To be presented at the Fifth National Passive Solar
Conference, Amherst, MA, October 19-26, 1980; and published
in the proceedings

A PROCEDURE FOR CALCULATING INTERIOR DAYLIGHT ILLUMINATION WITH A PROGRAMMABLE HAND CALCULATOR

Harvey J. Bryan and Robert D. Clear

October 1980

To Appear in Proceedings of the Fifth National Passive Solar Conference
Amherst, Massachusetts
October 19-26, 1980

A PROCEDURE FOR CALCULATING INTERIOR DAYLIGHT ILLUMINATION

WITH A PROGRAMMABLE HAND CALCULATOR

Harvey J. Bryan and Robert D. Clear

Lawrence Berkeley Laboratory
University of California
Berkeley, California   94720

A PROCEDURE FOR CALCULATING INTERIOR DAYLIGHT ILLUMINATION WITH A PROGRAMMABLE HAND CALCULATOR

Harvey J. Bryan and Robert D. Clear
Lawrence Berkeley Laboratory
University of California
Berkeley, California

ABSTRACT

A procedure is described for calculating interior daylight illumination using an inexpensive programmable hand calculator. The proposed procedure calculates illumination at any point within a room utilizing sky luminance distribution functions that are consistent with the CIE (Commission Internationale de l'Eclairage) Overcast and Clear Sky functions. This procedure separates the light reaching the point being considered into three components, these being (a) light directly from the sky, (b) light after being reflected from external, and (c) internal surfaces. Finally, two examples are presented in order to demonstrate the proposed procedure and indicate the speed with which the calculations may be performed.

1. INTRODUCTION

As design professionals focus on thermal load reduction in commercial buildings, lighting looms as the single largest energy consumer in these buildings. Daylighting is presently being proposed as one of the most promising energy conservation strategies for reducing these lighting loads.[1] Although substantial savings in both electrical energy and peak power demand exist, it is not obvious that projected savings can be achieved given the present state of daylighting design procedures. The major obstacle in the utilization of daylighting is the lack of accurate and simple procedures for daylighting analysis.

Present procedures for calculating the effects of daylight can be divided into two categories: simplified procedures (e.g., lumen method[2]) which often make assumptions which can result in questionable accuracy, and computer programs (e.g., Lumen II daylighting program[3]) which although highly accurate, require the preparation of detailed input data and access to time-sharing computer facilities. Therefore, it became necessary that a procedure be developed for calculating daylight which is highly accurate and yet not dependent on a computer environment. This objective can now be achieved with the aid of a programmable hand calculator.

Numerous procedures for calculating illumination from natural sources have been discussed in the literature.[4] However, none of these procedures have been introduced into a programmable hand calculator. The proposed procedure utilizes the daylight factor method, which is able to calculate daylight illumination at any point within a room. The procedure presented in this paper was developed specifically for the Texas Instruments TI-59 calculator, however with some modifications this procedure can be extended to other programmable hand calculators (e.g., HP-67).

2. THE DAYLIGHT FACTOR METHOD

The daylight factor method, which is recommended by the CIE, is used in Europe, particularly in Britain where the method was first developed. Today, the daylight factor is defined as the ratio between the daylight illumination at a point in the interior and the simultaneous exterior illumination available on a horizontal surface from an unobstructed sky (excluding direct sunlight) expressed as a percentage. The light reaching the point being considered is separated into three paths (see Figure 1): light directly from the sky (Sky Component or SC); light after reflection from external surfaces (Externally Reflected Component or ERC); and light after reflection from internal surfaces (Internally Reflected Component or IRC). The total for these three components gives the daylight factor, which is simply expressed as:

Daylight Factor = SC + ERC + IRC

3. THE SKY COMPONENT

The sky component is the ratio between the daylight illumination at a point in the interior which is received directly from the sky and the simultaneous exterior illumination available on a horizontal surface from an unobstructed sky.

3.1 Overcast Sky

The sky component from an overcast sky is a function of the sky luminance, the window transmission, the solid angle ($\sin\theta d\theta d\Psi$) and

Program for the overcast sky, this value is then converted into the ERC by correcting for the reduced luminance of the obstruction compared with the luminance of the sky. In cases where the obstruction is below 20°, its luminance is usually assumed to be one fifth that portion of the sky that is obscured; and for cases where the obstruction is above 20°, its luminance is usually assumed to be one tenth that portion of the sky that is obscured.

4.2 Clear Sky

The ERC from a clear sky has not yet been introduced into the programmable hand calculator procedure. However, horizontal shading devices can be modeled by assuming an effective window and neglecting the luminance of the under side of the device, which is usually small. Work is in progress to extend the programmable hand calculator procedure to include the ERC for more complex obstructions. The present procedure may be used with no loss in accuracy when there are no external obstructions visible from the reference point.

5. THE INTERNALLY REFLECTED COMPONENT

The IRC is the ratio between the daylight illumination at a point in the interior which is received after being interreflected off interior surfaces and the simultaneous exterior illumination available on a horizontal surface from an unobstructed sky. Presently there are two approaches for calculating the IRC, the split flux method and the more accurate finite difference method. The approach that will be used here is the split flux method, the simpler of the two.

The split flux method divides the light entering the room into two parts (see Figure 3), light received directly from the sky and that received directly from the ground. The light from the sky on entering the room is considered to be modified by the average reflectance of the floor and those parts of the walls below the mid-height of the window. The light from the ground is considered to be modified by the average reflectance of the ceiling and those parts of the walls above the mid-height of the window.

The formula for the average IRC is given as:[7]
$$IRC = \frac{T \times W}{A(1-R)} (f_s \cdot R_{fw} + f_g \cdot R_{cw}) \times 100\% \quad . . . (4)$$
where
T = Transmittance of glass
W = Area of window
A = Total area of ceiling, floor and walls including area of window
R = Average reflectance of the ceiling, floor and all walls, including window
R_{fw} = Average reflectance of the floor and those parts of the walls, below the plane of the mid-height of the window (excluding the window-wall)
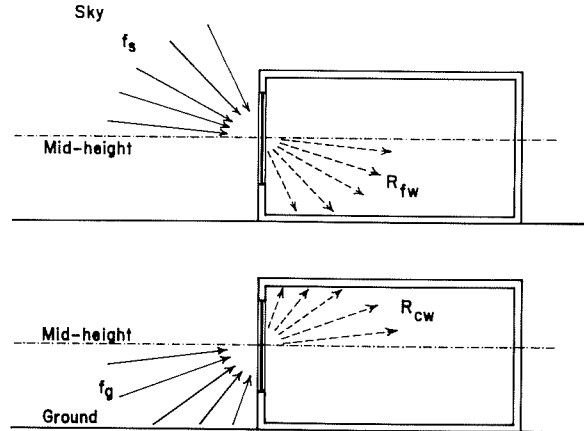

Fig. 3. The split flux concept

R_{cw} = Average reflectance of the ceiling and those parts of the walls, above the plane of the mid-height of the window (excluding the window-wall)
f_s = Window factor due to the light incident on the window from sky
f_g = Window factor due to the light incident on the window from ground

5.1 Overcast Sky

The IRC from an overcast sky is determined by equation (4). The various window (transmittance and area) and room (surface areas and reflectances for walls, ceiling and floor) conditions are determined from the design, while the window factors are determined as follows: f_s for various angles of obstructions have been precalculated and are documented separately, and f_g by multiplying the reflectances of ground surface (R_g) by the ground configuration factor (G_{cf}) which is always 0.5 for a horizontal ground plane. The various values are then entered into the IRC program which calculates the average IRC for the entire room.

5.2 Clear Sky

The IRC from a clear sky is also determined by equation (4). However, because direct solar illumination is contributing illumination to the ground, f_s and f_g are calculated differently. The window factors for f_s have been precalculated and are documented separately, while f_g is determined as follows:
$$f_g = \frac{(E_{sun} + E_{sky}) \times R_g \times G_{cf}}{E_{sky}} \quad . . . . (5)$$
where
E_{sun} = Illumination from the sun
E_{sky} = Illumination from the sky
R_g = Reflectance of ground surface
G_{cf} = Ground configuration factor (for a horizontal surface G_{cf} always = 0.5)

The various values are then entered into the IRC program which calculates the average IRC for the entire room.

7.2 Clear Sky

The sky component program took approximately 5 minutes for data entry and another 27 minutes (3 minutes/reference point) to calculate the 9 reference points, again the designer does not need to be present for the calculation phase if a PC-100 printer is used.

The externally reflected component was omitted because no obstructions are present.

The internally reflected component program took approximately 4 minutes for data entry and another minute to calculate.

The daylight factor was determined by adding the IRC of 6.4% to the 9 sky component values. Finally, the daylight factors are multiplied by the horizontal illumination available from an unobstructed clear sky (1374 fc) in order to determine the daylight illumination incident on the reference points. The daylight illumination for the 9 reference points are presented in Table 3.

A Lumen II daylighting analysis was then performed utilizing the same parameters as was assumed in the programmable hand calculator procedure. The daylighting illumination for the 9 reference points are presented in Table 4.

Table 3.
****
PROGRAMMABLE HAND CALCULATOR PROCEDURE
****

ILLUMINATION FROM CLEAR SKY

Y-COOR.	X-COORDINATES		
	5.0	10.0	15.0
15.0	109.9	109.9	105.8
10.0	138.8	140.1	127.8
5.0	232.2	239.1	196.5

Table 4.
****
LUMEN II DAYLIGHTING ANALYSIS
****

ILLUMINATION FROM CLEAR SKY

Y-COOR.	X-COORDINATES		
	5.0	10.0	15.0
15.0	117.1	115.4	111.6
10.0	144.5	141.2	128.9
5.0	235.3	236.6	189.7

The percent difference between the Programmable Hand Calculator Procedure and Lumen II are within ± 6% of each other, which suggests a satisfactory correlation.

8. CONCLUSION

An accurate, simple and relatively fast procedure for calculating daylight has been presented which links itself with internationally recommended practices. It is hoped that the application of this procedure will encourage the use of daylighting, as well as placing daylighting in a proper relationship to other design considerations. A lengthier version of this paper which includes a detailed program description, program listings, user instructions and several worked examples is available from the authors at: Windows and Daylighting Program, Lawrence Berkeley Laboratory, Building 90, Room 3111, Berkeley, California, 94720.

9. ACKNOWLEDGEMENTS

10. REFERENCES

1. U.S. Department of Energy, Office of Conservation and Solar Applications, "Energy Performance Standards for New Buildings; Proposed Rule, Part II," Federal Register, Vol. 44, No. 230, November 28, 1979, p. 68135.
2. IES Daylighting Committee, "Recommended Practice of Daylighting," LIGHTING DESIGN & APPLICATION, Vol. 9, No. 2, February 1979.
3. DiLaura, D.L., and Hauser, G.A. "On Calculating the Effects of Daylighting in Interior Spaces," JOURNAL OF THE ILLUMINATING ENGINEERING SOCIETY, Vol. 8, No. 1, October 1978, p.2.
4. CIE Technical Committee E-3.2, "Daylight: International Recommendations for the Calculation of Natural Daylight," CIE PUBLICATION No. 16, Commission Internationale de l'Eclairage, Paris, 1970.
5. Rivero, R. "Illuminacion Natural. Calculo del Factor de Dia Directo para Ventanas sin Vidrios y con Vidrios y para Cielos Uniformes y No Uniformes," Instituto de la Construccion de Edificios, Montevideo, 1958.
6. CIE Technical Committee 4.2, "Standardization of Luminance Distribution on Clear Skies," CIE PUBLICATION No. 22, Commission Internationale de l'Eclairage, Paris, 1973, p.7.
7. Hopkinson, R.G., Longmore, J., and Petherbridge, P., "An Empirical Formula for the Computation of the Indirect Component of the Daylight Factor," TRANSACTIONS OF THE ILLUMINATING ENGINEERING SOCIETY, London, Vol. 19, 1954, p.201.
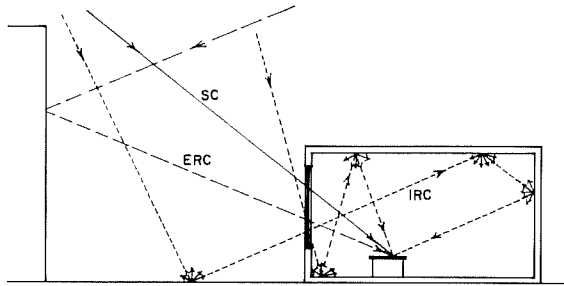
Fig. 1. Components of the daylight factor

Fig. 2. Angular coordinate system of window

the cosine correction ($\cos\theta$) for the projection onto the horizontal plane. Here θ,ℓ are the standard polar angles; θ equals the angle from the zenith ($\theta=90°$-altitude) and ℓ equals the azimuthal angle. For convenience we set $\ell=0$ to be the direction of the window normal. The CIE Standard Sky luminance function[4] is given by the following formula:

$$L(\theta)= \frac{L(0)}{3}(1 + 2\cos\theta) \quad\quad\quad (1)$$

where
 $L(\theta)$= Luminance of sky position at angle θ
 $L(0)$= Luminance of the zenith
  θ = Angle from the zenith to sky position

The illumination through a window is then divided by the illumination on a horizontal plane which is the integral of equation (1) times the cosine correction over the hemisphere. This integral has been done analytically and is equal to $7\pi/9\ L(0)$.

The window transmission corrected for angle of incidence loss must also be considered. Rivero's approximation for angle of incidence loss is used,[5] which is given by the following formula:

$$T(\Psi)= 1.018\ T(0)\ \cos\Psi(1 + \sin^3\Psi) \quad (2)$$

where
 $T(\Psi)$= Transmission of glass at angle Ψ to the normal of the window
 $T(0)$= Transmission at normal incidence

The sky component program does the integral over the product of equations (1),(2) and the cosine correction numerically. The logic of the program is made simpler by changing to a rectangular coordinate system Ω,ℓ (see Figure 2) so that the limits of the integral are independent and are given by the angular coordinates of the four corners of the window. The integral over ℓ is performed using the calculator's built in Simpson's rule integration routine (TI-59's ML-09 Program). The integral over Ω is performed by writing a separate Simpson's rule routine.

3.2 Clear Sky

The clear sky component calculation for the clear sky has the same form as the overcast sky calculation except for the sky luminance function and its associated inputs. The clear sky luminance function that will be used is
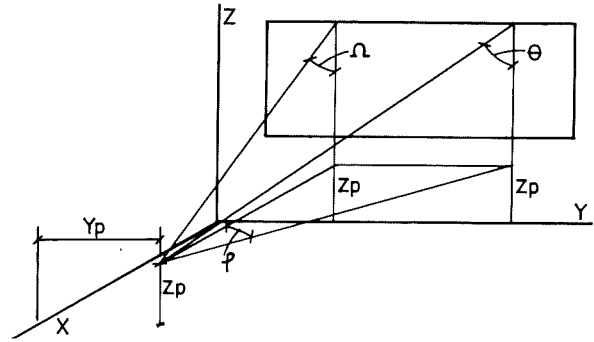
consistent with the function presently being considered by the CIE Technical Committee TC-4.2,[6] which is given by the following formula:

$$\frac{L(\theta)}{L(0)} = \frac{F_1}{F_2} \quad\quad\quad\quad\quad (3)$$

where
 $L(\theta)$= Luminance of sky position at angle θ
 $L(0)$= Luminance of the zenith
  $F_1 = (1-e^{-0.32\sec\theta})(0.91+10e^{-3\delta}+0.45\cos^2\delta)$
  $F_2 = 0.274(0.91+10e^{-3\theta_s}+0.45\cos^2\theta_s)$
  θ = Angle from the zenith to sky position
  θ_s = Angle from the zenith to the sun
  δ = Angle of the sun from $L(\theta)=\cos^{-1}$
    $(\sin\theta\ \sin\theta_s\ \cos(\ell+\ell_w-\ell_s)+\cos\theta\ \cos\theta_s)$

In addition the sky component calculation for the clear sky requires added inputs: θ_s, the angular zenith distance of the sun; ℓ_s, the angular azimuthal distance of the sun from south; ℓ_w, angular azimuthal distance of the window normal from south; and Nsc, normalization factor which is the illumination on a horizontal plane from the hemisphere (integral of equation (3) times the cosine correction over the hemisphere). This integral has been done numerically and is documented separately.

As in the overcast sky program the integration is done numerically over a rectangular coordinate system by an integral over the product of equations (2), (3) and the cosine correction.

4. THE EXTERNALLY REFLECTED COMPONENT

The externally reflected component is the ratio between the daylight illumination at a point in the interior which is received directly from external surfaces and the simultaneous exterior illumination available on a horizontal surface from an unobstructed sky.

4.1 Overcast Sky

The ERC from an overcast sky can be determined by considering the external obstruction that is visible from the reference point as an effective sky component (SCe) whose luminance is some fraction of the sky that is obscured. The effective sky component for any obstruction is calculated by the Sky Component

4

6. THE TOTAL DAYLIGHT FACTOR

The total daylight factor is the sum of the sky component (SC), the externally reflected component (ERC) if present and the internally reflected component (IRC) programs. However, in order that the daylight factor be useful it needs to be converted into daylight illumination. This is accomplished by determining the exterior sky illumination incident on a horizontal surface, this value is then multiplied by the daylight factor in order to get the interior daylight illumination.
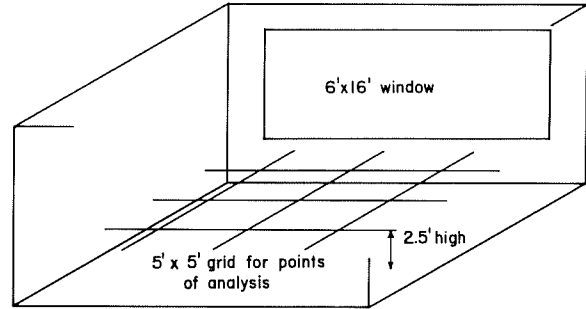
7. VALIDATION OF PROPOSED PROCEDURE

Both the overcast and clear sky procedures have been validated against the Lumen II computer program. The Lumen II program was selected because it is well documented, has undergone extensive testing, and its daylighting features have been compared favorably to a series of physical measurements.[3]

Figure 4 illustrates the 20'x20'x10' room with 6'x16' window 2.5' high that was used in both the overcast and clear sky examples. Within this room a 5'x5' grid (9 reference points) 2.5' high was selected for analyses. External and internal conditions were then established as criteria for these examples.

The external conditions for these examples assumed the following parameters.
● Solar Altitude: 40°
● Window Orientation: 90° west of sun
● Illumination from sky on ground: 1268 footcandles from overcast sky, and 1374 footcandles from clear sky (lumen II #'s)
● Illumination from sun on ground: 5192 footcandles (Lumen II #'s)
● Ground Reflectance: 20%
● Obstruction: None

The internal conditions for these examples assumed the following parameters.
● Room Length: 20 feet
● Room Width: 20 feet
● Room Height: 20 feet
● Sill Height: 2.5 feet
● Ceiling Reflectance: 70%
● Wall Reflectance: 60%
● Floor Reflectance: 40%
● Glazing Type: Clear, 85% Transmittance

7.1 Overcast Sky

The sky component program took approximately 5 minutes for data entry and another 11 minutes (1.2 minutes/reference point) to calculate the 9 reference points. However the designer does not need to be present for the calculation phase if a PC-100 printer is used.

The externally reflected component was omitted because no obstructions are present.

Fig. 4. Typical room arrangement

The internally reflected component program took approximately 4 minutes for data entry and another minute to calculate.

The daylight factor was then determined by adding the IRC of 2.6% to the 9 sky component values. Finally, the daylight factors are multiplied by the horizontal illumination available from an unobstructed overcast sky (1268 fc) in order to determine the daylight illumination incident on the reference points. The daylight illumination for the 9 reference points are presented in Table 1.

A Lumen II daylighting analysis was then performed utilizing the same parameters as was assumed in the programmable hand calculator procedure. The daylighting illumination for the 9 reference points are presented in Table 2.

Table 1.
****
PROGRAMMABLE HAND CALCULATOR PROCEDURE
****

ILLUMINATION FROM OVERCAST SKY

	X-COORDINATES		
Y-COOR.	5.0	10.0	15.0
15.0	44.4	46.9	44.4
10.0	64.7	71.0	64.7
5.0	150.9	171.2	150.9

Table 2.
****
LUMEN II DAYLIGHTING ANALYSIS
****

ILLUMINATION FROM OVERCAST SKY

	X-COORDINATES		
Y-COOR.	5.0	10.0	15.0
15.0	46.3	47.2	46.5
10.0	65.0	70.5	65.0
5.0	150.4	171.1	150.4

The percent difference between the Programmable Hand Calculator Procedure and Lumen II are within ± 5% of each other, which suggests a satisfactory correlation.