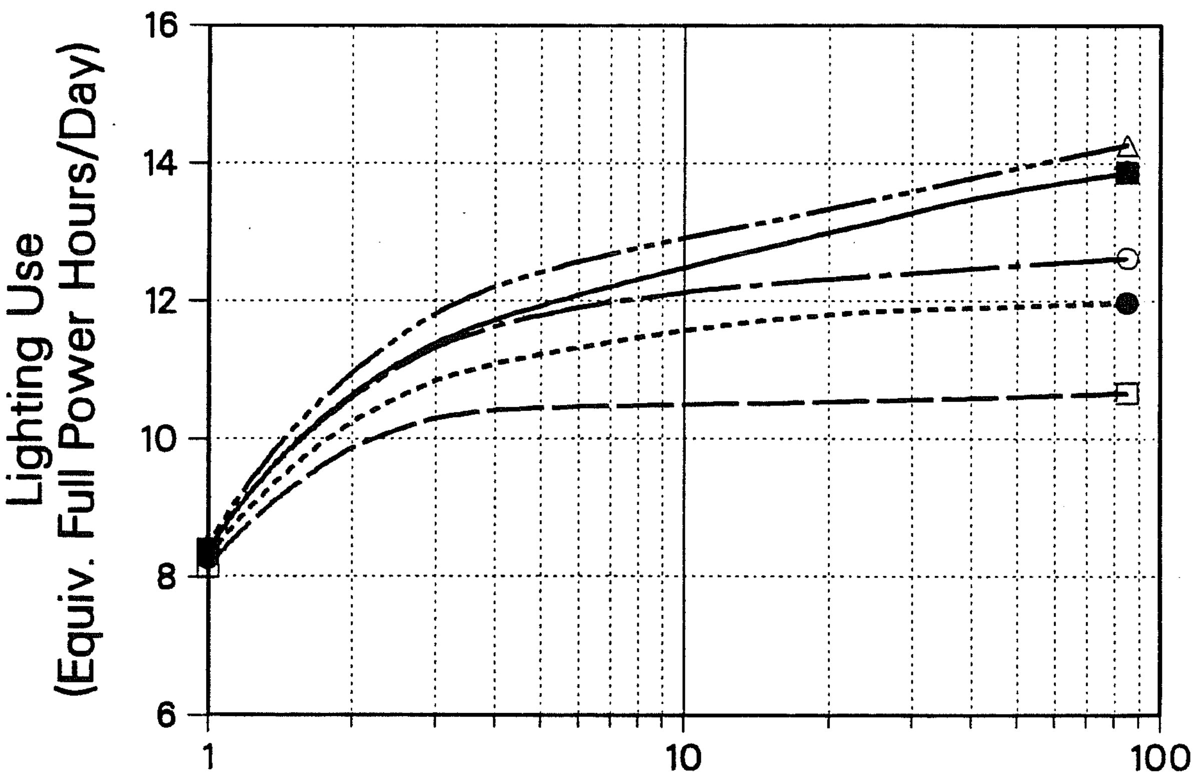


Figure 9. Lighting energy use per day as a function of number of occupants in a sector for four Gaussian switching probability distributions (see Fig. 8a) and experimentally derived distribution. Lighting use plotted as equivalent full power lighting hours per day for control schedule (a) with noon-time lighting reduction and (b) without.

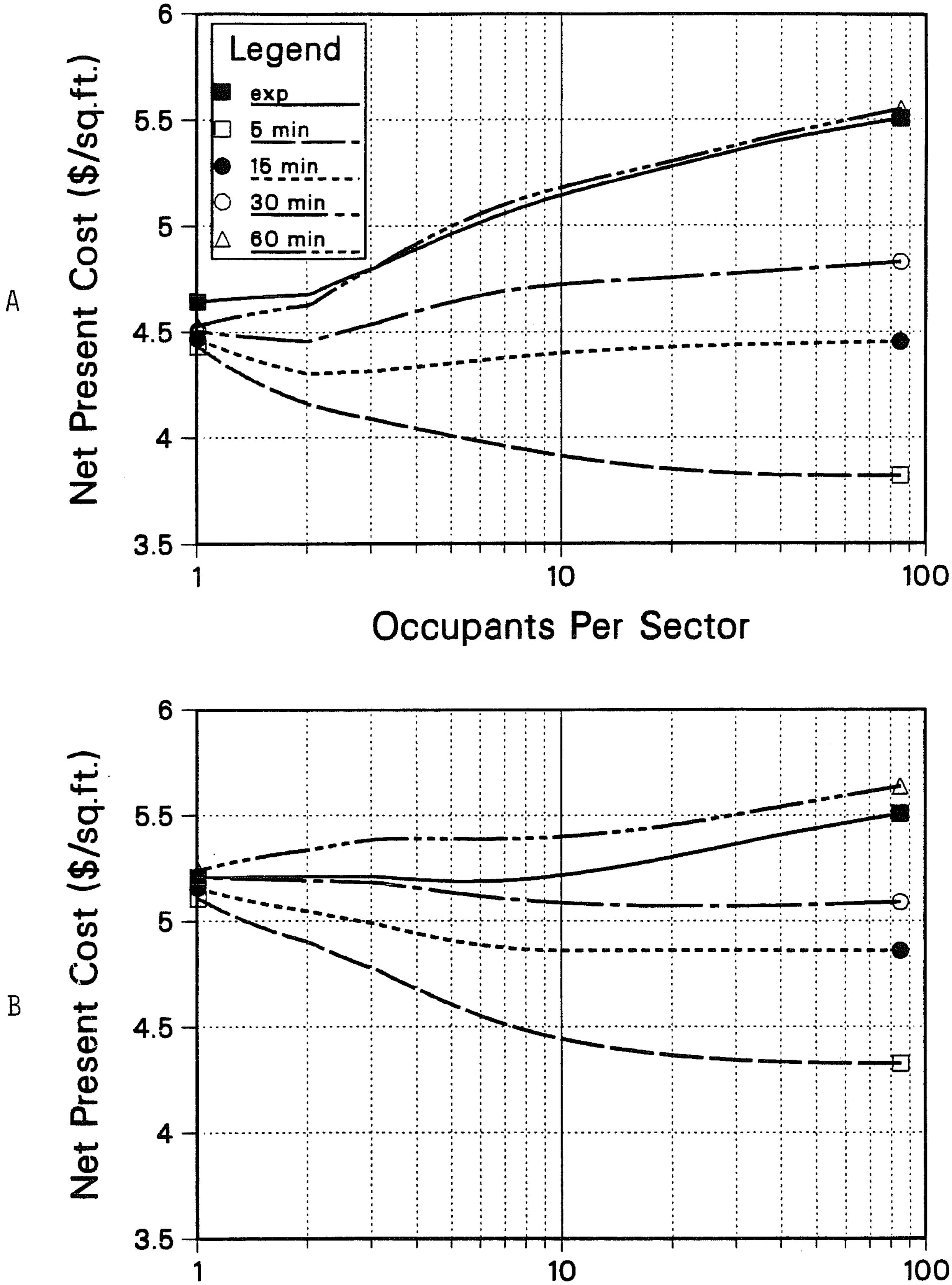


Figure 10. Net present cost of lighting system per square foot as a function of number of occupants per sector for four Gaussian switching probability distributions and experimentally measured distribution. Results are for control schedule (a) with noon light level reduction and (b) without, with wc = 0.2 x 10^{-3} \$/hr-ft^2, \$100 installed control point cost, and 110 ft^2/person.