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ABSTRACT

The bidirectional solar-optical properties of a fenestration system are necessary to accurately determine its luminous and thermal performance. Bidirectional transmittance and reflectance can be determined experimentally for fenestration systems of arbitrary complexity using a scanning radiometer, after which the total directional absorptance can be calculated. However, for the case of multilayer fenestration systems, this approach does not provide information about the net absorptance of each layer. Moreover, the same layers can be ordered in more than one way, resulting in fenestration systems with different solar-optical properties, the determination of which requires additional experimental procedures. This paper describes a mathematical model for the calculation of the bidirectional solar-optical properties of multi-layer fenestration systems, using the bidirectional solar-optical properties of each layer. The model is based on the representation of the bidirectional solar-optical properties using matrices. Matrix operations are then used to calculate the bidirectional solar-optical properties of any combination of layers, considering the interreflections between them. This approach offers two advantages: (1) the reduction of the experimental procedures to those required for the determination of the bidirectional transmittance and reflectance of fenestration layers, rather than complete fenestration systems, and (2) the determination of the net absorptance of each layer as part of the fenestration system, rather than the total absorptance of the complete fenestration system.

INTRODUCTION

A quantitative understanding of the solar-optical properties of fenestration systems is essential for accurate calculation of daylight illuminance levels, glare potential, solar heat gain, and thermal comfort. For clear, tinted, or reflective glass, for each direction of incoming radiation, there are only two specific directions of outgoing radiant flux: the direction of the transmitted radiation and the direction of the specularly reflected radiation. In this case, the solar-optical properties are expressed as simple functions of the incident angle of the incoming radiation. However, very little is known about the properties of fenestration systems that are optically more complex, such as systems that incorporate diffusive glass, venetian blinds, horizontal or vertical louvers, solar screens, etc. In this case, for each direction of incoming radiation, there is a particular 4 π distribution of outgoing radiant flux, either transmitted or reflected by the fenestration system. For a complete description of the radiant behavior of such complex fenestration components, it is necessary to express their solar-optical properties as functions of both the incoming and the outgoing directions of the radiant flux.

For the purposes of determining such bidirectional properties, a scanning radiometer is under development at the Lawrence Berkeley Laboratory. This facility has been designed to measure the solar and visible bidirectional transmittance and reflectance of fenestration components and systems of arbitrary complexity (Spitzglas 1986). Determining the bidirectional properties of actual fenestration systems through direct measurement allows us to avoid the assumptions about the geometry and the texture of fenestration components that are commonly used in mathematical modeling. However, various combinations of even the most common fenestration components can produce thousands of optically different fenestration systems. Measuring all such combinations is practically impossible; moreover, the scanning radiometer provides no information about the net absorptance of individual layers as they perform as parts of fenestration systems. The layer-by-layer absorption of solar radiation, which ultimately contributes to solar heat gain through re-conduction, is a complicated function of the distribution of the incident radiation and the nature of the interreflections between the fenestration layers.

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A mathematical procedure is therefore required to determine the overall optical properties of a fenestration system from the properties of each individual layer. This paper describes such a procedure. It is based on a matrix representation of the bidirectional properties of fenestration layers and systems. A computer program named TRA (Transmittance Reflectance Absorptance) was developed as an application of the method. The output of TRA serves as input to the daylighting calculation model SUPERLITE (Selkowitz et al. 1982; Modest 1982; Windows and Daylighting Group 1985) for determining daylight illuminance and luminance distributions and as input to heat transfer calculation models such as WINDOW-2.0 (Rubin et al. 1986) and DOE-2 (Building Energy Simulation Group 1984, 1985) for determining solar heat gain. This approach offers the capability to determine the hourly, seasonal, or annual luminous and thermal performance of fenestration systems of arbitrary complexity under varying environmental conditions in an accurate and consistent way.

BIDIRECTIONAL SOLAR-OPTICAL PROPERTIES

The bidirectional solar-optical properties of an optical element describe the fraction of the incoming radiant flux incident in direction (ζ_{in},θ_{in}) that leaves in each direction $(\zeta_{out},\theta_{out})$ (Figure 1). Fenestration systems usually define a plane that separates the environment into two hemispheres. If the incoming and the outgoing directions are in the same hemisphere, the solar-optical property is called *reflectance* ; otherwise, it is called *transmittance*. The fraction that is neither transmitted nor reflected is called *absorptance*. The solar-optical properties that describe the alteration of the incoming radiation by a fenestration system are then expressed by the following functions:

$$\text{front transmittance} = ft\,(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in}), \tag{1a}$$
$$\text{front reflectance} = fr\,(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in}), \tag{1b}$$
$$\text{front absorptance} = fa\,(\zeta_{in},\theta_{in}), \tag{1c}$$
$$\text{back transmittance} = bt\,(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in}), \tag{1d}$$
$$\text{back reflectance} = br\,(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in}), \text{ and} \tag{1e}$$
$$\text{back absorptance} = ba\,(\zeta_{in},\theta_{in}), \tag{1f}$$

where "front" represents the outside hemisphere, referring to radiation coming from outdoors, and "back" represents the inside hemisphere, referring to radiation coming from indoors.

The bidirectional solar-optical properties can be represented in matrix form by dividing the incoming and outgoing hemispheres into small solid-angle elements centered around a discrete set of *m* incoming directions $((\zeta_{in},\theta_{in})_j, j=1,m)$ and a discrete set of *n* outgoing directions $((\zeta_{out},\theta_{out})_i, i=1,n)$. An arbitrary incoming direction can then be associated with the single index *j* and an outgoing direction with the single index *i*. The transmittance function $ft(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in})$, for example, can then be represented by the matrix:

$$\begin{vmatrix} ft(1,1) & ft(1,2) & ft(1,3) & \dots & ft(1,m) \\ ft(2,1) & ft(2,2) & ft(2,3) & \dots & ft(2,m) \\ ft(3,1) & ft(3,2) & ft(3,3) & \dots & ft(3,m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ ft(n,1) & ft(n,2) & ft(n,3) & \dots & ft(n,m) \end{vmatrix} \tag{2}$$

where $ft(i,j) = ft((\zeta_{out},\theta_{out})_i, (\zeta_{in},\theta_{in})_j)$.

The transmitted flux T(i) in direction *i* is obtained from the incident flux distribution (I(j), j=1,m) by the following summation:

$$T(i) = \sum_{j=1}^{m} ft(i,j)\cdot I(j) \qquad \text{where } i=1,\dots,n \tag{3}$$

The transmitted flux in the *n* outgoing directions can also be determined by the following expression, which is the equivalent of Equation 3 in matrix format:

$$\begin{vmatrix} T(1) \\ T(2) \\ T(3) \\ \vdots \\ T(n) \end{vmatrix} = \begin{vmatrix} ft(1,1) & ft(1,2) & ft(1,3) & \dots & ft(1,m) \\ ft(2,1) & ft(2,2) & ft(2,3) & \dots & ft(2,m) \\ ft(3,1) & ft(3,2) & ft(3,3) & \dots & ft(3,m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ ft(n,1) & ft(n,2) & ft(n,3) & \dots & ft(n,m) \end{vmatrix} \cdot \begin{vmatrix} I(1) \\ I(2) \\ I(3) \\ \vdots \\ I(m) \end{vmatrix} \tag{4}$$

or, using boldface for the matrices, this may be written in very compact form as:

$$\mathbf{T} = \mathbf{ft} \cdot \mathbf{I} \tag{5}$$

Similarly, the reflected flux is given by $\mathbf{R}=\mathbf{fr}\cdot\mathbf{I}$ and the total absorbed radiation is $\mathbf{fa}\cdot\mathbf{I}$ where **fa** is the row vector:

$$\begin{vmatrix} fa(1) & fa(2) & fa(3) & \dots & fa(m) \end{vmatrix} \tag{6}$$

We note that, of the six properties in Equations 1a through 1f, only **ft**, **fr**, and **br** need to be measured, since **bt** can be obtained from **ft** by optical reciprocity, and the absorptances **fa** and **ba** can be calculated from the corresponding transmittance and reflectance matrices.

MULTILAYER FENESTRATION SYSTEMS

The power of the matrix approach is evident when we apply it to determining the net bidirectional properties of a multilayer fenestration system when the transmittance and reflectance distributions for the constituent layers are known. We consider the outgoing flux from each layer to be an incoming flux to the next layer in sequence (Figure 2). For a two-layer system, let $\mathbf{ft_1}$ and $\mathbf{ft_2}$ be the front transmittance matrices for layers 1 and 2, respectively. For incident flux **I**, using Equation 5, the transmitted flux $\mathbf{T_1}$ through layer 1 is determined by:

$$\mathbf{T_1} = \mathbf{ft_1} \cdot \mathbf{I} \tag{7}$$

Flux $\mathbf{T_1}$ is incident on layer 2, so, without interreflections, the transmitted flux $\mathbf{T_2}$ through layer 2 is determined by:

$$\mathbf{T_2} = \mathbf{ft_2} \cdot \mathbf{T_1} = \mathbf{ft_2} \cdot (\mathbf{ft_1} \cdot \mathbf{I}) \tag{8}$$

or, using the standard associative property of matrix multiplication:

$$\mathbf{T_2} = (\mathbf{ft_2} \cdot \mathbf{ft_1}) \cdot \mathbf{I} = \mathbf{ft} \cdot \mathbf{I} \tag{9}$$

where

$$\mathbf{ft} = \mathbf{ft_2} \cdot \mathbf{ft_1} \tag{10}$$

The overall bidirectional transmittance of the two-layer system can thus be expressed in terms of the product of the transmittance matrices of the constituent layers.

We illustrate the details of the various matrix multiplications that take place for a two-layer system by considering the simple case of only two incoming and two outgoing directions (Figure 3). Let the incident flux be:

$$\mathbf{I} = \begin{vmatrix} I(1) \\ I(2) \end{vmatrix} \tag{11}$$

The flux $\mathbf{T_1}$ transmitted into the gap between the two layers is $\mathbf{T_1}=\mathbf{ft_1}\cdot\mathbf{I}$, or, by expanding the matrices:

$$\begin{vmatrix} T_1(1) \\ T_1(2) \end{vmatrix} = \begin{vmatrix} ft_1(1,1) & ft_1(1,2) \\ ft_1(2,1) & ft_1(2,2) \end{vmatrix} \cdot \begin{vmatrix} I(1) \\ I(2) \end{vmatrix} = \begin{vmatrix} ft_1(1,1)\cdot I(1)+ft_1(1,2)\cdot I(2) \\ ft_1(2,1)\cdot I(1)+ft_1(2,2)\cdot I(2) \end{vmatrix} \tag{12}$$

Thus $T_1(1)$, for example, consists of a fraction $ft_1(1,1)$ of $I(1)$, and a fraction $ft_1(1,2)$ of $I(2)$.† Neglecting interreflections, the flux $\mathbf{T_2}$ transmitted through the second layer is $\mathbf{T_2=ft_2 \cdot T_1}$, or, by expanding the matrices:

$$\begin{vmatrix} T_2(1) \\ T_2(2) \end{vmatrix} = \begin{vmatrix} ft_2(1,1) & ft_2(1,2) \\ ft_2(2,1) & ft_2(2,2) \end{vmatrix} \cdot \begin{vmatrix} T_1(1) \\ T_1(2) \end{vmatrix} \tag{13a}$$

$$\begin{vmatrix} T_2(1) \\ T_2(2) \end{vmatrix} = \begin{vmatrix} ft_2(1,1) & ft_2(1,2) \\ ft_2(2,1) & ft_2(2,2) \end{vmatrix} \cdot \begin{vmatrix} ft_1(1,1)\cdot I(1)+ft_1(1,2)\cdot I(2) \\ ft_1(2,1)\cdot I(1)+ft_1(2,2)\cdot I(2) \end{vmatrix} \tag{13b}$$

$$\begin{vmatrix} T_2(1) \\ T_2(2) \end{vmatrix} = \begin{vmatrix} ft_2(1,1)\cdot ft_1(1,1)\cdot I(1)+ft_2(1,1)\cdot ft_1(1,2)\cdot I(2)+ft_2(1,2)\cdot ft_1(2,1)\cdot I(1)+ft_2(1,2)\cdot ft_1(2,2)\cdot I(2) \\ ft_2(2,1)\cdot ft_1(1,1)\cdot I(1)+ft_2(2,1)\cdot ft_1(1,2)\cdot I(2)+ft_2(2,2)\cdot ft_1(2,1)\cdot I(1)+ft_2(2,2)\cdot ft_1(2,2)\cdot I(2) \end{vmatrix} \tag{13c}$$

Returning to the general case of an arbitrary number of incoming and outgoing directions, we consider Equation 9 with one interreflection between the two layers and we obtain (Figure 4):

$$\mathbf{ft = ft_2 \cdot ft_1 + ft_2 \cdot br_1 \cdot fr_2 \cdot ft_1 = ft_2 \cdot (1+br_1 \cdot fr_2) \cdot ft_1} \tag{14}$$

Including second- and higher-order interreflections, we obtain the infinite series:

$$\mathbf{ft = ft_2 \cdot (1 + (br_1 \cdot fr_2) + (br_1 \cdot fr_2)^2 + (br_1 \cdot fr_2)^3 + \ldots) \cdot ft_1} \tag{15}$$

which is equivalent to:

$$\mathbf{ft = ft_2 \cdot (1 - br_1 \cdot fr_2)^{-1} \cdot ft_1} \tag{16}$$

where $\mathbf{(1-br_1 \cdot fr_2)^{-1}}$ is the matrix inverse of $\mathbf{(1-br_1 \cdot fr_2)}$.

The matrix expressions for the other bidirectional properties of a two-layer fenestration system are listed in Table 1. Expressions for three or more layers can be obtained by sequential application of the two-layer relationships. Spectrally, selective layers can be handled by carrying through the matrix analysis for each of several wavelength bands.

The matrix approach described will be validated, and limitations such as edge effects will be evaluated using scanning radiometer (Spitzglas 1986) data on single layers and their combinations.

CONCLUSIONS

For accurate determination of the luminous and thermal performance of a total fenestration system, it is necessary to know its bidirectional solar-optical properties. These properties can be directly calculated only for optically simple layers (primarily glazing). Bidirectional transmittance and reflectance can be determined experimentally for fenestration system of arbitrary complexity using a scanning radiometer, after which directional absorptance can be calculated. However, the scanning radiometer provides no information about the net absorptance of the layers of the fenestration system.

Representing bidirectional properties of fenestration layers using matrices is simple and convenient. Matrix operations can then be used to determine bidirectional solar-optical properties of multilayer systems, based on the properties of their layers. For determining the properties of multilayer systems, this approach has the advantages over experimental procedures of minimizing time and cost and, most important, providing additional information on the radiation absorbed by each layer of the system.

The computer program TRA, an application of the matrix representation approach, generates input for appropriate daylighting calculation models, e.g. SUPERLITE, and heat transfer calculation models, e.g., WINDOW 2.0 and DOE-2, to determine the hourly, seasonal, and annual luminous and thermal performance of fenestration systems of arbitrary complexity for varying environmental conditions. These capabilities for more accurate and consistent modeling of realistic fenestration systems should contribute to better comparisons and optimizations of fenestration systems for use in buildings.

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† For ordinary clear, tinted, or reflective glass the matrix ft_1 would be diagonal, i.e., $ft_1(1,2)$ =m $ft_1(2,1) = 0$. This would result in $T_1(1) = ft_1(1,1) \cdot I(1)$ and $T_1(2) = ft_1(2,2) \cdot I(2)$.

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TABLE 1

Bidirectional Properties of a Two-Layer Fenestration System

Expressed in Terms of Matrices Representing the Properties of the Individual Layers,

where $\mathbf{interf=(1\text{-}br_1 \cdot fr_2)^{-1}}$ and $\mathbf{interb=(1\text{-}fr_2 \cdot br_1)^{-1}}$.

Property of fenestration system	Mathematical expression
front transmittance of system	$\mathbf{ft = ft_2 \cdot interf \cdot ft_1}$
front reflectance of system	$\mathbf{fr = fr_1 + bt_1 \cdot fr_2 \cdot interf \cdot ft_1}$
total absorptance of layer 1 for front flux	$\mathbf{ta_1f = fa_1 + ba_1 \cdot fr_2 \cdot interf \cdot ft_1}$
total absorptance of layer 2 for front flux	$\mathbf{ta_2f = fa_2 \cdot interf \cdot ft_1}$
back transmittance of system	$\mathbf{bt = bt_1 \cdot interb \cdot bt_2}$
back reflectance of system	$\mathbf{br = br_2 + ft_2 \cdot br_1 \cdot interb \cdot bt_2}$
total absorptance of layer 1 for back flux	$\mathbf{ta_1b = ba_1 \cdot interb \cdot bt_2}$
total absorptance of layer 2 for back flux	$\mathbf{ta_2b = ba_2 + fa_2 \cdot br_2 \cdot interb \cdot bt_2}$

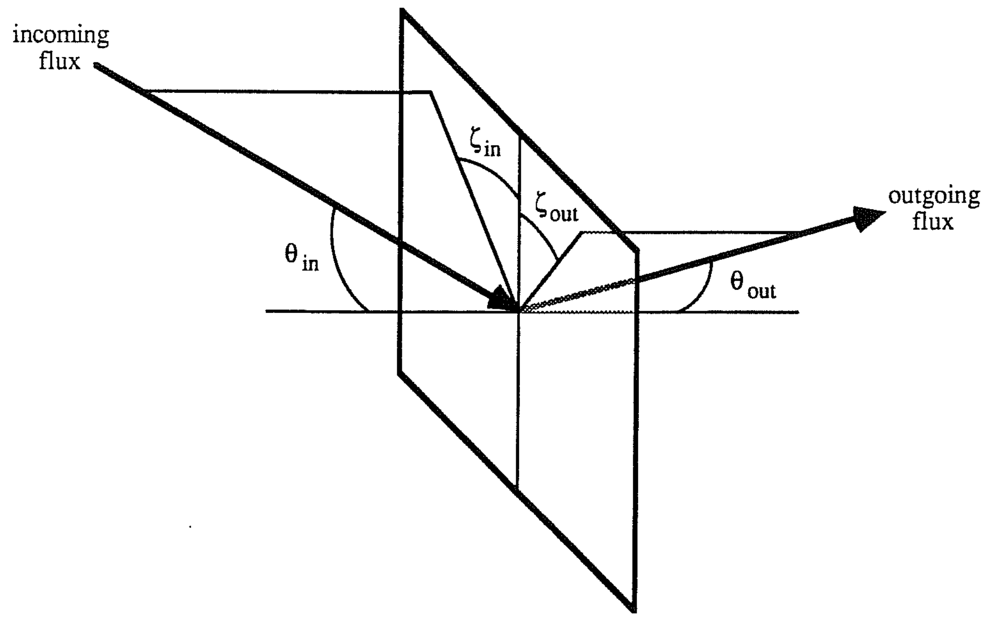

Figure 1. Schematic of an optical element showing the pairs of angles (ζ_{in}, θ_{in}) and (ζ_{out}, θ_{out}) that specify the directions of incoming and outgoing radiant flux.

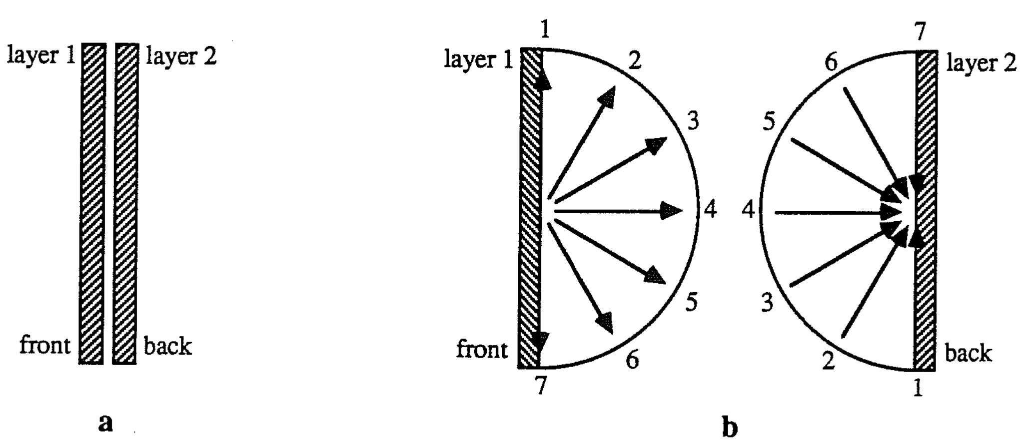

Figure 2. Schematic section of a two-layer fenestration system (a). The outgoing directional fluxes from one layer are considered to be incoming directional fluxes to the other layer (b).

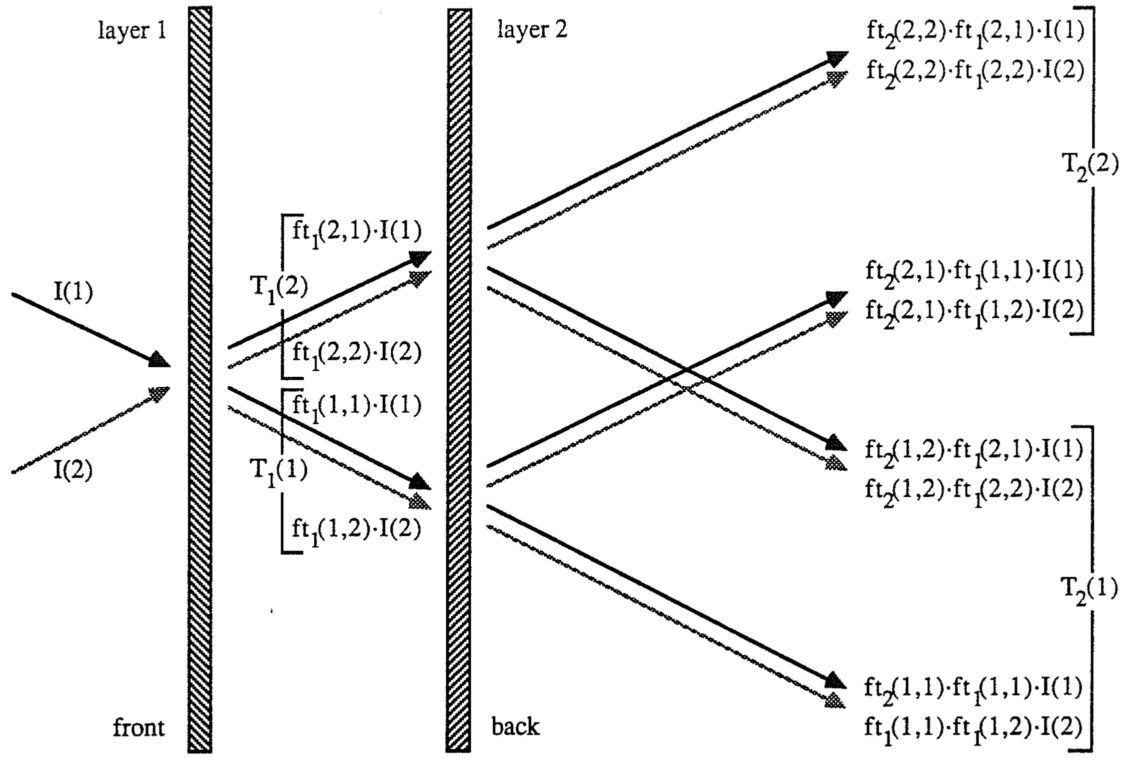

Figure 3. Schematic section of a two-layer fenestration system. A simplified case is illustrated, showing how the components of the incident flux propagate through the system. Two incoming and two outgoing directions are considered for each layer.

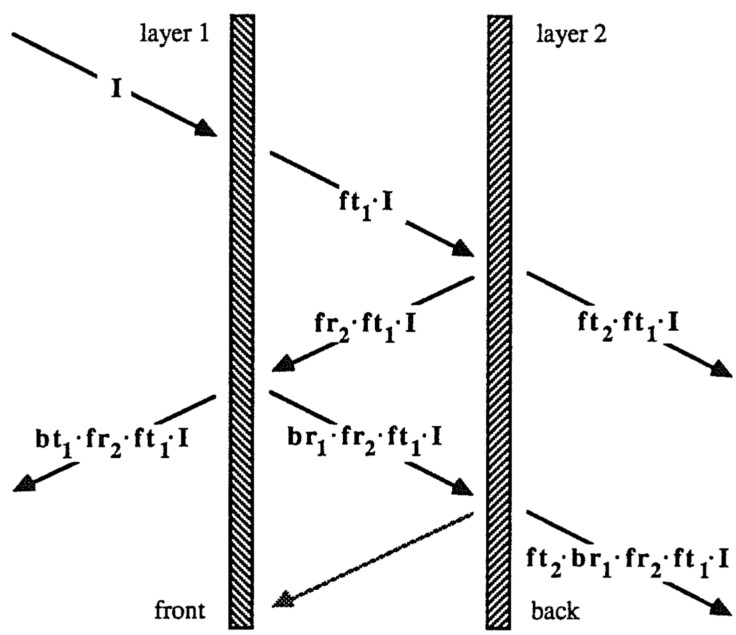

Figure 4. Schematic section of a two-layer fenestration system, showing the interreflections between the two layers, expressed as matrix multiplications. The matrix **I** represents the distribution of the incoming radiant flux.